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A Simple Analog Controller for a Magnetic Levitation Kit

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***Abstract*— A simple proportional-derivative analog controller is proposed for a stabilization of a magnetic levitation kit which is widely adopted as a control education kit. We present a dynamic model of a magnetic levitation system and systematic procedures for a controller synthesis and an actual implementation. From experiments we confirmed a good performance of our closed loop levitation system.**

Keywords—Magnetic Levitation; Controller Design; PD Controller

I. INTRODUCTION

A magnetic levitation system is one of the most popular systems in control education [1-5]. This is because it clearly and intuitively shows a real control action and thus effectively demonstrates the importance of control engineering.

As a matter of fact in many control laboratories worldwide a magnetic levitation system is widely chosen as a control demonstration kit or a student project. Moreover, the control problem of a magnetic levitation system is not merely important for control education but there are many industrial applications of magnetic levitation techniques.

When a magnetic levitation system is used for a student project or a lab experiment topic, it is highly desirable that a control circuit is simple enough that students with little experiences in electronics can actually implement the control circuit in a limited time. It is also required that a levitation system is low-cost and easy to maintain.

A low-cost magnetic levitation kit was proposed in [3]. However the controller circuit in [3] includes a fan-management IC and an H-bridge motor driver IC. The functions of those components in a controller circuit are not easy to understand.

In addition, in [5], a simple analog PD (proportional and derivative) controller was proposed for a magnetic levitation testbed. The position sensor used for measuring the location of a floating object in [5] was an optical sensor. Even though an optical sensor has a merit of having a simple dynamics and showing good robustness to electrical noises, a levitation system combined with an optical sensor requires an external frame to install/align optical sender/receiver and inevitably its size is bigger and it gets far from being low-cost in general. Furthermore, as a minor disadvantage, an optical sensor frame hides the view of a levitating object.

A good alternative to an optical sensor is a Hall Effect sensor which is typically installed underneath of an electromagnet. A main disadvantage of a Hall sensor however is that it is subject to not only the position of a levitating object but also the electromagnet (actuator). This results in an intricate dynamics of a whole magnetic levitation system.

In this paper, we report that a simple PD controller can work for a magnetic levitation system with a Hall sensor.

II. SYSTEM MODEL

We have designed an analog controller for a commercial electromagnetic levitation kit from Zeltom© [6]. This kit has an electromagnet combined with a linear Hall Effect sensor A1324 from Allegro ©. Originally this kit has a digital controller board including a microcontroller, which was replaced with our simple analog controller.

A. Mathematical Dynamic Model

In our system, as shown in Fig. 1, a Hall sensor is attached beneath an electromagnet. Define d as a distance between the sensor and the mass center of a floating magnet ball. The force f_m between the electromagnet and a magnetic ball is given as

$$f_m = k\,\frac{i}{d^4} \qquad (1)$$

where $i(t)$ denotes the current across the electromagnet and k is a constant.

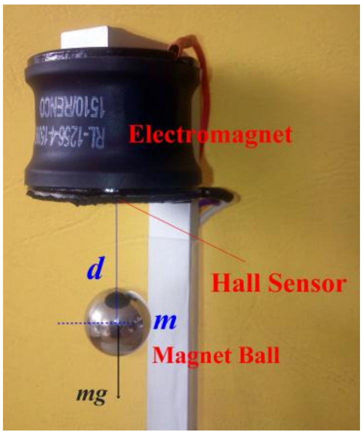

Fig. 1. Magnetic Levitation Plant

From a force balancing equation, we have

$$m\frac{d^2}{dt^2}[d(t)] = mg - k\,\frac{i(t)}{d^4}, \qquad (2)$$

In addition, an electrical dynamics of the electromagnet can be expressed by

$$v(t) = Ri(t) + L\,\frac{di(t)}{dt}, \qquad (3)$$

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where R, L are resistance and inductance of the electromagnet.

