A separate Exponential Ratio- type Estimator of finite Population Mean under Power Transformation

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***Abstract*: In this paper, a separate ratio-type exponential estimator for estimating the finite population mean has been proposed. Mathematical expressions for the bias and mean square error (MSE) of the proposed estimator have been derived to the first order of approximation. Theoretical conditions have been obtained under which the proposed estimator is more efficient than the estimators under study. Numerical illustration has also been carried out to compare the efficiency of proposed estimator and found that the proposed estimator was more efficient.**

Keywords Bias, Exponential estimator, Mean square error, Separate estimator, Stratified sampling.

1. INTRODUCTION

The problem of estimation of population parameters has been an important issue in sample survey and many methods have been used in order to improve the efficiency of the estimators. In survey sampling, it is well established that the use of auxiliary information results in substantial gain in efficiency over the estimators which do not use such information. In some cases, in addition to mean of auxiliary variable, various other parameters related to auxiliary variable such as coefficient variation, correlation coefficient etc are used to estimate the population parameter. Sisodia and Dwivedi (1981), Rao (1991), Upadhayaya and Singh (1999), developed various estimators to improve the ratio estimators in simple random sampling. Kadilar and Cingi (2003) modified the various estimators under stratified random sampling. Bahl and Tuteja (1991) introduced ratio and product-type exponential estimators which perform better than the classical ratio and product estimators respectively. Singh *et al.* (2008) proposed a ratio and product-type exponential estimators which were more efficient than the Bahl and Tuteja (1991) estimators. Upadhyaya *et al.* (2011), Singh and Ahmed (2014), Singh and Ahmed (2015a, 2015b), Singh (2016) did remarkable work in this direction. Singh *et al.* (2018) suggested combined ratio-type exponential estimator of population mean which was equally efficient as combine linear regression estimator.

2. NOTATIONS

Let $U=\{U_1,.....,U_N\}$ be a finite population of size N which are partitioned into K distinct strata with i^{th} stratum containing N_i units $(i=1,2.....,k)$ such that $\sum_{i=1}^{K} N_i = N$. Let a sample of size n_i units $(i=1,2,....,k)$ be drawn from the population using simple random sampling preferably without replacement (SRSWOR). Such that $\sum_{i=1}^{k} n_i = n$.

Let (y_{ij}, x_{ij}) be the observed values of (Y, X) on the j^{th} unit of the i^{th} stratum $(j=1,2,...,N_i)$ Moreover, the population means of the variables Y and X in the ith stratum are $\bar{Y}_i = \frac{1}{N_i}\sum_{j=1}^{N_i} Y_{ij}, \bar{X}_i = \frac{1}{N_i}\sum_{j=1}^{N_i} X_{ij}$ and the corresponding sample means of the variable Y and X in the i^{th} stratum are $\bar{y}_i = \frac{1}{n_i}\sum_{i=1}^{n_i} y_{ij}, \bar{x}_i = \frac{1}{n_i}\sum_{i=1}^{n_i} x_{ij}$ respectively.

Estimators of the variable Y and X, in stratified random sampling are given by;

$\bar{y}_{st} = \sum_{i=1}^{k} W_i \bar{y}_i$ and $\bar{x}_{st} = \sum_{i=1}^{k} W_i \bar{x}_i$ are the unbiased estimators of the population means $\bar{Y} = \sum_{i=1}^{k} W_i \bar{Y}_i$ and $\bar{X} = \sum_{i=1}^{k} W_i \bar{X}_i$ respectively. Where $W_i = \frac{N_i}{N}$ denotes the stratum weight.

Let ρ be the correlation coefficient between the study variable and the auxiliary variable.

