A Statistical Analysis of Carbon Dioxide Emission From Different Attributes in West Bengal, India

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***Abstract*— Global warming is one of the most important environmental tribulations ever to confront human society and most important responsible green house gas for global warming is carbon dioxide. Increased emission of carbon dioxide from different sources to the environment is much alarming amongst which solid fuels and liquid fuels are the most important one but gaseous fuels and cement industry also add fuel to the flame. In this frightening situation, a mathematical model has been developed for the emissions of carbon dioxide based on above four attributes for the state West Bengal, India. A statistical approach, namely least square method is applied to study the behavior of the said attributes in the developed model utilizing the data set of twenty one years in West Bengal. The solutions of the environmental model match well with the real historical data from where future prediction for the emission of carbon dioxide can be made.**

***Keywords*—Carbon dioxide emission; global warming; least square method; fuel.**

I. INTRODUCTION

One of the major serious issues nowadays is global warming which has emerged most important environmental tribulations ever to confront humanity. This burning problem is inextricably linked to the process of development and economic growth. The main green house gases which cause global warming are carbon dioxide (CO_2), methane (CH_4), water vapor (H_2O), nitrous oxide (N_2O) and ozone (O_3). Global warming has serious implications for all aspects of human life, including infectious diseases [1]. Among the green house gases, CO_2 is the most important that is being affected by human activities. The concentration of CO_2 in the earth's atmosphere was about 280±10 parts per million by volume (ppmv) in 1750 [2]. By 1999 it was 367 ppmv and rising by about 1.5 ppmv per year. If emission continues in this rate, the concentration will reach 500 ppmv by the end of twenty first century which is very much alarming for the existence of living system [3].

Emission of CO_2 from different sources in India is also frightening. Nandi and Basak [4] in earlier works studied the emission of carbon dioxide from different attributes in Indian perspective through search method. India is the fourth largest emitter of CO_2 in the world (Source: Oak Ridge National Laboratory (ORNL), USA). Different states in India are responsible for this undesirable emission of CO_2. A state wise analytical study for total carbon dioxide emission has also been made by Basak and Nandi [5] using search method. West Bengal is one of the major contributors among identified states. Undesirable emission of CO_2 from different sources varies from state to state. Main sources for this emission are solid fuels, liquid fuels and gaseous fuels and cement industry. Cement manufacturing industry releases CO_2 as it uses essentially 100% calcium oxide which is obtained by burning calcium carbonate during calcinations.

Several studies have been done by different researchers on the emissions of green house gases in India. Some scientists deal with emissions in the regional level in the country also. Earlier Parikh and Gokarn [6] attempted for the estimation of emission levels in various sectors of the economy for the year 1983–84. The trends of CO_2, SO_2 and NOx between the periods 1973–74, 1983–/84, 1991–92 and 1996–97 have been analyzed [7]. On the other hand, a time series estimate of indirect carbon emissions per unit of power consumption was provided by [8]. Ghoshal and Bhattacharyya [9] made a detailed survey regarding state level CO_2 emissions of India during the year 1980-2000. Estimated emission of CO_2 by mathematical modeling has been attempted by many researchers. Basak and Nandi [10] formulated mathematical models of total CO_2 emission in some eastern and northern states of India and also predicted future emission in that states. Rust [11] demonstrated the connections between fossil fuel emissions, atmospheric CO_2 concentrations and global temperatures by coupled mathematical models for their measured time series. Tokos et al [12] developed differential equations for the emission of CO_2 based on six attributable/ variables. Jin et al [13] made a dynamic evolutionary model of carbon emissions in Yangtze Delta, China and they showed that due to excessive dependency of fossil fuels, carbon emission has risen dramatically after year 2000. In 1990, Goreau [14] briefly mentioned that the rate of change of CO_2 emissions in the atmosphere could be studied using differential equations.

In the present study, we developed a mathematical model for the emissions of carbon dioxide in the state West Bengal, India based on four attributes. Here, data set of twenty one years has been utilized to study the behavior of the said attributes in the developed model by a statistical approach. From the analytical solution, the CO_2 emissions by various sources is to be estimated for short and long range of time so that remedial measures can be taken to reduce the emissions as far as practicable without compromising economic growth.