Now consider the following perturbations

$$\begin{aligned} i(t) &= i_e + \Delta i(t), \\ d(t) &= d_e + \Delta d(t), \\ v(t) &= v_e + \Delta v(t). \end{aligned} \tag{4}$$

Under this perturbation, the dynamics (2) and (3) around an operating point (i_e, d_e, v_e) can be linearized as

$$\begin{aligned} m\frac{d^2}{dt^2}(\Delta d) &= -\frac{k}{d_e^4}\Delta i + \frac{4ki_e}{d_e^5}\Delta d, \\ \frac{d\Delta i}{dt} &= -\frac{R}{L}\Delta i + \frac{1}{L}\Delta v. \end{aligned} \tag{5}$$

After eliminating Δi in (5) and applying Laplace transforms, we obtain the transfer function from Δv to Δd given as

$$\frac{\Delta D(s)}{\Delta V(s)} = \frac{-\frac{gR}{v_e}}{(Ls+R)\left(s^2 - \frac{4ki_e}{md_e^5}\right)} \tag{6}$$

where $\Delta V(s)$ and $\Delta D(s)$ denote the Laplace transforms of $\Delta v(t)$ and $\Delta d(t)$, respectively.

Our Hall sensor has a voltage output of the form

$$z(t) = \alpha + \frac{\beta}{d^2} + \gamma\, i(t) \tag{7}$$

where α, β, γ are constant sensor parameters. A linearization of (7) around $z(t)=z_e + \Delta z$ results in

$$\Delta z = -\frac{2\beta}{d_e^3}\Delta d + \gamma\, \Delta i. \tag{8}$$

Applying Laplace transform to (8) and using $\Delta I(s) = \Delta V(s)/(Ls+R)$ from (3) and the representation in (6), we obtain a relation between the electromagnet voltage $\Delta V(s)$ and a sensor voltage perturbation $\Delta Z(s)$ as follows;

$$\frac{\Delta Z(s)}{\Delta V(s)} = \frac{\gamma\left(s^2 - \frac{4ki_e}{md_e^5}\right) + \frac{2\beta k}{md_e^7}}{(Ls+R)\left(s^2 - \frac{4ki_e}{md_e^5}\right)} \tag{9}$$

From (6) and (9), we can describe our levitation system as a block diagram shown in Fig. 2 where "Ref" denotes a reference sensor voltage and an *open loop transfer function* is given as

$$G(s)H(s) = \frac{\Delta Z(s)}{\Delta V(s)} = \frac{\gamma\left(s^2 - \frac{4ki_e}{md_e^5}\right) + \frac{2\beta k}{md_e^7}}{(Ls+R)\left(s^2 - \frac{4ki_e}{md_e^5}\right)}$$

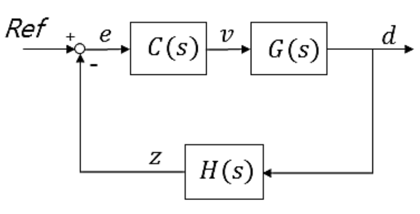

Fig. 2. Block Diagram

TABLE I. SYSTEM PARAMETERS

	parameter	value	unit
Sensor	β	2.92	V
	γ	0.48	V/A
Electro -magnet	k	17.31×10^{-9}	kgm^5/s^2A
	R	2.6	Ω
	L	15.0×10^{-3}	H
Operating Points	i_e	0.41	A
	d_e	27.0	mm

By substituting system parameters in Table I into (9), we finally obtain an explicit representation

$$G(s)H(s) = \frac{31.94\, s^2 + 1888}{s^3 + 173\, s^2 - 108.4\, s - 1.875 \times 10^4} \tag{10}$$

Some parameters in Table I are taken from a technical note of Zeltom© [7].

III. CONTROLLER DESIGN

Firstly we note from the root locus plot in Fig. 3 that our open loop transfer function $G(s)H(s)$ cannot be stabilized with a pure proportional controller $C(s) = k_p \in \mathbb{R}$ in Fig. 2.

Motivated by this fact and, at the same time, in order to minimize the complexities of a controller synthesis and its implementation, we prefer the simplest stabilizing controller. It turns out that a standard PD (proportional and derivative) controller can do the work. The same controller structure was also proposed in [5].

