A Simulated Annealing Algorithm For The Optimization Of Surface Finish In Dry Turning Of SS 420 Materials

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Abstract

Determination of optimal cutting parameters is one of the most important elements in any process planning of metal parts. This paper presents a development of a simulated annealing algorithm (SA) and its application to optimize the cutting parameters for predicting the surface roughness is proposed. Optimization of cutting parameters and prediction of surface roughness is concerned with a highly constrained nonlinear dynamic optimization problem that can only be fully solved by complete enumeration. Optimisation in turning means the termination of the optimal set of machining parameters to satisfy the objectives within the operational constraints. Simulated annealing combines a downhill search with a random search. In simulated annealing (SA) method, an exponential cooling schedule based on Newtonian cooling process is employed and experimentation is done on choosing the number of iterations (m) at each step. The SA approach is applied to predict the influence of tool geometry (nose radius) and cutting parameters (feed, speed and depth of cut) on surface roughness in dry turning of SS 420 materials conditions based on Taguchi’s orthogonal array method.


1. Introduction

Optimization of cutting parameters is usually a difficult work [1], where the following aspects are required: knowledge of machining; empirical equations relating the tool life, forces, power, surface finish, etc., to develop realistic constrains; specification of machine tool capabilities; development of an effective optimization criterion; and knowledge of mathematical and numerical optimization techniques [2]. The selection of optimal cutting parameters, like depth of cut, feed and speed, is a very important issue for every machining process. In workshop practice, cutting parameters are selected from machining databases or specialized handbooks, but the range given in these sources are actually starting values, and are not the optimal values [3]. Regarding machining operations, these considerations usually lead the designers to seek optimum machining parameters in order to minimize machining costs. In turning, cutting speed, feed rate and cutting depth are the most important of these parameters. These are determined based on the constraints imposed by the specifications of the machine, cutting tool and the
engaged work piece. The number of modeled constraints has a direct clear relevance to the complexity of problem. In several case studies, the number of constraints is limited in order to facilitate finding the optimum [4-9]. In some others, additional considerations such as the chip-tool interface temperature are taken into account [10]. The above mentioned parameters are of discrete nature or imposed to discretization due to practical limitations. Therefore, the discrete or continuous-discrete search space of the problem as well as the complex objective function necessitates the employment of methods other than classic approaches.

In any optimization procedure, it is a crucial aspect to identify the output of chief importance, the so-called optimization objective or optimization criterion. There is no generalized solution method that can be used for all machining optimisation problems (Chang et.al.1991). The earlier optimisation techniques may only be useful for a specific problem and inclined to obtain local optimal solution. In this paper, simulated annealing is employed as it normally exhibits fast convergence and straightforward implementation, to obtain the optimal parameters in turning processes. The SA algorithm has been demonstrated that it has the capability of escaping from the local optima. Simulated annealing (SA) is one of the stochastic search algorithms, which is designed using a spin glass model by the Kirkpatrick [12]. It has been used in wide areas from the combinatorial problems to the real world problems because it performs well on most of optimization problems, especially on complex problems [13,14,15].The power of SA originates from the good selection scheme and annealing technique. Generally SA used two kinds of selection scheme. One is the Metropolis algorithm and the other is the logistic selection algorithm [16]. Originally any kind of selection that satisfies the detailed balance equation can be used as a selection scheme because the detailed balance equation guarantees the convergence of SA [17]. Another reason why SA performs well is annealing, that is, the gradual temperature reducing technique. On the contrary, studies on evolutionary algorithms have shown that these methods can be efficiently used to eliminate most of the above-mentioned difficulties of classical methods [18].

Surface roughness is defined as the irregularities on any material surface resulting from machining operations. Average roughness $Ra$ is theoretically derived as the arithmetic average value of departure of the profile from the mean line along a sampling length [19]. Surface roughness is an essential requirement in determining the surface quality of a product. In this paper the optimum process parameter on surface roughness on the machining of SS420 material was investigated.
2. Experimental details

When developing models on the basis of experimental data, careful planning of experimentation is essential. The factors considered for experimentation and analysis were cutting speed, feed rate, depth of cut and nose radius.

2.1 Parameter design based on the Taguchi method

Modeling provides reliable equations obtained from the data of properly designed experiments. Therefore, it is essential to have a well-designed set of experiments. A well-designed experiment can substantially reduce the number of experiments required. Several types of experimental results have been reported. In this research, the design suggested by Taguchi is used.