II. METHODOLOGY

To generate mathematical models of CO_2 emissions in West Bengal, we consider the different sources of CO_2 emissions like solid fuels (S), liquid fuels (L), gaseous fuels (G) and cement (C) industry. In our model, for each of the attributes, the third degree polynomial model is formulated through least square method. The third degree polynomial fitted to CO_2 emission [13] may be written as

$$y = a + b.x + c.x^2 + d.x^3 \quad (1)$$

where y represents total CO_2 emission and x represents year. Given data $(x_1,y_1), (x_2,y_2),\ldots,(x_n,y_n)$, we may define an error associated with can be presented as

$$E = \sum_{i=1}^{n} (y_i - a - bx_i - cx_i^2 - dx_i^3)^2$$

Equating to zero, the partial derivatives with respect to a, b, c, d can be written as

$$\frac{\partial E}{\partial a} = 0 = -2\sum (y_i - a - bx_i - cx_i^2 - dx_i^3)$$

$$\frac{\partial E}{\partial b} = 0 = -2\sum x_i(y_i - a - bx_i - cx_i^2 - dx_i^3)$$

$$\frac{\partial E}{\partial c} = 0 = -2\sum x_i^2(y_i - a - bx_i - cx_i^2 - dx_i^3)$$

$$\frac{\partial E}{\partial d} = 0 = -2\sum x_i^3(y_i - a - bx_i - cx_i^2 - dx_i^3)$$

The corresponding normal equations are

$$\sum y_i = na + b\Sigma x_i + c\Sigma x_i^2 + d\Sigma x_i^3$$

$$\sum x_i y_i = a\Sigma x_i + b\Sigma x_i^2 + c\Sigma x_i^3 + d\Sigma x_i^4$$

$$\sum x_i^2 y_i = a\Sigma x_i^2 + b\Sigma x_i^3 + c\Sigma x_i^4 + d\Sigma x_i^5$$

$$\sum x_i^3 y_i = a\Sigma x_i^3 + b\Sigma x_i^4 + c\Sigma x_i^5 + d\Sigma x_i^6 \quad (2)$$

For given set of points (x_i, y_i); $(i=1,2,\ldots,n)$, the equation (2) can be solved for a, b, c, d. Equation (1) is the third degree polynomial best fit. It has been found that in all the cases, the values of the 2^{nd} order derivatives come out to be '+'ve at the points a, b, c. d. These provide minimization of E. Thus, the third degree fitted polynomial of carbon dioxide emission is estimated as

$$y = a + b.x + c.x^2 + d.x^3.$$

For evaluating model performance, we use two statistical criteria, R^2 (adjusted R^2) and residual analysis. The coefficient of determination R^2 is defined as the proportion of the total response variation that is explained by the model. It provides an overall measure of how well the model fits. In general, R^2 may be represented by the following term

$$R^2 \equiv 1 - \frac{SS_{\rm err}}{SS_{\rm tot}}.$$

where

$$SS_{\rm tot} = \sum_i (y_i - \bar{y})^2,\ SS_{\rm reg} = \sum_i (f_i - \bar{y})^2,$$ and

$$SS_{\rm err} = \sum_i (y_i - f_i)^2$$

SS_{tot} = Total sum of squares (proportional to the sample variance)
SS_{reg} = The regression sum of squares..
SS_{err} = The sum of squares of residuals.
and $SS_{tot} = SS_{reg} + SS_{err}$
$\bar{y}$ = The mean of the observed data and may be represented by

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

where i is the number of observations.
The adjusted R^2 is defined as

$$\bar{R}^2 = 1 - (1 - R^2)\frac{n-1}{n-p-1} = R^2 - (1 - R^2)\frac{p}{n-p-1}$$

where *p* is the total number of repressors in the linear model (not counting the constant term), and *n* is the sample size. R^2 adjusted will adjust for degrees of freedom of the model.