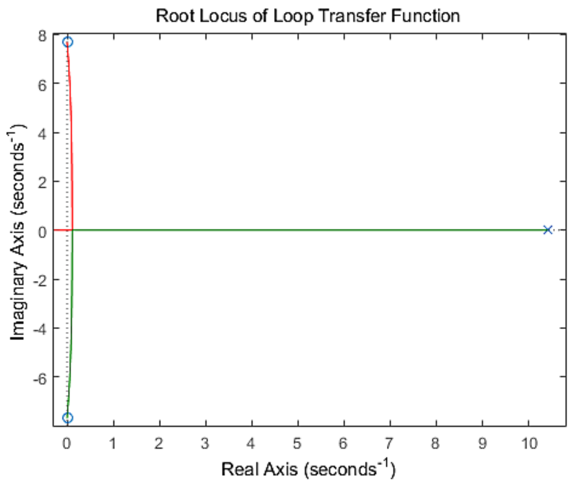

Fig. 3. Open Loop Root Locus

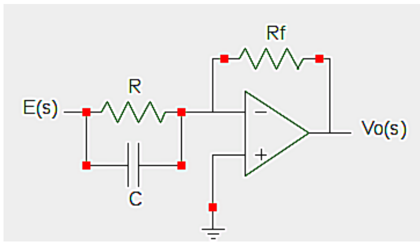

Fig. 4. PD controller

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Our PD controller has the structure shown in Fig. 4 where $E(s)$ denote the Laplace transform of an error signal between a reference voltage "Ref" and an actual sensor signal $z(t)$.

The transfer function of the PD controller is given as

$$C(s) := \frac{V(s)}{E(s)} = -\frac{R_f}{R}\,(RC\,s + 1) \qquad (11)$$

The minus sign in the controller (11) is to be cancelled out because our differential circuit to obtain $\Delta E(s)$ also has a minus sign.

When $R = 5\,k\Omega$ and $C = 10\,\mu F$, for an example, the controller (11) becomes

$$C(s) := -k_c\,(0.05\,s + 1). \qquad (12)$$

The controller gain k_c is a tuning parameter determined by R_f in (11). The root locus of a transfer function $(0.05\,s + 1)G(s)H(s)$ is shown in Fig. 5 for $1 \le k_c \le \infty$. It follow from Fig. 5 that the closed loop system is stable provided that $k_c \ge 8.94$ holds.

A circuit diagram of our analog controller is given in Fig. 6 and a breadboard implementation is shown in Fig. 7. Note that we have ignored the dynamic behaviors of a Darlington transistor TIP102 in the above controller design as we have experimentally verified that the voltage across the electromagnet is almost same as the controller voltage.

