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A Simplified Adomian Decomposition Framework Based on the Sumudu Transform for Differential Equation Models

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Abstract - This paper presents the use of the **Sumudu Transform** combined with the **modified Adomian Decomposition Method (ADM)** to obtain approximate analytical solutions for different types of mathematical models. The method is applied to both **linear and nonlinear ordinary differential equations**, and **numerical results** are provided to demonstrate its accuracy by comparing the approximate solutions with exact ones whenever available.

Keywords - Sumudu Transform ; non-linear ; linear ; ADM; ODF.

I. INTRODUCTION

Recent research has shown a growing interest in enhancing the capabilities of homotopy methods through integration with integral transforms, particularly the Sumudu transform, which has proven effective in simplifying the mathematical structure of both fractional and nonlinear differential equations. The work of Bulut and Belgacem (2013) laid the foundation for applying the Sumudu transform to fractional differential equations, followed by Kumar and Daftardar-Gejji (2018), who developed the iterative Sumudu Transform Method (STIM) to extend its application to fractional partial differential equations. Subsequently, Prakash et al. (2018) and Rehman et al. (2018) introduced hybrid models combining Sumudu transforms with homotopy perturbation and double Sumudu transform methods to efficiently solve Burgers and KdV equations. Moreover, Mustafa (2023) enhanced this approach by improving the iterative schemes associated with the Sumudu transform, resulting in higher accuracy and stability for complex nonlinear systems.

In parallel, other integral transforms, such as the Zakai-type transform, have also been utilised, as highlighted in the study "Mathematical Modelling in Engineering via MADM", which demonstrates the advantages of combining these transforms with modified Adomian Decomposition Methods—particularly MADM (Reliable Modification of Adomian Decomposition Method) and TADM (Modified Adomian Decomposition Method by expressed in Taylor series) (see Al-Mazmumy et al)to improve solution accuracy and reduce the number of required iterations.

In the present work, we have modified the homotopy method by integrating it with the Sumudu transform to enhance the efficiency of analytical solutions for both linear and nonlinear equations. This modification reformulates the homotopy steps within the Sumudu transform space, allowing for a gradual reduction of nonlinearity and a smoother extraction of solution components compared to traditional approaches. Application of the modified method to a set of differential equations—both homogeneous and non-homogeneous—demonstrated a significant reduction in the number of iterations and an increase in solution accuracy.

Furthermore, a detailed comparison with exact solutions confirmed that the modified method rapidly converges to the true solution, with minimal error, even in nonlinear cases that typically challenge standard perturbation techniques. This confirms that integrating the Sumudu transform with the homotopy framework provides a powerful approach for solving complex mathematical models without requiring excessive simplifications or additional assumptions.

Overall, this integration reflects a promising research trend toward developing **hybrid methods** that combine the advantages of integral transforms with the flexibility of homotopy techniques, providing accurate, stable, and widely applicable solutions for complex differential models.

II. SUMUDU TRANSFORM – DEFINITION

The **Sumudu Transform** is an integral transform, similar in purpose to the Laplace Transform, used to solve differential equations and other problems in engineering, physics, and applied mathematics. It converts a function of time $f(t)$ into a function of a new variable u, often simplifying calculations and preserving the dimensionality of the original problem.

Mathematically, for a function $f(t)$ defined for $t \geq 0$

$$S\{f(t)\}(u) = \int_0^\infty f(ut).e^{-t}dt$$

where: t is the original time variable, u is the Sumudu transform variable, e^{-t} ensures convergence of the integral.

The **Sumudu Transform,** like the Laplace Transform, it helps turn differential equations into algebraic equations, making them easier to solve, and also preserves the "unit" of the original function. That is, if f(t) has units of seconds, u keeps compatible dimensions, unlike the Laplace transform variable s, which is reciprocal time.

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III. METHODOLOGY OF SUMUDU ADOMIAN DECOMPOSITION METHOD (SADM)

Consider the general nonlinear ordinary differential equation of the form:

$$L(y(x)) + R(y(x)) + N(y(x)) = g(x) \quad (1)$$

The initial condition is given by $y^{(k)}(0) = h_k(x)$

where L is an operator of the highest derivative,

$R(y(x))$ is the remainder of the differential operator.

$N(y(x))$ is the nonlinear term, $g(x)$ Can any function.

By taking the S-Transform on both sides of Eq. (1), we have:

$$\boldsymbol{S}[L(y(x))] + \boldsymbol{S}[R(y(x))] + \boldsymbol{S}[N(y(x))] = \boldsymbol{S}[g(x)] \quad (2)$$

Then:

$$\boldsymbol{S}[L(y(x))] = \boldsymbol{S}[g(x)] - \boldsymbol{S}[R(y(x))] - \boldsymbol{S}[N(y(x))]$$

$$\frac{Y(v)}{v^n} - \sum_{k=0}^{n-1} v^{-n+k} y^{(k)}(0) = \boldsymbol{S}[g(x)] - \boldsymbol{S}[R(y(x))] - \boldsymbol{S}[N(y(x))]$$

$$Y(v) = v^n \sum_{k=0}^{n-1} v^{-n+k} y^{(k)}(0) + v^n \boldsymbol{S}[g(x)] - v^n \boldsymbol{S}[R(y(x))] - v^n \boldsymbol{S}[N(y(x))] \quad (3)$$