2.1.1 Orthogonal array experiment

Classical experimental design methods are too complex and are not easy to use. A large number of experiments have to be carried out when the number of process parameters increases. To solve this problem, the Taguchi method uses a special design of orthogonal arrays to study the entire parameter space with only a small number of experiments. According to the Taguchi method, a robust design and an $L_{27}$ orthogonal array are employed for the experimentation. Four machining parameters are considered as controlling factors – namely, cutting speed, feed rate, depth of cut and nose radius and each parameter has three levels – namely low, medium and high, denoted by 1, 2 and 3, respectively. Table 1 shows the cutting parameters and their levels considered for the experimentation. The experimental design considered for the investigation to achieve an optimal surface finish during the turning of SS 420 steel is based on the $L_{27}$ orthogonal array. Based on this, a total number of 27 experiments in dry machining is done, each having a different combination of levels of factors were carried out.

#####Table 1 – Three level tables with four factors

2.2 Experimental details

The experiment is performed on SS 420 of size 25 mm diameters which contains 12% of chromium sufficient enough to give corrosion resistance property and good
ductility. Its chemical composition is given as 0.15% C, 12.0-14.0% Cr, < 1.0% Si, <0.04% P, <1.0% Mn, <0.03% S and remaining as Fe. The cutting tool for turning with rhombic tooling system is uncoated tungsten carbide having zero rake angle, 7° clearance angle and 55° cutting edge angle and of nose radii 0.4, 0.8 and 1.2 have been used for experiment. The different sets of experiments are performed using a Kirloskar centre lathe. The machined surface is measured at three different positions and the average values are taken using a RUGOSURF 10G surface texture measuring instrument.

3. Cutting process model

3.1. Decision variables

In the constructed optimization problem, four decision variables are considered: cutting speed (V), feed (F), cutting depth (D) and cutting tool nose radius (R). These are the important cutting parameters of the process.

3.2. Mathematical model

In this work, the experimental results were used for modeling using response surface methodology. The purpose of developing mathematical models was to relate the machining responses to the parameters and thereby to facilitate the optimization of the machining process. With these mathematical models, the objective function and process constraints can be formulated, and the optimization problem can then be solved by using the Simulated algorithm (SA).

3.2.1 Mathematical formulation

Response Surface Methodology (RSM) is a combination of mathematical and statistical techniques for empirical model building and optimization. By conducting experiments and applying regression analysis, a model of the response to certain independent input variables can be obtained. The mathematical models commonly used are represented by:

\[ Y = \phi(V, F, D, R) + \varepsilon \]

where \( Y \) is the machining response (surface finish), \( \phi \) is the response function
and V, F, D, R are turning variables and ε is the error that is normally distributed about the observed response Y with a zero mean.

The general second-order polynomial response is as given below:

\[ Y_u = \beta_0 + \sum_{i=1}^{k} \beta_i x_{iu} + \sum_{i=1}^{k} \beta_i x_{iu}^2 + \sum_{i} \sum_{j} \beta_{ij} x_{iu} x_{ju} \]  

where \( Y_u \) represents the corresponding response, the surface roughness \( R_a \) in the present research. The code values of \( i^{th} \) machining parameters for \( u^{th} \) experiment are represented by \( x_{iu} \). The values of \( k \) indicate the number of machining parameters. The terms \( \beta_i, \beta_{ii} \) and \( \beta_{ij} \) are the second order regression co-efficient. The second term under the summation sign of this polynomial equation attributes to linear effects, whereas the third term of the above equation corresponds to the higher order effects and lastly the fourth term of the equation includes the interactive effects of the parameters.

It also confirms that this model provides an excellent explanation of the relationship between the independent factors and the response arithmetic average roughness (\( R_a \)). The second order response surface representing the surface roughness, \( R_a \) can be expressed as a function of cutting parameters such as feed (F), cutting speed (V), depth of cut (D) and nose radius (R).

\[ R_a = \beta_0 + \beta_1(F) + \beta_2(D) + \beta_3(V) + \beta_4(R) + \beta_5(FD) + \beta_6(FV) + \beta_7(FR) + \beta_8(DV) + \beta_9(DR) + \beta_{10}(VR) + \beta_{11}(F^2) + \beta_{12}(D^2) + \beta_{13}(V^2) + \beta_{14}(R^2) \]  

To obtain practical predictive quantitative relationships, it is necessary to model the turning responses and the process variables. In the present work, the mathematical models were developed on the basis of dry machining experimental results as shown in Table 2. The experimental results were used to model the response using response surface methodology.