For the formulation and analysis of our model, the following CO_2 emission data (Table 1) is used. The data reflects the contribution of different fuels for CO_2 emission in West Bengal. For our analysis, we consider twenty one year's data from 1980 to 2000.

Table 1: Real data of CO_2 emission from different attributes in West Bengal ('000 MT of carbon)

Year	Solid (S)	Liquid (L)	Gas (G)	Cement (C)
1980	8365.76	2720.74	82.79	292.08
1981	8991.07	2961.60	96.26	346.29
1982	9134.05	2997.07	140.28	359.64
1983	8880.37	2797.58	150.22	367.67
1984	8242.40	2744.25	162.11	379.05
1985	9043.93	3025.77	188.87	432.98
1986	8997.42	2933.30	251.02	440.88
1987	9479.26	2962.35	276.80	441.82
1988	10170.10	3062.24	303.67	481.08
1989	10067.69	3144.72	341.01	502.24
1990	10666.79	3312.79	427.94	546.31
1991	12656.39	3880.91	530.11	625.02
1992	11974.91	3974.44	498.73	553.77
1993	13001.71	4104.64	516.30	612.19
1994	14343.95	4544.94	776.56	674.38
1995	14871.95	4925.16	833.72	728.70
1996	14175.31	4923.06	951.34	782.73
1997	13291.36	4440.06	774.36	747.37
1998	11926.62	4266.10	798.89	706.57
1999	16072.77	5381.04	985.19	952.35
2000	15784.52	5588.59	969.59	953.23

III. RESULTS AND DISCUSSION

Now we formulate and analyze model for four attributes one by one. The general mathematical model for the solid fuel in West Bengal is given by $y(S) = a + b.x + c.x^2 + d.x^3$ where x represents year in the equation. The particular solution of the equation is given by

$y(S) = -146879.156 + 14.7859 x - 0.0873 x^2 + 6.025 \times 10^{-5} x^3$

The graphical representation of the real data and the solution of the equation for the emission of carbon dioxide from solid fuel is given by figure 1.

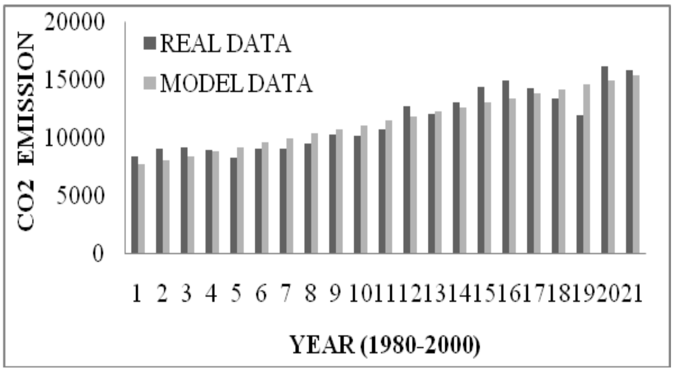

Figure 1: Comparative emission of CO_2 due to solid fuels (Real data vs model data)

Here, the rate of change of CO_2 emission is compared with the model data and it is evident from Figure1 that our CO_2 emission model matches well with the actual status of CO_2 emission from solid fuels. Therefore, from the figure, it is possible to predict the CO_2 emission from the solid fuel for short and medium terms of time. The calculated values for the solid fuel model of R^2 and R^2 adjusted are given by Table 2 below.

Table 2: R^2 and R^2 adjusted values for emission from solid fuel

R^2	R^2 adjusted
0.8443	0.8168

It is observed that the values of R^2 and R^2 adjusted are sufficiently high. Thus it may be concluded that the observed data matches reasonably well with the model. Furthermore, the residual analysis on the proposed differential equation of solid fuels is given in Table 3 below.