3. EXISTING ESTIMATORS

The separate ratio estimator for the population mean $\bar{Y}$ is defined as;

$$\bar{y}_{sr} = \sum_{i=1}^{k} W_i \left(\frac{\bar{y}_i}{\bar{x}_i}\right) \bar{X}_i \quad (1)$$

and its bias and MSE are given as;

$$B\left(\bar{y}_{sr}\right)=\sum_{i=1}^{k}W_i\theta_i\frac{1}{X_i}\left(R_iS_{xi}^2-S_{YXi}\right)$$

$$MSE\left(\bar{y}_{sr}\right)=\sum_{i=1}^{k}W_i^2\theta_i\left(S_{yi}^2+R_i^2S_{xi}^2-2R_iS_{YXi}\right)\quad(2)$$

The separate product estimator for the population mean $\bar{Y}$ is define as;

$$\bar{y}_{sp}=\sum_{i=1}^{k}W_i\left(\frac{\bar{y}_i.\bar{x}_i}{\bar{X}_i}\right)\quad(3)$$

and its Bias and MSE are given as;

$$B\left(\bar{y}_{sp}\right)=\sum_{i=1}^{k}W_i\theta_i\frac{1}{X_i}S_{YXi}$$

$$MSE\left(\bar{y}_{sp}\right)=\sum_{i=1}^{k}W_i^2\theta_i\left(S_{yi}^2+R_i^2S_{xi}^2+2R_iS_{YXi}\right)\quad(4)$$

The separate linear regression estimator for the population mean $\bar{Y}$ is defined as;

$$\bar{y}_{sl}=\sum_{i=1}^{k}W_i\left[\bar{y}_i+b_i\left(\bar{X}_i-\bar{x}_i\right)\right]\quad(5)$$

And its MSE is given as;

$$MSE(\bar{y}_{sl})=\sum_{i=1}^{k}W_i^2\theta_iS^2_{Yi}\left(1-\rho^2_{YXi}\right)\quad(6)$$

4. ADAPTED ESTIMATORS

Bahl and Tuteja (1991) ratio-type exponential estimator for the population mean $\bar{Y}$ under stratified random sampling can be defined as:

$$\bar{y}_{stBTR}=\sum_{i=1}^{k}W_i\bar{y}_i\exp\left(\frac{\bar{X}_i-\bar{x}_i}{\bar{X}_i+\bar{x}_i}\right)\quad(7)$$

its bias and MSE are given as;

$$Bais\left(\bar{y}_{stBTR}\right)=\sum_{i=1}^{k}W_i\theta_i\frac{1}{\bar{X}_i}\left(\frac{3}{8}R_iS_{Xi}^2-\frac{1}{2}S_{XYi}\right)$$

$$MSE\left(\bar{y}_{stBTR}\right)=\sum_{i=1}^{k}W_i^2\theta_i\left(S_{yi}^2+\frac{1}{4}R_i^2S_{xi}^2-R_iS_{YXi}\right)\quad(8)$$

Bahl and Tuteja (1991) product-type exponential estimator for the population mean $\bar{Y}$ under stratified random sampling can be defined as:

$$\bar{y}_{stBTP}=\sum_{i=1}^{k}W_i\bar{y}_i\exp\left(\frac{\bar{x}_i-\bar{X}_i}{\bar{x}_i+\bar{X}_i}\right)\quad(9)$$

and bias and MSE are given as;

$$Bais\left(\bar{y}_{stBTP}\right)=\sum_{i=1}^{k}W_i\theta_i\frac{1}{\bar{X}_i}\left(\frac{1}{2}S_{XYi}-\frac{1}{8}R_iS_{Xi}^2\right)$$

$$MSE\left(\bar{y}_{stBTP}\right)=\sum_{i=1}^{k}W_i^2\theta_i\left(S_{yi}^2+\frac{1}{4}R_i^2S_{xi}^2+R_iS_{YXi}\right)\quad(10)$$

Singh *et al* (2018) estimator

A combined ratio-type exponential estimator for population mean under stratified random sampling using information on single auxiliary variable has been suggested by Singh *et al* (2018) given as:

$$\bar{y}_S=\bar{y}_{st}\left[\frac{\bar{x}_{st}}{\bar{X}}\right]^{\alpha}\exp\left[\frac{\bar{X}-\bar{x}_{st}}{\bar{X}+\bar{x}_{st}}\right]$$ where α is a constant. (11)