Then, taking the inverse S-Transform:

$$y(x) = \boldsymbol{G}(\boldsymbol{x}) - \boldsymbol{S}^{-1}\left[v^n \boldsymbol{S}[R(y(x))] + v^n \boldsymbol{S}[N(y(x))]\right] \quad (13);$$

$$\boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{S}^{-1}\left(v^n \sum_{k=0}^{n-1} v^{-n+k} y^{(k)}(0) + v^n \boldsymbol{S}[g(x)]\right)$$

The methodology consists of approximating the solution of (1) as an infinite Series given:

$$y(x) = \sum_{n=0}^{\infty} y_n(x)$$

However, the nonlinear term $F(y(x))$ is decomposed as

$$F(y(x)) = \sum_{n=0}^{\infty} A_n(x)$$

$$A_n(x) = \frac{1}{n!}\frac{d^n}{d\lambda^n}[F(y(\lambda))]_{\lambda=0}$$

$$\sum_{n=0}^{\infty} y_n(x) = \boldsymbol{G}(\boldsymbol{x}) - \boldsymbol{S}^{-1}\left[v^n \boldsymbol{S}\left[R\sum_{n=0}^{\infty} y_n(x) + \sum_{n=0}^{\infty} A_n(x)\right]\right]$$

Then:

$$y_1 = -\boldsymbol{S}^{-1}[v^n \boldsymbol{S}[R(y_0) + A_0]]$$

$$y_2 = -\boldsymbol{S}^{-1}[v^n \boldsymbol{S}[R(y_1) + A_1]]$$

$$y_3 = -\boldsymbol{S}^{-1}[v^n \boldsymbol{S}[R(y_2) + A_2]]$$

Eventually, we have the general recursive relation as follows:

$$\begin{cases} y_0 = G(x) \\ y_{n+1} = -\boldsymbol{S}^{-1}[v^n \boldsymbol{S}[R(y_n) + A_n]] \ \boldsymbol{n \geq 0} \end{cases}$$

Hence, the exact or approximate solution is given by:

$$y(x) = \sum_{n=0}^{\infty} y_n(x)$$

IV. APPLICATIONS

In this section, we solve several example problems using the **new method** and present the corresponding **numerical results**. The results are illustrated with **graphs** to make the solutions easier to understand. We also compare the performance of the new method with the **classical Adomian Decomposition Method (ADM)**, **TADM**, and **MADM** to highlight its **accuracy and efficiency**.

Application (1): Find the solution?

$$\begin{cases} \boldsymbol{\frac{d^2y}{dx^2} = x + y} \\ \boldsymbol{y(0) = 1 \ \ , \dot{y}(0) = 0} \end{cases}$$

Solution:

Using ADM:

The approximate solution $y \approx y_0 + y_1 + y_2 + y_3$

$$y(x) \approx \boldsymbol{1 + \frac{x^3}{3!} + \frac{x^2}{2!} + \frac{x^5}{5!} + \frac{x^4}{4!} + \frac{x^7}{7!} + \frac{x^6}{6!} + \frac{x^9}{9!}}$$

Using MADM:

$$\sum_{n=0}^{\infty} y_n(x) = \boldsymbol{1 + \frac{1}{3!}x^3 + L^{-1}}\left(\sum_{n=0}^{\infty} y_n\right)$$

We have $F(x,t) = f_1 + f_2 = \boldsymbol{1 + \frac{1}{3!}x^3}$

We consider $f_1 = 1 \ \ , \ f_2 = \frac{1}{3!}\boldsymbol{x^3}$

Then the Recursive Relations**:**

$$\begin{cases} y_0 = \boldsymbol{1} \\ \boldsymbol{y_1} = \boldsymbol{\frac{x^3}{3!} + L^{-1}}(y_0) \\ y_{n+2} = \boldsymbol{L^{-1}}(y_{n+1}) \qquad \boldsymbol{n \geq 0} \end{cases}$$

$$y_1 = \boldsymbol{\frac{x^3}{3!} + L^{-1}}(y_0) = \boldsymbol{L^{-1}(1) = \frac{x^2}{2!} + \frac{x^3}{3!}}$$

$$y_2 = \boldsymbol{L^{-1}}(y_1) = \boldsymbol{L^{-1}}\left(\boldsymbol{\frac{x^2}{2!} + \frac{x^3}{3!}}\right) = \boldsymbol{\frac{x^4}{4!} + \frac{x^5}{5!}}$$

$$y_3 = \boldsymbol{L^{-1}}(y_1) = \boldsymbol{L^{-1}}\left(\boldsymbol{\frac{x^4}{4!} + \frac{x^5}{5!}}\right) = \boldsymbol{\frac{x^6}{6!} + \frac{x^7}{7!}}$$

The approximate solution $y \approx y_0 + y_1 + y_2 + y_3$

$$y(x) \approx \boldsymbol{1 + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!}}$$

Using SADM:

take the Sumudu transform**:**

$$s(\boldsymbol{Ly}) = \boldsymbol{S(x) + S(y)}$$

$$\frac{Y(v)}{v^2} - \frac{y(0)}{v^2} - \frac{\dot{y}(0)}{v} = \boldsymbol{S(x) + S(y)}$$