Table 3: Residual analysis for emission of CO_2 from solid fuels in West Bengal ('000 MT of carbon)

Year	Real data	Model data	Residual
1980	8365.76	7622.87	742.89
1981	8991.07	8000.66	990.40
1982	9134.05	8378.96	755.08
1983	8880.37	8757.86	122.50
1984	8242.40	9137.26	-894.86
1985	9043.93	9517.23	-473.30
1986	8997.42	9897.74	-900.31
1987	9479.26	10278.77	-799.50
1988	10170.10	10660.34	-490.24
1989	10067.69	11042.48	-974.79
1990	10666.79	11425.16	-758.36
1991	12656.39	11808.36	848.02
1992	11974.91	12192.14	-217.22
1993	13001.71	12576.45	425.26
1994	14343.95	12961.29	1382.66
1995	14871.95	13346.70	1525.25
1996	14175.31	13732.64	442.66
1997	13291.36	14119.15	-827.78
1998	11926.62	14506.20	-2579.57
1999	16072.77	14893.77	1178.99
2000	15784.52	15281.92	502.59
Mean of residuals			0.018
Standard error of residuals (SE)			31.90

It is evident from the above table that the residuals are small compared to the data and so is the standard error. These results support the good quality of the proposed model for solid fuels. Here our predicted value of emission of CO_2 from solid fuels for West Bengal in 2015 and 2020 are 20810.45 and 22978.12 ('000 MT of carbon) respectively.

Now, the general mathematical model for the liquid fuel of West Bengal is given by
$y(L) = a + b.x + c.x^2 + d.x^3$ where x represents year in the equation. The particular solution of the above equation is given by
$y(L) = -47781.3359 + 7.1765 x - 0.0383 x^2 + 2.398 \times 10^{-5} x^3$

The graphical representation of the real data and the solution of the model equation for the emission of carbon dioxide from liquid fuels are given by figure 2.

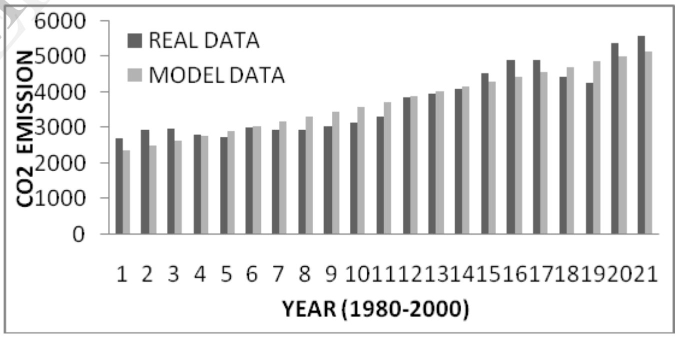

Figure 2: Comparative emission of CO_2 due to liquid fuels (Real data vs model data)

From the figure, it is seen that the rate of change of emission of model data and real data are quite closer. So by comparing the model data and actual data from the above figure, one can estimate the CO_2 emission from the liquid fuel for short and medium terms of time.

The calculated values for the liquid fuel model of R^2 and R^2 adjusted are given by Table 4 below.

Table 4: R^2 and R^2 adjusted values for emission from liquid fuel

R^2	R^2 adjusted
0.8590	0.8342

From the value of R^2 and adjusted R^2, it can be concluded that we have developed a good model for the emission of carbon dioxide. Furthermore, the residual analysis on the proposed differential equation of liquid fuels is given in Table 5 below.

Table 5: Residual analysis for emission of CO_2 from liquid fuels in West Bengal ('000 MT of carbon)

Year	Real data	Model data	Residual
1980	2720.74	2368.11	362.62
1981	2961.60	2495.70	465.90
1982	2997.07	2633.48	363.58
1983	2797.58	2771.49	26.08
1984	2744.25	2909.70	-165.34
1985	3025.77	3048.13	-22.35
1986	2933.30	3186.76	-253.45
1987	2962.35	3325.59	-363.24
1988	3062.24	3464.64	-402.39
1989	3144.72	3603.90	-459.17
1990	3312.79	3743.36	-430.57
1991	3880.91	3883.03	-2.12
1992	3974.44	4022.92	-48.48
1993	4104.64	4163.02	-58.38
1994	4544.94	4303.32	241.61
1995	4925.16	4443.84	481.32
1996	4923.06	4584.56	338.49
1997	4440.06	4725.51	-285.45
1998	4266.10	4866.66	-600.55
1999	5381.04	5008.01	373.03
2000	5588.59	5149.58	439.00
Mean of residuals			0.0063
Standard error of residuals (SE)			18.70

It is clear from the table that the residuals are small compared to the data and standard error is negligibly small. These results prove good quality of the proposed model for liquid fuels. Here our predicted value of emission of CO_2 from liquid fuels for West Bengal in 2015 and 2020 are 7354.11 and 8099.57 ('000 MT of carbon) respectively.