Bias and MSE of t_S are obtained as

$$B\left(\bar{y}_S\right)=\bar{Y}\left[\frac{(2\alpha-1)}{2}\frac{1}{\bar{X}\bar{Y}}COV\left(\bar{y}_{st}\bar{x}_{st}\right)+\left(\frac{4\alpha^2-8\alpha+3}{8}\right)\frac{1}{\bar{X}^2}V\left(\bar{x}_{st}\right)\right]$$

$$MSE\left(\bar{y}_S\right)=\sum_{i=1}^{k}W_i^2\theta_i\left\{\begin{matrix}S_{Yi}^2+\frac{(2\alpha-1)^2}{4}R^2S_{Xi}^2\\+(2\alpha-1)RS_{YXi}\end{matrix}\right\}$$

For the optimum value of α, the minimum MSE of t_S is obtained as

$$MSE\left(\bar{y}_S\right)=\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)W_i^2S^2_{Yi}\left(1-\rho^{*2}_{YX}\right)\quad(12)$$

That is the MSE of combined linear regression estimator.

5. THE SUGGESTED ESTIMATOR

In stratified random sampling, we suggest a separate ratio-type exponential estimator as

$$t_P=\sum_{i=1}^{K}W_i\bar{y}_i\left(\frac{\bar{x}_i}{\bar{X}_i}\right)^{\alpha_i}\exp\left(\frac{\bar{X}_i-\bar{x}_i}{\bar{X}_i+\bar{x}_i}\right)\quad(13)$$

where α_i is a constant.

6. BIAS AND MEAN SQUARE ERROR OF THE SUGGESTED ESTIMATOR

In order to obtain the Bias and MSE to the first order of approximation, let us define

$\bar{y}_i=\bar{Y}_i(1+e_{0i})$ and $\bar{x}_i=\bar{X}_i(1+e_{1i})$

Therefore

$$\left.\begin{aligned} &E(e_{0i}) = E(e_{1i}) = 0 \\ &E(e_0^2) = \left(\frac{1}{n_i} - \frac{1}{N_i}\right)\frac{S_{Yi}^2}{\bar{Y}_i^2} \\ &E(e_{1i}^2) = \left(\frac{1}{n_i} - \frac{1}{N_i}\right)\frac{S_{Xi}^2}{\bar{X}_i^2} \\ &E(e_{0i}e_{1i}) = \left(\frac{1}{n_i} - \frac{1}{N_i}\right)\frac{S_{YXi}}{\bar{Y}_i\bar{X}_i} \end{aligned}\right\} \quad (15)$$

Now, by substituting the values of $\bar{x}_i$ and $\bar{y}_i$ from (14) into (13) and on solving, the bias of proposed estimator is obtained as;

$$Bais\left(t_P\right) = \sum_{i=1}^{K} W_i \left(\frac{1}{n_i} - \frac{1}{N_i}\right)\frac{1}{\bar{X}_i} \left\{\left(\frac{4\alpha_i^2 - 8\alpha_i + 3}{8}\right) R_i S_{Xi}^2 + \left(\frac{2\alpha_i - 1}{2}\right) S_{YXi}\right\} \quad (16)$$

Now, MSE of the proposed estimator t_P can be obtained as;

$$MSE\left(t_P\right) = \sum_{i=1}^{K} W_i^2 \left(\frac{1}{n_i} - \frac{1}{N_i}\right) \left\{S_{Yi}^2 + \frac{\left(2\alpha_i - 1\right)^2}{4} R_i^2 S_{Xi}^2 + \left(2\alpha_i - 1\right) R_i S_{YXi}\right\} \quad (17)$$

To obtain the optimum mean squared error, $\dfrac{\partial \text{MSE}\left[t_P\right]}{\partial \alpha_i} = 0$ gives $\alpha_{i(opt)} = \dfrac{1}{2} - \dfrac{\rho_i C_{Yi}}{C_{Xi}}$

and minimum MSE of t_P is obtained as

$$MSE\left(t_P\right)_{\min} = \sum_{i=1}^{k} \left(\frac{1}{n_i} - \frac{1}{N_i}\right) W_i^2 S_{Yi}^2 \left(1 - \rho_i^2\right) \quad (18)$$

That is the MSE of the separate linear regression estimator.