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$$\frac{Y(v)}{v^2} = \frac{1}{v^2} + \boldsymbol{S}(\boldsymbol{x}) + \boldsymbol{S}(\boldsymbol{y})$$
$$Y(v) = 1 + v^2\boldsymbol{S}(\boldsymbol{x}) + v^2\boldsymbol{S}(\boldsymbol{y})$$
$$Y(v) = 1 + v^3 + \boldsymbol{S}(\boldsymbol{y})$$
$$S(Y(v)) = S(1 + v^3) + \boldsymbol{S}^{-1}(v^2\boldsymbol{S}(\boldsymbol{y}))$$
$$y(x) = 1 + \frac{x^3}{3!} + \boldsymbol{S}^{-1}[v^2\boldsymbol{S}(\boldsymbol{y})]$$
$$suppose: \; y(x) = \sum_{n=0}^{\infty} y_n(x)$$
$$\sum_{n=0}^{\infty} y_n(x) = \boldsymbol{1} + \frac{\boldsymbol{x}^3}{\boldsymbol{3}!} + \boldsymbol{S}^{-1}\left[v^2\boldsymbol{S}\left(\sum_{n=0}^{\infty} y_n\right)\right]$$

Then the Recursive Relations:

$$\begin{cases} y_0 = \boldsymbol{1} + \dfrac{\boldsymbol{x}^3}{\boldsymbol{3}!} \\ y_{n+1} = \boldsymbol{S}^{-1}[v^2\boldsymbol{S}(y_n)] \qquad n \geq 0 \end{cases}$$

$$y_1 = \boldsymbol{S}^{-1}[v^2\boldsymbol{S}(y_0)] = \boldsymbol{S}^{-1}\left[v^2\boldsymbol{S}\left(\boldsymbol{1} + \frac{\boldsymbol{x}^3}{\boldsymbol{3}!}\right)\right]$$
$$= \boldsymbol{S}^{-1}[v^2(\boldsymbol{1} + \boldsymbol{v}^3)]$$
$$= \boldsymbol{S}^{-1}[v^2 + \boldsymbol{v}^5] = \frac{\boldsymbol{x}^2}{\boldsymbol{2}!} + \frac{\boldsymbol{x}^5}{\boldsymbol{5}!}$$
$$y_2 = \boldsymbol{S}^{-1}[v^2\boldsymbol{S}(y_1)] = \boldsymbol{S}^{-1}\left[v^2\boldsymbol{S}\left(\frac{\boldsymbol{x}^2}{\boldsymbol{2}!} + \frac{\boldsymbol{x}^5}{\boldsymbol{5}!}\right)\right]$$
$$= \boldsymbol{S}^{-1}[v^2(v^2 + \boldsymbol{v}^5)] = \boldsymbol{S}^{-1}[v^4 + \boldsymbol{v}^7] = \frac{\boldsymbol{x}^4}{\boldsymbol{4}!} + \frac{\boldsymbol{x}^7}{\boldsymbol{7}!}$$
$$y_3 = \boldsymbol{S}^{-1}[v^2\boldsymbol{S}(y_2)] = \boldsymbol{S}^{-1}\left[v^2\boldsymbol{S}\left(\frac{\boldsymbol{x}^4}{\boldsymbol{4}!} + \frac{\boldsymbol{x}^7}{\boldsymbol{7}!}\right)\right]$$
$$= \boldsymbol{S}^{-1}[v^2(v^4 + \boldsymbol{v}^7)] = \boldsymbol{S}^{-1}[v^6 + \boldsymbol{v}^9] = \frac{\boldsymbol{x}^6}{\boldsymbol{6}!} + \frac{\boldsymbol{x}^9}{\boldsymbol{9}!}$$

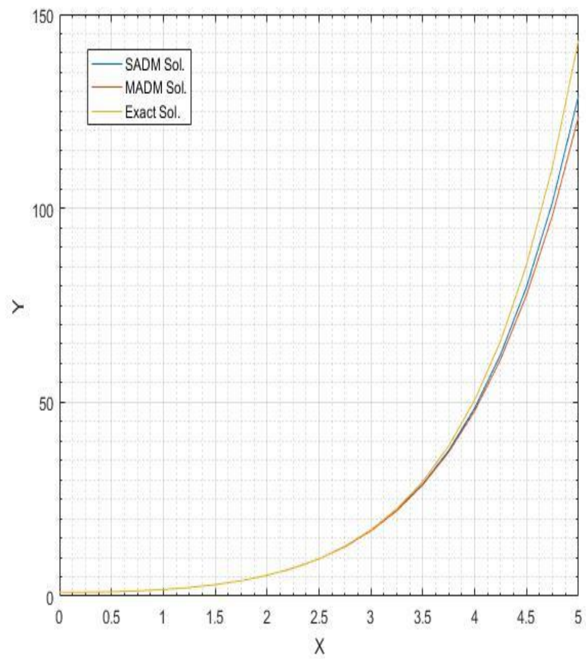
The approximate solution $y(x) \approx y_0 + y_1 + y_2 + y_3$

$$y(x) = \boldsymbol{1} + \frac{\boldsymbol{x}^2}{\boldsymbol{2}!} + \frac{\boldsymbol{x}^3}{\boldsymbol{3}!} + \frac{\boldsymbol{x}^4}{\boldsymbol{4}!} + \frac{\boldsymbol{x}^5}{\boldsymbol{5}!} + \frac{\boldsymbol{x}^6}{\boldsymbol{6}!} + \frac{\boldsymbol{x}^7}{\boldsymbol{7}!} + \frac{\boldsymbol{x}^9}{\boldsymbol{9}!}$$

TABLE(1): Comparison of SADM/ADM Approximate Solution with MADM, Exact Solution, and Absolute Error