The general mathematical model for the gaseous fuel of West Bengal is given by

$y\,(G) = a + b.x + c.x^2 + d.x^3$ where x represents year in the equation. The particular solution of the above equation is given by

$y(G) = -19394.8613 + 2.1649\ x - 0.0118\ x^2 + 7.9237 \times 10^{-6}\ x^3$.

The graphical representation of the actual data and the solution of the differential equation for the emission of carbon dioxide from gaseous fuels are given by figure 3.

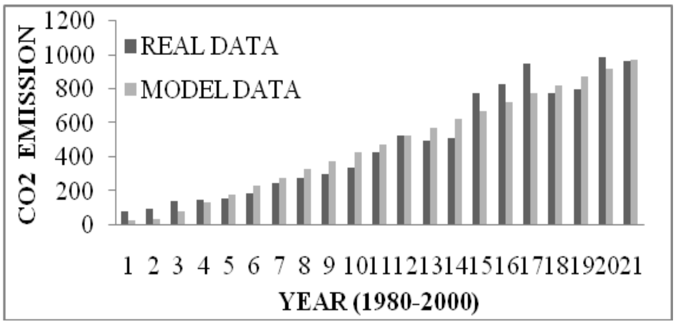

Figure 3: Comparative emission of CO_2 due to gaseous fuels (Real data vs model data)

Here, again the rate of change of CO_2 emission can be compared with the model data and it is seen that the actual curve and empirical curve matches well. So from the figure that one can estimate the CO_2 emission from the gaseous fuel for short and medium terms of time. For maintaining the quality of the proposed analytical model, here, we use two statistical criteria, R^2, adjusted R^2 and residual analysis.

The calculated values for the gaseous fuel model of R^2 and R^2 adjusted are given by Table 6 below.

Table 6: R^2 and R^2 adjusted values for emission from gaseous fuel

R^2	R^2 adjusted
0.9087	0.8926

It is shown from the above values that an excellent correlation of R^2 and adjusted R^2 is obtained. Furthermore, the residual analysis we performed on the proposed differential equation of gaseous fuels is given in Table 7 below.

Table 7: Residual analysis for emission of CO_2 from gaseous fuels in West Bengal ('000 MT of carbon)

Year	Real data	Model data	Residual
1980	82.79	30.25	52.54
1981	96.26	37.74	58.51
1982	140.28	86.32	53.96
1983	150.22	134.98	15.24
1984	162.11	183.70	-21.58
1985	188.87	232.50	-43.63
1986	251.02	281.37	-30.34
1987	276.80	330.30	-53.50
1988	303.67	379.31	-75.64
1989	341.01	428.39	-87.38
1990	427.94	477.54	-49.60
1991	530.11	526.76	3.34
1992	498.73	576.06	-77.32
1993	516.30	625.42	-109.11
1994	776.56	674.85	101.70
1995	833.72	724.36	109.35
1996	951.34	773.93	277.40
1997	774.36	823.58	-149.22
1998	798.89	873.30	-74.41
1999	985.19	923.09	62.09
2000	969.59	972.96	-3.36
Mean of residuals			0.0027
Standard error of residuals (SE)			9.82

From the above table, it can be said that small residuals suggest good quality of the proposed model for gaseous fuels. Here our predicted value of emission of CO_2 from gaseous fuels for West Bengal in 2015 and 2020 are 1707.06 and 1947.88 ('000 MT of carbon) respectively.