7. THEORETICAL EFFICIENCY COMPARISON

In this section, conditions have been found under which the proposed estimator is more efficient than existing estimators under study.

i. Comparison of the separate ratio estimator with the proposed estimator

$MSE\left(t_P\right) < MSE\left(\bar{y}_{sr}\right)$ if and only if

$$\min\left[\left(\frac{3}{2} - 2\rho_i \frac{C_{Yi}}{C_{Xi}}\right)_, -\frac{1}{2}\right] \text{(14)} < \alpha_i < \max\left[\left(\frac{3}{2} - 2\rho_i \frac{C_{Yi}}{C_{Xi}}\right)_, -\frac{1}{2}\right] \quad (19)$$

Therefore the proposed estimator is more efficient than $\bar{y}_{sr}$ if condition (19) is satisfied.

ii. Comparison of the separate product estimator with the proposed estimator

$MSE\left(t_P\right) < MSE\left(\bar{y}_{sp}\right)$ if

$$\min\left[\frac{3}{2}, -\left(\frac{1}{2} + 2\rho_i \frac{C_{Yi}}{C_{Xi}}\right)\right] < \alpha_i < \max\left[\frac{3}{2}, -\left(\frac{1}{2} + 2\rho_i \frac{C_{Yi}}{C_{Xi}}\right)\right] \quad (20)$$

Therefore the proposed estimator is more efficient than $\bar{y}_{sp}$ if condition (20) is satisfied.

iii. Comparison of the Bahl and Tuteja (1991) ratio-type exponential estimator under stratified random sampling with the proposed estimator

$MSE\left(t_P\right) < MSE\left(\bar{y}_{stBTR}\right)$ if

$$\min\left(0, 1 - 2\rho_i \frac{C_{Yi}}{C_{Xi}}\right) < \alpha_i < \max\left(0, 1 - 2\rho_i \frac{C_{Yi}}{C_{Xi}}\right) \quad (21)$$

Therefore the proposed estimator is more efficient than $\bar{y}_{stBTR}$ if condition (21) is satisfied.

iv. Comparison of the Bahl and Tuteja (1991) product-type exponential estimator under stratified random sampling with the proposed estimator

$MSE\left(t_P\right) < MSE\left(\bar{y}_{stBTP}\right)$ if

$$\min\left(1, -2\rho_i \frac{C_{Yi}}{C_{Xi}}\right) < \alpha_i < \max\left(1, -2\rho_i \frac{C_{Yi}}{C_{Xi}}\right) \quad (22)$$

Therefore the proposed estimator is more efficient than $\bar{y}_{stBTP}$ if condition (22) is satisfied.

These Theoretical conditions obtained above have been supported numerically in table 2 and found to be satisfied.

8. NUMERICAL EFFICIENCY COMPARISON

For numerical illustration the data of Kadilar and Cingi (2003) is used which is given in table 1. Y represents apple production amount (as variable of interest) and X represents number of apple trees (as auxiliary variable) in 854 villages of Turkey in 1999 (Source: Institute of Statistics, Republic of Turkey). The data is stratified by regions of Turkey and from each stratum (region) the samples (villages) are selected by using Neyman allocation.