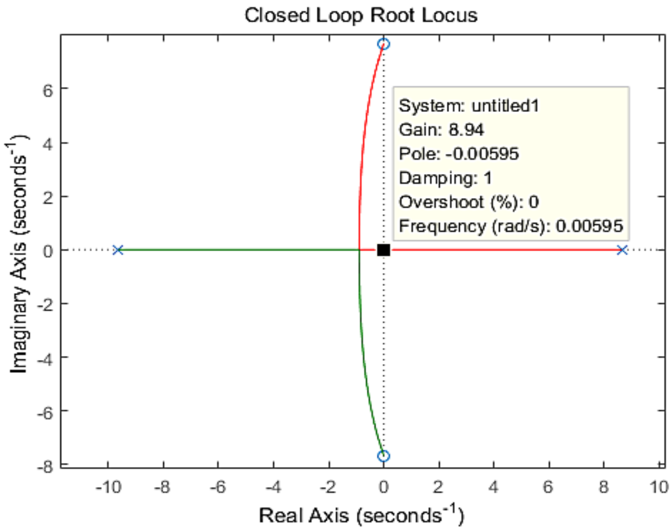

Fig. 5. Closed Loop Root Locus

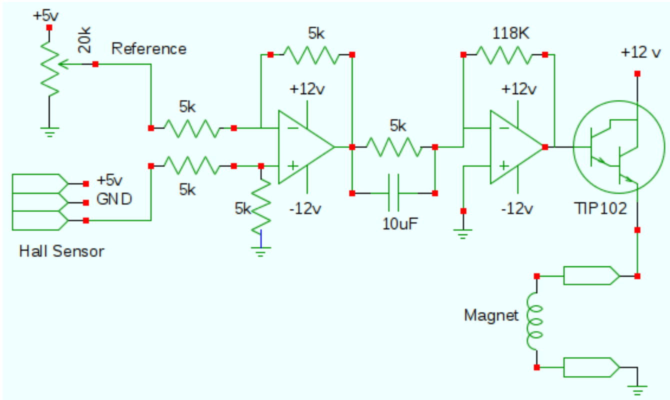

Fig. 6. Overall Circuit Diagram

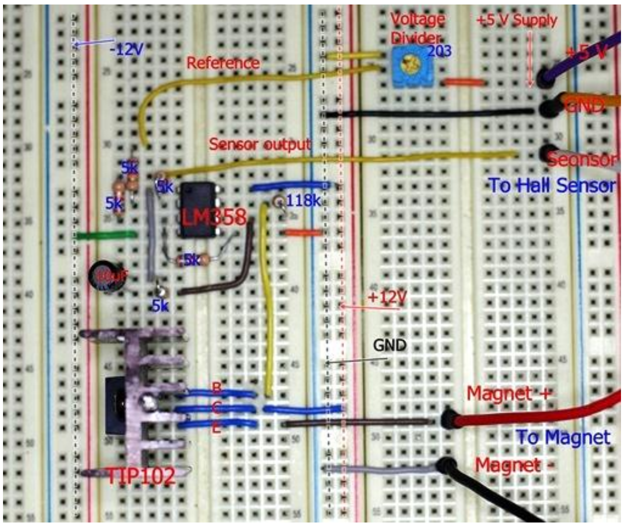

Fig. 7. Analog Controller Implementation

IV. EXPERIMENTS

By changing controller parameters with different values of resistor R_f and capacitor C in (11), we have verified the performance of our closed loop levitation system.

In overall, for wide ranges of R_f and C, our magnetic levitation system remains stable as theoretically predicted from Fig. 5. An example with of $R_f = 118\,k\Omega$ and $C = 10\,\mu F$ is shown in Fig. 8. The closed system in this case has a gain margin of infinity and a phase margin of 31.6° (deg).

With a fixed $R_f = 118\,k\Omega$, the magnitude of capacitor C can be increased up to $470\,\mu F$, not breaking a stable levitation. Similarly, with a fixed $C = 10\,\mu F$, the resistance R_f can be increased up to several hundred $k\Omega$.

A more interesting observation is that, as decreasing the capacitor C down to zero, the levitating magnet ball shows larger vibrations and when $C = 0\,\mu F$ a stable levitation could not achieved.

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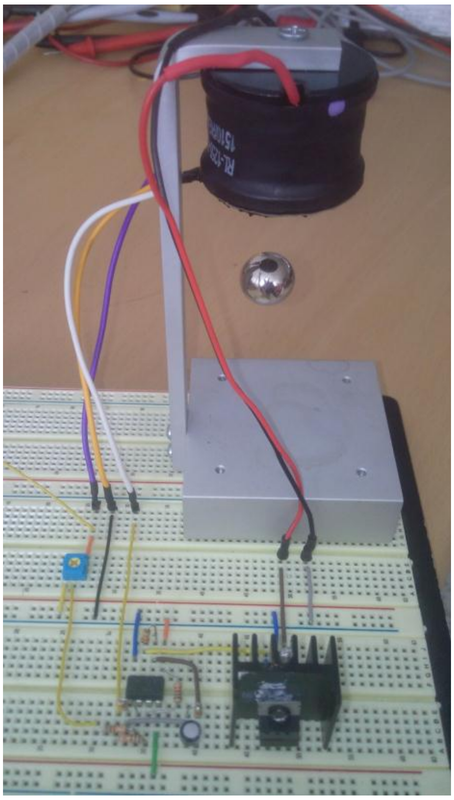

Fig. 8. A Levitation Test

These observations are well consistent with our theoretical predictions from the root locus in Fig. 5 of the previous sections.

Finally, note that by changing the reference voltage with a potentiometer (trimmer) in Fig. 6 and Fig. 7, the equilibrium position of a levitating magnet ball can be raised or lowered.

V. CONCLUSION

We have shown that a simple PD analog controller can stabilize an electromagnetic levitation kit equipped with a Hall Effect sensor. Systematic procedures for system modeling, a controller synthesis and an implementation with an operational amplifier are discussed. From experiments we have confirmed that our simple controller can provide a good performance as theoretically predicted.

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