X	SADM (ADM)	MADM	EX	error1	error2
0	1	1	1	0	0
0.25	1.034025	1.034025	1.03E+00	3.66E-10	3.76E-10
0.5	1.148721	1.148721	1.15E+00	8.46E-08	8.93E-08
1	1.718257	1.718254	1.72E+00	1.46E-05	1.62E-05
1.25	2.240192	2.240172	2.24E+00	6.73E-05	7.64E-05
1.5	2.981035	2.980929	2.981689	0.000219	0.000255
2	5.382363	5.380952	5.389056	0.001242	0.001504
2.25	7.220299	7.216226	7.237736	0.002409	0.002972
2.5	9.641271	9.630759	9.682494	0.004257	0.005343
3	16.90067	16.84643	17.08554	0.01082	0.013995
3.25	22.18086	22.06938	22.54034	0.015948	0.020894
3.5	28.94718	28.72998	29.61545	0.022565	0.029899
4	48.52875	47.80635	50.59815	0.040899	0.055176
4.25	62.37482	61.12819	65.85541	0.052852	0.071782
4.5	79.80807	77.72287	85.51713	0.066759	0.091143
5	129.0013	123.619	143.4132	0.100492	0.138022

Figure (1): Comparison of SADM/ADM Approximation with MADM and Exact Solution



Application (2): Find the solution?

$$\begin{cases} \boldsymbol{y}^{(14)} = \boldsymbol{2e}^{\boldsymbol{x}} - \boldsymbol{y} \\ \boldsymbol{y}^{(k)}(\boldsymbol{0}) = \boldsymbol{1} \;\; ; \boldsymbol{k} = \boldsymbol{0}, \boldsymbol{1}, \ldots, \boldsymbol{13} \end{cases}$$

Solution:

Using ADM*:*

The approximate solution $\quad y \approx y_0 + y_1 + y_2$

$$= 2e^x - \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^{41}}{41!}\right)$$

Using TADM

$$y = 2e^x - 1 - \frac{x}{1!} - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!} - \frac{x^5}{5!} - \frac{x^6}{6!} - \frac{x^7}{7!} - \frac{x^8}{8!} - \frac{x^9}{9!} - \frac{x^{10}}{10!} - \frac{x^{11}}{11!} - \frac{x^{12}}{12!} - \frac{x^{13}}{13!} - \boldsymbol{L}^{-1}(\boldsymbol{y})$$

We have a Taylor series for $e^x = 1 + \frac{x}{1!} + \cdots + \frac{x^5}{5!} + \cdots$

$$y = 2\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots\right) - 1 - \frac{x}{1!} - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!} - \frac{x^5}{5!} - \frac{x^6}{6!} - \frac{x^7}{7!} - \frac{x^8}{8!} - \frac{x^9}{9!} - \frac{x^{10}}{10!} - \frac{x^{11}}{11!} - \frac{x^{12}}{12!} - \frac{x^{13}}{13!} - \boldsymbol{L}^{-1}(\boldsymbol{y})$$

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$$\boldsymbol{y} = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+\frac{x^6}{6!}+\frac{x^7}{7!}+\frac{x^8}{8!}+\frac{x^9}{9!}+\frac{x^{10}}{10!}+\frac{x^{11}}{11!}+\frac{x^{12}}{12!}+\frac{x^{13}}{13!}+2\frac{x^{14}}{14!}+2\frac{x^{15}}{15!}+2\frac{x^{16}}{16!}-\boldsymbol{L}^{-1}(\boldsymbol{y})$$

$$suppose:\ y(x)=\sum_{n=0}^{\infty} y_n(x)$$

$$\sum_{n=0}^{\infty} y_n(x) = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+\frac{x^6}{6!}+\frac{x^7}{7!}+\frac{x^8}{8!}+\frac{x^9}{9!}+\frac{x^{10}}{10!}+\frac{x^{11}}{11!}+\frac{x^{12}}{12!}+\frac{x^{13}}{13!}+2\frac{x^{14}}{14!}+2\frac{x^{15}}{15!}+2\frac{x^{16}}{16!}+\cdots-\boldsymbol{L}^{-1}\left(\sum_{n=0}^{\infty} y_n(x)\right)$$

We consider $f_0=1$, $f_1=\frac{x}{1!}$, $f_2=\frac{x^2}{2!}, f_3=\frac{x^3}{3!}$, … ,

Then the Recursive Relations:

$$\begin{cases} y_0 = 1 \\ y_{n+1} = f_{n+1} - \boldsymbol{L}^{-1}(y_n) \quad n \geq 0 \end{cases}$$

$$y_1 = f_1 - \boldsymbol{L}^{-1}(y_0) = \frac{x}{1!} - \boldsymbol{L}^{-1}(1)$$

$$= \frac{x}{1!} - \left(\frac{x^{13}}{13!}\right) = \frac{x}{1!} - \frac{x^{13}}{13!}$$

$$y_2 = f_2 - \boldsymbol{L}^{-1}(y_1) = \frac{x^2}{2!} - \boldsymbol{L}^{-1}\left(\frac{x}{1!}-\frac{x^{13}}{13!}\right)$$

$$= \frac{x^2}{2!} - \frac{x^{14}}{14!} + \frac{x^{26}}{26!}$$

The approximate solution $y \approx y_0 + y_1 + y_2 + y_3$

$$y \approx 1 + \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^{13}}{13!} - \frac{x^{14}}{14!} + \frac{x^{26}}{26!}$$