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Table 1: Data Statistics

	N_i	n_i	$\overline{X}_i$	$\overline{Y}_i$	S_{Xi}	S_{Yi}	ρ_i	θ_i	W_i^2
Stratum1	106	9	24375	1536	49189	6425	0.82	0.102	0.015
Stratum 2	106	17	27421	2212	57461	11552	0.86	0.049	0.015
Stratum 3	94	38	72409	9384	160757	29907	0.90	0.016	0.012
Stratum 4	171	67	74365	5588	285603	28643	0.99	0.009	0.04
Stratum 5	204	7	26441	967	45403	2390	0.71	0.138	0.057
Stratum 6	173	2	9844	404	18794	946	0.89	0.006	0.041

Table 2: Theoretical comparison supported numerically using data set provided in Table 1

$MSE(t_P) < MSE(\overline{y}_{sr})$	α	α_1 = -1.1997	-1.8994 < -1.1997 < -0.5	Satisfied
		α_2 = -1.6433	-2.7866 < -1.6433 < -0.5	Satisfied
		α_3 = -0.7920	-1.0839 < -0.7920 < -0.5	Satisfied
		α_4 = -0.8213	-1.1426 < -0.8213 < -0.5	Satisfied
		α_5 = -0.5220	-0.5439 < -0.5220 < -0.5	Satisfied
		α_6 = -0.5916	-0.6831 < -0.5916< -0.5	Satisfied
$MSE(t_P) < MSE(\overline{y}_{sp})$	α	α_1 = -1.1997	-3.8994 < -1.1997 < 1.5	Satisfied
		α_2 = -1.6433	-4.7866 < -1.6433 < 1.5	Satisfied
		α_3 = -0.7920	-3.0839 < -0.7920 < 1.5	Satisfied
		α_4 = -0.8213	-3.1426 < -0.8213 < 1.5	Satisfied
		α_5 = -0.5220	-2.5439 < -0.5220 < 1.5	Satisfied
		α_6 = -0.5916	-2.6831 < -0.5916 < 1.5	Satisfied
$MSE(t_P) < MSE(\overline{y}_{stBTR})$	α	α_1 = -1.1997	-2.3994 < -1.1997 < 0	Satisfied
		α_2 = -1.6433	-3.2866 < -1.6433 < 0	Satisfied
		α_3 = -0.7920	-1.5839 < -0.7920 < 0	Satisfied
		α_4 = -0.8213	-1.6426 < -0.8213 < 0	Satisfied
		α_5 = -0.5220	-1.0439 < -0.5220 < 0	Satisfied
		α_6 = -0.5916	-1.1831 < -0.5916 < 0	Satisfied
$MSE(t_P) < MSE(\overline{y}_{stBTP})$	α	α_1 = -1.1997	-3.3994 < -1.1997 < 1	Satisfied
		α_2 = -1.6433	-4.2866 < -1.6433 < 1	Satisfied
		α_3 = -0.7920	-2.5839 < -0.7920 < 1	Satisfied
		α_4 = -0.8213	-2.6426 < -0.8213 < 1	Satisfied
		α_5 = -0.5220	-2.0439 < -0.5220 < 1	Satisfied
		α_6 = -0.5916	-2.1831 < -0.5916 < 1	Satisfied

Table 3: MSE and PRE of the Proposed Estimator with other estimators

ESTIMATORS	MSE	PRE
$\bar{y}_{st}$	673477.7	100
$\bar{y}_{sr}$	159137.4	423.2
$\bar{y}_{sp}$	1790757	37.6
$\bar{y}_{stBTR}$	340940	197.5
$\bar{y}_{stBTP}$	1156750	58.2
$\bar{y}_{S}$	202122.1	333.2
t_p	**107065.8**	**629**

9. CONCLUSION

In the present study, Singh et al (2018) estimator has been improved by modifying it as separate ratio-type exponential estimator and its properties have been studied. Theoretical conditions have been obtained under which the proposed estimator is more efficient than the estimators under study and supported numerically as mentioned in table 2. Table 3 reveals that the proposed estimator t_p has smallest mean square than the conventional ratio, product and Bahl and tuteja (1991) estimators under stratified random sampling as well as Singh et al (2018) estimator. Therefore, t_p is more efficient than the other existing estimators for estimating the finite population mean.

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