The general mathematical model for the cement industry in West Bengal is given by

$y\,(C) = a + b.x + c.x^2 + d.x^3$ where x represents year in the equation. The solution of the above equation is given by

$y(C) = -12629.6377 + 2.3358x - 0.0075x^2 + 4.862 \times 10^{-6} x^3$ The graphical representation of the real data and the solution of the differential equation for the emission of carbon dioxide from cement industry are given by figure 4.

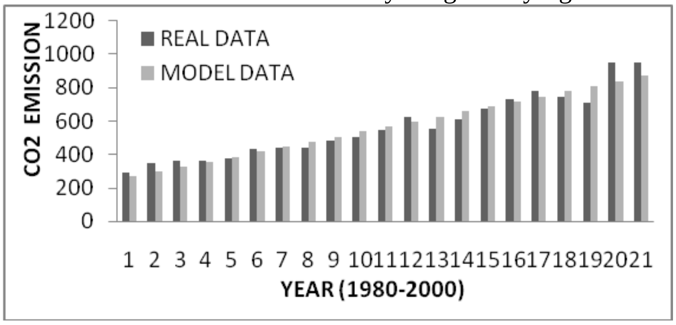

Figure 4: Comparative emission of CO_2 from cement industry (Real data vs model data)

Here, again the rate of change of CO_2 emission can be compared with the model data and it can be explained from the figure that one can estimate the CO_2 emission from cement manufacturing industry for short and medium terms of time. For maintaining the quality of the proposed analytical model, here, we use two statistical criteria, R^2 (adjusted R^2 and residual analysis. The calculated values for the cement industry R^2 and R^2 adjusted are given by Table 8 below.

Table 8: R^2 and R^2 adjusted values for emission from cement industry

R^2	R^2 adjusted
0.9348	0.9233

It is shown from the above values that we have identified good models. Furthermore, the residual analysis we performed on the proposed differential equation of cement industry is given in Table 9 below.

Table 9: Residual analysis for emission of CO_2 from cement industry in West Bengal ('000 MT of carbon)

Year	Real data	Model data	Residual
1980	292.08	267.08	24.99
1981	346.29	296.85	49.43
1982	359.64	326.67	32.97
1983	367.67	356.53	3.01
1984	379.05	386.43	-7.37
1985	432.98	416.38	16.6
1986	440.88	446.36	-5.48
1987	441.82	476.39	-34.57
1988	481.08	506.47	-25.42
1989	502.24	536.58	-34.34
1990	546.31	566.74	-20.43
1991	625.02	596.94	28.08
1992	553.77	627.19	-73.42
1993	612.19	657.48	-45.29
1994	674.38	687.81	-13.42
1995	728.70	718.18	10.51
1996	782.73	748.60	34.13
1997	747.37	779.06	-31.69
1998	706.57	809.57	-102.99
1999	952.35	840.12	112.23
2000	953.23	870.71	82.52
Mean of residuals			0.0028
Standard error of residuals (SE)			7.02

As seen from the table, the residuals are small compared to the data and so is the standard error. These results attest to the good quality of the proposed model for cement industry. Here our predicted value of emission of CO_2 from cement industry for West Bengal in 2015 and 2020 are 1311.08 and 1469.33 ('000 MT of carbon) respectively.

IV. CONCLUSION

We have developed mathematical models based on the emission of carbon dioxide for each of the four main attributable variables in West Bengal using actual data from 1980 to 2000. The main sources of emission are gaseous fuels, liquid fuels, solid fuels and cement. Here we adopted least square method for the formulation of our model. We have used two different statistical procedures, namely R^2 (R^2 adjusted) and residual analysis to evaluate the quality of the proposed models. Analyzing the model by using regression analysis method, it illustrates that the model matches well with the actual status of carbon emission from four main attributable variables. All these statistical procedures advocate to the quality of the proposed models. Finally, we predict the short and medium term total carbon dioxide emissions trend in West Bengal by using our model. Proper framing of emission strategies and policies are immediately necessary to restrain the rapid increasing of emission. Our models provide a theoretical basis for the further study in future on the undesirable situation of carbon emissions and it may be useful for intended planning and formulating policies to reduce emission of global warming gases.

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