Using SADM:

Take the Sumudu **transform:**

$$S(y^{(14)}) = S(2e^x) - S(y(x))$$

$$\frac{Y(v)}{v^{14}} - \frac{y(0)}{v^{14}} - \frac{\dot{y}(0)}{v^{13}} - \frac{y^{(2)}(0)}{v^{12}} - \frac{y^{(3)}(0)}{v^{11}} - \frac{y^{(4)}(0)}{v^{10}} - \frac{y^{(5)}(0)}{v^9} - \frac{y^{(6)}(0)}{v^8} - \frac{y^{(7)}(0)}{v^7} - \frac{y^{(8)}(0)}{v^6} - \frac{y^{(9)}(0)}{v^5} - \frac{y^{(10)}(0)}{v^4} - \frac{y^{(11)}(0)}{v^3} - \frac{y^{(12)}(0)}{v^2} - \frac{y^{(13)}(0)}{v} = \frac{2}{1-v} - S(y(x))$$

$$\frac{Y(v)}{v^{14}} - \frac{1}{v^{14}} - \frac{1}{v^{13}} - \frac{1}{v^{12}} - \frac{1}{v^{11}} - \frac{1}{v^{10}} - \frac{1}{v^9} - \frac{1}{v^8} - \frac{1}{v^7} - \frac{1}{v^6} - \frac{1}{v^5} - \frac{1}{v^4} - \frac{1}{v^3} - \frac{1}{v^2} - \frac{1}{v} = \frac{2}{1-v} - S(y(x))$$

$$Y(v) = 1+v+v^2+v^3+v^4+v^5+v^6+v^7+v^8+v^9+v^{10}+v^{11}+v^{12}+v^{13}+\frac{2v^{14}}{1-v} - v^{14}S(y(x))$$

$$Y(v) = -1-v-v^2-v^3-v^4-v^5-v^6-v^7-v^8-v^9-v^{10}-v^{11}-v^{12}-v^{13}+\frac{2}{1-v} - v^{14}S(y(x))$$

$$S^{-1}[Y(v)] = S^{-1}\left[-1-v-v^2-v^3-v^4-v^5-v^6-v^7-v^8-v^9-v^{10}-v^{11}-v^{12}-v^{13}+\frac{2}{1-v}\right] - S^{-1}[v^{14}S(y(x))]$$

$$y(x) = 2e^x - 1 - \frac{x}{1!} - \frac{x^2}{2!} - \cdots - \frac{x^{13}}{13!} - S^{-1}\big(v^{14}S(y(x))\big)$$

$$suppose:\ y(x)=\sum_{n=0}^{\infty} y_n(x)$$

Then the Recursive Relations:

$$\begin{cases} y_0 = 2e^x - 1 - \frac{x}{1!} - \frac{x^2}{2!} - \cdots - \frac{x^{13}}{13!}; \\ y_{n+1}(x) = -S^{-1}\left(v^{14}.S(y_n(x))\right) \end{cases}$$

$$y_1 = -S^{-1}[v^{14}.S(y_0)]$$

$$= -S^{-1}\left[v^{14}.S\left(2e^x - 1 - \frac{x}{1!} - \frac{x^2}{2!} - \cdots - \frac{x^{13}}{13!}\right)\right]$$

$$= -S^{-1}\left[\frac{2v^{14}}{1-v} - v^{14} - v^{15} - v^{16} - v^{17} - v^{18} - v^{19} - v^{20} - v^{21} - v^{22} - v^{23} - v^{24} - v^{25} - v^{26} - v^{27}\right]$$

$$= -S^{-1}\left[2\left(-1-v-v^2-v^3-v^4-v^5-v^6-v^7-v^8-v^9-v^{10}-v^{11}-v^{12}-v^{13}+\frac{1}{1-v}\right) - v^{14} - v^{15} - v^{16} - v^{17} - v^{18} - v^{19} - v^{20} - v^{21} - v^{22} - v^{23} - v^{24} - v^{25} - v^{26} - v^{27}\right]$$

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$$= -S^{-1}\Big[-2.1 - 2v - 2v^2 - 2v^3 - 2v^4 - 2v^5 - 2v^6 - 2v^7 - 2v^8 - 2v^9 - 2v^{10} - 2v^{11} - 2v^{12} - 2v^{13} + \frac{2}{1-v} - v^{14} - v^{15} - v^{16} - v^{17} - v^{18} - v^{19} - v^{20} - v^{21} - v^{22} - v^{23} - v^{24} - v^{25} - v^{26} - v^{27}\Big]$$

$$= -\left[2e^x - 2 - \frac{2x}{1!} - \frac{2x^2}{2!} - \frac{2x^3}{3!} - \frac{2x^4}{4!} - \cdots - \frac{x^{27}}{27!}\right]$$

$$= -2e^x + 2 + \frac{2x}{1!} + \frac{2x^2}{2!} + \frac{2x^3}{3!} + \frac{2x^4}{4!} + \cdots + \frac{x^{27}}{27!}$$

$$y_2 = -S^{-1}[v^{14}.S(y_1)]$$

$$= -S^{-1}\left[v^{14}.S\left(-2e^x + 2 + \frac{2x}{1!} + \frac{2x^2}{2!} + \frac{2x^3}{3!} + \frac{2x^4}{4!} + \cdots + \frac{x^{27}}{27!}\right)\right]$$