### Table 2 – Experimental results of arithmetic average roughness, \( R_a \) ###

Optimization of machining parameters enhances not only the economics of machining, but also the product quality to a great extent. An effort has been made to
estimate the optimum tool geometry and machining conditions to produce the best possible surface quality within the chosen constraints in dry machining.

In order to optimize the present problem using simulated annealing algorithm (SA), the constrained optimization problem is stated as follows:

From the observed data for surface roughness, the response function has been determined using RSM and fitness function, defined as Minimize,

\[ R_u = -4.89 + 2.49F - 38.0D + 0.599V + 3.27R - 5.38F \cdot D + 0.0140F \cdot V - 18.2F \cdot R + 0.0097D \cdot V + 15.8D \cdot R - 0.232V \cdot R + 80.5F^2 + 16.5D^2 - 0.00318V^2 \]

subject to

39.269 m/min \( \leq V \leq \) 94.247 m/min

0.059 mm/rev \( \leq F \leq \) 0.26 mm/rev

0.4 mm \( \leq D \leq \) 1.2 mm

0.4 mm \( \leq R \leq \) 1.2 mm

\[ x_{il} \leq x_i \leq x_{iu} \]

where \( x_{il} \) and \( x_{iu} \) are the upper and lower bounds of process variables \( x_i \). \( x_1, x_2, x_3, x_4 \) are the cutting speed, feed, depth of cut and nose radius respectively. In order to optimize the present problem using SAs, the following parameters have been selected to obtain optimal solutions with less computational effort.

Maximum number of generations = 1000

Population Size =

Initial Temperature, \( T_0 = 1 \)

Final Temperature, \( T_f = 1 \times 10^{-20} \)

Cooling rate = 0.9

4. PROPOSED ALGORITHM

This section describes the proposed SA. First, a brief overview of the SA is provided then the solution procedures of the proposed SA are stated.

4.1. SIMULATED ANNEALING METHOD (SA)
Simulated annealing presents an optimization technique that can: (a) process cost functions possessing quite arbitrary degrees of nonlinearities, discontinuities, and stochasticity; (b) process quite arbitrary boundary conditions and constraints imposed on these cost functions; (c) be implemented quite easily with the degree of coding quite minimal relative to other nonlinear optimization algorithms; (d) statistically guarantee finding an optimal solution. Simulated annealing combines a downhill search with a random search. In order not to be trapped in a locally optimum region, this procedure sometimes accepts movements in directions other than steepest ascend or descend. The acceptance of an uphill rather that a downhill direction is controlled by a sequence of random variables with a controlled probability. Simulated annealing (SA) [20] is a powerful stochastic search method applicable to a wide range of problems for which little prior knowledge is available. It can produce high-quality solutions for hard combinatorial optimization [21].

The process of slow cooling is known as annealing in metallurgical process. The simulated annealing procedure simulates this process of slow cooling of molten metal to achieve the minimum function value of surface roughness in the problem of minimization. It is a point-by-point method. The algorithm begins with an initial point and a high temperature T. A second point is taken at random in the vicinity of the initial point and the difference in the function values (∆E) at these two points is calculated. Suppose that initially we have a point $x^i$ in the search space and that the cost at that point is $f(x^i)$. A new point $x^{i+1}$ is randomly generated that is "nearby" in some sense; we will call this a "trial point". The cost there is $f(x^{i+1})$. Next we decide whether to move to $x^{i+1}$, that is whether to replace $x^i$ by $x^{i+1}$ as the current approximation. If $f(x^{i+1}) < f(x)$ then the move is definitely accepted. If $f(x^{i+1}) \geq f(x)$ then the move is accepted with a probability of

$$P_{move\_accepted} = \exp\left(\frac{f(x^{i+1}) - f(x^i)}{T}\right)$$

This completes an iteration of this simulated annealing procedure. In the next generation another point is created at random in the neighborhood of the current point and the Metropolis algorithm is used to accept or reject it. In order to simulate the thermal equilibrium at every temperature the
number of points \( n \) is usually tested at a particular temperature before reducing the temperature. The algorithm is terminated when a sufficiently small temperature is obtained are a small enough change in function value is obtained.

The structure of the proposed simulated annealing algorithm (SA) is as follows.