$$= -S^{-1}\Big[-\frac{2v^{14}}{1-v} + 2v^{14} + 2v^{15} + 2v^{16} + 2v^{17} + 2v^{18} + 2v^{19} + 2v^{20} + 2v^{21} + 2v^{22} + 2v^{23} + 2v^{24} + 2v^{25} + 2v^{26} + 2v^{27} + v^{28} + v^{29} + v^{30} + v^{31} + v^{32} + v^{33} + v^{34} + v^{35} + v^{36} + v^{37} + v^{38} + v^{39} + v^{40} + v^{41}\Big]$$

$$= -S^{-1}\Big[-2\Big(-1 - v - v^2 - v^3 - v^4 - v^5 - v^6 - v^7 - v^8 - v^9 - v^{10} - v^{11} - v^{12} - v^{13} + \frac{1}{1-v}\Big) + 2v^{14} + 2v^{15} + 2v^{16} + 2v^{17} + 2v^{18} + 2v^{19} + 2v^{20} + 2v^{21} + 2v^{22} + 2v^{23} + 2v^{24} + 2v^{25} + 2v^{26} + 2v^{27} + v^{28} + v^{29} + v^{30} + v^{31} + v^{32} + v^{33} + v^{34} + v^{35} + v^{36} + v^{37} + v^{38} + v^{39} + v^{40} + v^{41}\Big]$$

$$= S^{-1}\Big[-2.1 - 2v - 2v^2 - 2v^3 - 2v^4 - 2v^5 - 2v^6 - 2v^7 - 2v^8 - 2v^9 - 2v^{10} - 2v^{11} - 2v^{12} - 2v^{13} + \frac{2}{1-v} - 2v^{14} - 2v^{15} - 2v^{16} - 2v^{17} - 2v^{18} - 2v^{19} - 2v^{20} - 2v^{21} - 2v^{22} - 2v^{23} - 2v^{24} - 2v^{25} - 2v^{26} - 2v^{27} - v^{28} - v^{29} - v^{30} - v^{31} - v^{32} - v^{33} - v^{34} - v^{35} - v^{36} - v^{37} - v^{38} - v^{39} - v^{40} - v^{41}\Big]$$

$$= 2e^x - 2.1 - \frac{2x}{1!} - \frac{2x^2}{2!} - \frac{2x^3}{3!} - \frac{2x^4}{4!} - \frac{2x^5}{5!} - \frac{2x^6}{6!} - \frac{2x^7}{7!} - \frac{2x^8}{8!} - \frac{2x^9}{9!} - \frac{2x^{10}}{10!} - \frac{2x^{11}}{11!} - \frac{2x^{12}}{12!} - \frac{2x^{13}}{13!} - \frac{2x^{14}}{14!} - \frac{2x^{15}}{15!} - \frac{2x^{16}}{16!} - \frac{2x^{17}}{17!} - \frac{2x^{18}}{18!} - \frac{2x^{19}}{19!} - \frac{2x^{20}}{20!} - \frac{2x^{21}}{21!} - \frac{2x^{22}}{22!} - \frac{2x^{23}}{23!} - \frac{2x^{24}}{24!} - \frac{2x^{25}}{25!} - \frac{2x^{26}}{26!} - \frac{2x^{27}}{27!} - \frac{x^{28}}{28!} - \frac{x^{29}}{29!} - \frac{x^{30}}{30!} - \frac{x^{31}}{31!} - \frac{x^{32}}{32!} - \frac{x^{33}}{33!} - \frac{x^{34}}{34!} - \frac{x^{35}}{35!} - \frac{x^{36}}{36!} - \frac{x^{37}}{37!} - \frac{x^{38}}{38!} - \frac{x^{39}}{39!} - \frac{x^{40}}{40!} - \frac{x^{41}}{41!}$$

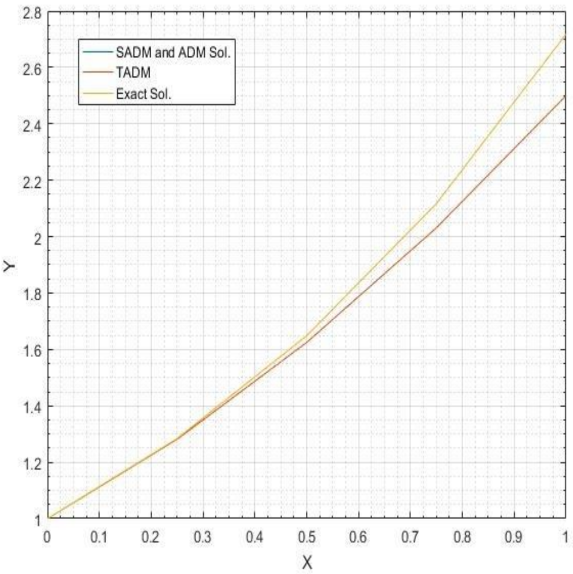
The approximate solution $y(x) \approx y_0 + y_1 + y_2$

$$= 2e^x - \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^{41}}{41!}\right)$$

TABLE(2): Comparison of SADM/ADM Approximate Solution with TADM, Exact Solution, and Absolute Error

x	SADM (ADM)	TADM	Exact	Error 1	Error 2
0	1	1	1	0	0
0.25	1.284025	1.28125	1.284025	0	0.002161
0.5	1.648721	1.625	1.648721	0	0.014388
0.75	2.117	2.03125	2.117	0	0.040505
1	2.718282	2.5	2.718282	0	0.080301

Figure (2): Comparison of SADM/ADM Approximation with TADM and Exact Solution



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Application (3) : Find the solution?

$$\begin{cases} \dot{y} - y^2 - 1 = 0 \\ y(0) = 0 \end{cases}$$

Solution:

Using ADM:

The approximate solution $y(x) \approx y_0 + y_1 + y_2 + y_3$

$$y(x) \approx x + \frac{x^3}{3} + 2\frac{x^5}{15} + \frac{17x^7}{945}$$

Using SADM

Take the Sumudu **transform:**

$S(\dot{y}) - S(y^2) - S(1) = 0$

$S(\dot{y}) = S(1) + S(y^2)$

$\frac{Y(v)}{v} - \frac{y(0)}{v} = 1 + S(y^2)$

$Y(v) = 1.v + v.S(y^2)$

$S^{-1}(Y(v)) = S^{-1}(v) + S^{-1}(v.S(y^2))$

$y(x) = x + S^{-1}(v.S(y^2))$

Now suppose: $y(x) = \sum_{n=0}^{\infty} y_n(x) \quad , y^2 = \sum_{n=0}^{\infty} A_n(x)$

$$\sum_{n=0}^{\infty} y_n(x) = x + S^{-1}\left[v.S\left(\sum_{n=0}^{\infty} A_n(x)\right)\right]$$

Then the Recursive Relations:

$$\begin{cases} y_0 = x \\ y_{n+1}(x) = S^{-1}(v.A_n(x)) \quad n \geq 0 \end{cases}$$

$A_0(x) = (y_0)^2$

$y_1 = S^{-1}[v.S(A_0(x))] = S^{-1}[v.S(x)] = S^{-1}[v.v^2] = S^{-1}[v^3]$

$= \frac{x^3}{3}$

$A_1(x) = 2y_0y_1 = 2.x.\frac{x^3}{3} = \frac{2x^4}{3}$

$y_2 = S^{-1}[v.S(A_1(x))] = S^{-1}\left[v.S\left(\frac{2x^4}{3}\right)\right] = S^{-1}\left[v.\frac{2 \times 4! \times v^4}{3}\right]$

$= 16.S^{-1}[v^5] = 16\frac{x^5}{5!} = \frac{2x^5}{15}$

$A_2(x) = 2y_0y_2 + (y_1)^2$

$= 2.x.\frac{2x^5}{15} + \left(\frac{x^3}{3}\right)^2 = \frac{4x^6}{15} + \frac{x^6}{9} = \frac{17x^6}{45}$

$y_3 = S^{-1}[v.S(A_2(x))] = S^{-1}\left[v.S\left(\frac{17x^6}{45}\right)\right]$

$= S^{-1}\left[v.\frac{17 \times 6! \times v^6}{45}\right]$

$= \frac{17 \times 6!}{45} S^{-1}[v^7] = \frac{17 \times 6!}{45} \times \frac{x^7}{7!} = \frac{17x^7}{315}$

The approximate solution $y(x) \approx y_0 + y_1 + y_2 + y_3 + y_4$

$$y(x) \approx x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315}$$

Remark that we can obtain the exact solution if we continue using Recursive Relations i.e

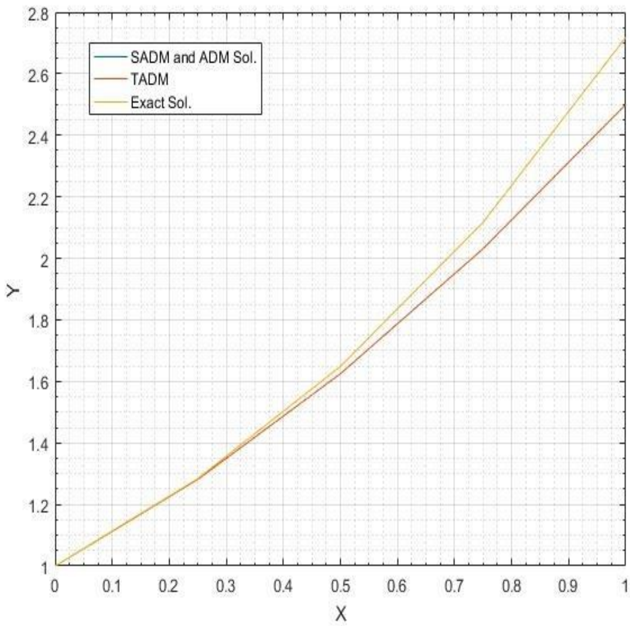
$$y(x) = y_0 + y_1 + y_2 + y_3 + +y_4 + \cdots = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots$$

$$= \tan(x)$$

TABLE(3): Comparison of SADM/ADM Approximate Solution with Exact Solution and Absolute Error

x	SADM/ADM	Exact	Error
0	0	0	0
0.1	0.100335	1.00E-01	2.19E-10
0.2	0.20271	2.03E-01	5.61E-08
0.3	0.309336	3.09E-01	1.44E-06
0.4	0.422787	0.422793	1.45E-05
0.5	0.546255	5.46E-01	8.70E-05
0.6	0.683879	6.84E-01	3.77E-04
0.7	0.841187	8.42E-01	1.31E-03
0.8	1.025675	1.03E+00	3.85E-03
0.9	1.247545	1.26E+00	1.00E-02
1	1.520635	1.557408	0.023612

Figure(3) : Comparison of SADM/ADM Approximation with Exact Solution



V. CONCLUSION

The **Adomian Decomposition Method (ADM)** and its variant, the **S-Transform Adomian Decomposition Method (SADM)**, provide powerful tools for obtaining analytical or approximate solutions to linear and nonlinear differential equations. While both methods produce the same series solution, **SADM offers significant advantages**: it simplifies calculations, eliminates the need for repeated integration, and is particularly efficient for **high-order or complex equations**.