**STEP-1**

Choose a start point \( x \) and set a high starting temperature \( T \), set the iteration count \( K=1 \)

**STEP-2**

Evaluate objective function \( E=f(x) \)

**STEP-3**

Find new point \( X_{i(k+1)} = X_{i(k)} + \lambda_i(X_{i max} - X_{i min}) \)

\[ \lambda_i \in (-1,1) \]

\[ \lambda_i = \text{sign}(\cup_{-0.5}^{0.5})T(1+\frac{1}{T_i})^{[2U_{i-1}]-1} \]

, \( U_i = \) random variable between 0 and 1

Select \( \Delta_i \) with probability determined by \( g(\Delta_i,T) \)

Set the new point \( X_{\text{new}} = X + \Delta_x \)

\[ g(\Delta_i,T) = (2\pi T)^{-n/2} \exp\left[ -\frac{[\Delta_i]^2}{2T} \right] \]

, \( n = \) dimension of space under exploration. The new point should be between the maximum and minimum limit

**STEP-4**

Calculate the new value of the objective function using fitness equation.

\[ E_{\text{new}} = f(X_{\text{new}}) \]
STEP-5

Set X to $X_{\text{new}}$ and E to $E_{\text{new}}$ with probability determined by acceptance function $h(\Delta E, T) = \frac{1}{1 + \exp(\Delta E/C T)}$, $T$= Current temperature, $C$=System dependent constant, $\Delta E = E_{\text{new}} - E$

STEP-6

Reduce the temperature according to annealing schedule.

$T = T_0 \times \alpha$, $\alpha$=cooling ratio usually between 0 and 1

STEP-7

Increment the iteration count $K$, if $K$ reaches the maximum stop iteration; otherwise go back to STEP-3.

Process 1 : Initialisation

Choose an initial point $x^{(0)}$ and a termination criterion $\alpha$, Set $T$ a sufficiently high value, number of iterations to be performed at a particular temperature $n$, and set $t = 0$

Process 2: Generation of neighborhood seed and evaluation

Process 3: Calculation of uphill and downhill move acceptance parameter $\Delta E$

The new seed is selected by calculating the value of $\Delta E$. The difference between the best seed in the neighborhood seed and the initial seed.

If $\Delta E = f(x^{t+1}) - f(x^t) < 0$, set $t=t+1$; Else create random number $(r)$ in the range $(0,1)$.

If $r \leq P_r$, set $t=t+1$, otherwise go to step 2.

If $|x^{t+1} - x^t| < \alpha$ and $T$ is small, terminate; Else

If $(t \mod n) = 0$ then lower $T$ according to a cooling schedule go to step 2.

The SA code was developed using MATLAB. The input machining parameter levels were fed to the SA program. The SA program uses number of iterations to predict the values of tool
geometry and cutting conditions for minimization of surface roughness. Table 3 shows the comparison of minimum values of surface roughness with respect to input machining parameters for SA. It is possible to determine the conditions at which the turning operation has to be carried out in order to get the optimum surface finish. The performance of SA is described in figure 1. The application of a simulated algorithm approach to obtain optimal machining conditions will be quite useful at the computer-aided process planning (CAPP) stage in the production of high-quality goods with tight tolerances by a variety of automated machining operations, and in adaptive control machine tools. With the known boundaries of surface roughness and machining conditions, machining can be performed with a relatively high rate of success with the selected machining conditions.

#Table 3- Output values of the simulated annealing algorithm with respect to input machining parameters ####

#### Fig.1 Cooling diagram f SA ####

6 Conclusions

The effect of various parameters such as cutting speed, feed rate, depth of cut and nose radius, has been studied in the turning of SS 420 steel material. By incorporating the tool geometry in the model, the validity of the model has been enhanced. The optimization, carried out in this work, gives an opportunity for the user to select the best tool geometry and cutting condition so as to get the optimum surface quality. To model the machining process, several important operational constraints have been considered. These constraints were taken to account in order to make the model more realistic. A major advantage of SA is its flexibility and robustness as a global search method. This algorithm is a powerful technique in optimization of problems with discrete search space and multiple local optima. It can deal with highly nonlinear problems and non-differentiable functions as well as functions with multiple local optima. The computational results clearly demonstrated that the proposed solution procedure is quite capable in solving such complicated problems effectively and efficiently.

References


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**Fig.1.** Cooling diagram of SAA

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Table 2 – Experimental results of arithmetic average roughness, Ra

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Table 3 – Output values of the genetic and simulated annealing algorithms with respect to input machining parameters

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