REFERENCES

[1] T. A. Abbasi, "Improved Adomian decomposition method for nonlinear initial value problems," *Appl. Math. Comput.*, vol. 215, no. 5, pp. 2034–2045, 2010.

[2] Z. A. Abdo and Y. Q. Hasan, "Modified ADM for higher-order Emden–Fowler equations," *AJPAS*, vol. 12, no. 1, pp. 45–59, 2020.

[3] G. Adomian, "A review of the decomposition method and some recent applications," *J. Math. Anal. Appl.*, vol. 135, no. 2, pp. 501–545, 1988.

[4] G. Adomian, "A review of the decomposition method in applied mathematics," *J. Math. Anal. Appl.*, vol. 135, no. 2, pp. 501–544, 1988.

[5] G. Adomian, "A review of the decomposition method and its application to stochastic continuous-time dynamical systems," *Math. Comput. Simul.*, vol. 32, pp. 19–42, 1990.

[6] E. U. Agom and F. O. Ogunfiditimi, "Modified Adomian polynomial for nonlinear functional with integer exponent," *J. Math.*, vol. 11, no. 6, pp. 40–45, 2015.

[7] E. U. Agom and F. O. Ogunfiditimi, "Modified Adomian polynomials for nonlinear functions with integer exponents," *Int. J. Math. Math. Sci.*, vol. 2015, pp. 1–12, 2015.

[8] S. Al Baghdadi, "A comparative study of Adomian decomposition method and variational iteration method in solving nonlinear differential equations," *Open Access J.*, 2024.

[9] S. Aljuhani *et al.*, "Application of Adomian decomposition method for solving nonlinear Boussinesq equations using Maple," *Sci. Res. Publ.*, vol. 12, no. 5, pp. 1–15, 2023.

[10] S. Aljuhani *et al.*, "Solving nonlinear Boussinesq equations using Adomian decomposition method with Maple," *SCIRP J.*, 2023.

[11] [11] M. Al-Mazmumy, "Enhanced ADM using inverse integral operators for ODEs," *Mathematics*, vol. 11, no. 12, p. 698, 2022.

[12] M. Al-Mazmumy, "ADM combined with Taylor series and orthogonal polynomials for fractional PDEs," *AIMS Math.*, vol. 9, no. 10, pp. 1475–1490, 2024.

[13] M. Al-Mazmumy and S. O. Almuhalbedi, "Solution of nonlinear integro-differential equations by two-step Adomian decomposition method (TSAM)," *Int. J. Mod. Nonlinear Theory Appl.*, vol. 5, no. 4, pp. 248–255, 2016.

[14] M. Almazmumy, S. O. Al-Muhalbedi, and M. Rashid, "Two-step Adomian decomposition method for initial value problems in ordinary differential equations," *Open Access J. Math.*, vol. 4, no. 3, pp. 120–132, 2012.

[15] M. Al-Mazmumy *et al.*, "Modified Adomian decomposition method for initial-boundary value problems of fractional PDEs," *AIMS Math.*, vol. 10, no. 3, pp. 1250–1275, 2025.

[16] S. Al-Mazmumy *et al.*, "Modified Adomian decomposition method for initial-boundary value problems of fractional PDEs," *AIMS Math.*, 2025.

[17] F. Almeida *et al.*, "Semi-analytical solutions of fractional Riccati differential equations using Adomian decomposition," *PPHMJ Open Access J.*, 2023.

[18] F. Almeida *et al.*, "Semi-analytical solutions of fractional Riccati differential equations using Adomian decomposition method," *J. Math. Res.*, vol. 15, no. 2, pp. 45–60, 2023.

[19] J. Bazar, E. Babolian, G. Kember, A. Nouri, and R. Islam, "An alternate algorithm for computing Adomian polynomials in special cases," *Appl. Math. Comput.*, vol. 138, nos. 2–3, pp. 523–529, 2003.

[20] J. Bazar, E. Babolian, A. Nouri, and S. Islam, "Alternative algorithm for computing Adomian polynomials," *Appl. Math. Comput.*, vol. 140, nos. 2–3, pp. 303–309, 2003.

[21] F. B. M. Belgacem and A. A. Karaballi, "Sumudu transform: Fundamental properties, investigations and applications," *J. Appl. Math. Stoch. Anal.*, vol. 2006, pp. 1–10, 2006.

[22] J. Biazar and S. M. Shafiof, "A simple algorithm for calculating Adomian polynomials," *Int. J. Contemp. Math. Sci.*, vol. 2, no. 20, pp. 975–982, 2007.

[23] J. Biazar and S. M. Shafiof, "Simplified algorithm for multivariable Adomian polynomials," *Comput. Math. Appl.*, vol. 54, no. 4, pp. 451–457, 2007.

[24] E. Centre and D. Uk, "On the Adomian decomposition method for the solution of differential equations," *Appl. Math.*, vol. 1, no. 2, pp. 20–29, 2013.

[25] B. Daoud, "Modified Adomian decomposition method for boundary layer convective heat transfer problems," *J. Comput. Phys.*, vol. 356, pp. 203–218, 2018.