International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 9 Issue 10, October-2020

A Selection of Prior Binomial, Two Point Prior Binomial and Gamma Poisson based on Lot Size indexed by Average Lot Quality, Limiting Quality Level & Acceptance Quality Level

G Uma* & M Nirmala**
*& **Assistant Professor, PSG College of Arts & Science,
Coimbatore, TN, India

Abstract:- **The theory of prior distributions is connected with theory of process control. Bayesian methods try to incorporate prior process knowledge to account for the variation in the sampling scheme. Though most of the acceptance sampling literature is based on classical methodology, there have been a few attempts to make the Bayesian Paradigm to explain acceptance sampling techniques, most notably by Hald.A (1981). The basic idea behind the Bayesian approach for calculating the sample size is to use prior information about the parameter 'p'. The Prior distribution is a key part of Bayesian Sampling plan to represent the information about an uncertain parameter. This chapter gives the study on Two Point Prior Binomial distribution and is used as a baseline distribution for designing the Single Sampling Plan to determine the average acceptance and rejection cost with AQL and LQL. Necessary Operating Characteristic curves are sketched by highlighting with illustrations.**

Key Words:- Prior Binomial, Two Point Prior Binomial, Gamma Poisson, AQL, LQL, ALQ and OC

INTRODUCTION

The basic assumption underlying the theory of conventional sampling plans by attributes is that the lot or process fraction nonconforming is a constant, which intensively means that the production process is stable. However, in practice, the lots of products produced from a process may have quality variations due to random fluctuations. The variations in the lots can be separated into two namely within - lot and between – lot variations. When the between – lot variations more than the within – lot variations, the proportion of nonconforming units in the lots will vary continuously. In such cases, the decision on the submitted lots should be made with the consideration of the between – lot variations and hence the conventional sampling schemes cannot be employed.

A complete statistical model for basic sampling inspection contains three components

1. The prior distribution is the expected distribution of submitted lots according to quality.
2. The cost of sampling inspection, acceptance and rejection.
3. A class of sampling plan that usually defined by means of the restriction designed to give a production against acceptance lot of poor quality.

The Cost

Most of the research studies concentrated on sampling plans based on prior distribution and costs design by assuming that costs for guarantee and repair as well as marginal costs for sampling and rejecting an item.

In this case the main factors are inspecting and rejecting lots, passing defective items, extra costs for repairing defective items are not considered. Since all the defective items have to be repaired either during the stage of sampling or when they are detected by the customer; by keeping the costs as constant and therefore it is irrelevant for our purposes. If there is a difference in the repair costs it will be integrated in the costs for passing defective items.

Prior Binomial Distribution

The Binomial distribution is frequently used to model the number of successes in a sample size 'n' drawn with replacement from a population of size 'N'. For larger 'n', the binomial distribution is a good approximation and widely used in a manufacturing process.

The probability of observing 'x', x = 0, 1, 2….n, non conforming units among the 'n' units of the sample is approximated by the prior binomial distribution. The probability density function is given by 0

$\mathrm{p(x: n, p) = nc_x\, p^x\, q^{n-x}};\quad \mathrm{x = 0,1 \ldots \ldots \ldots n.}\ 0 < \mathrm{p} < 1, q = 1 - \mathrm{p}$ (1)

Conditions of Application

Binomial distribution has the following four conditions:

1. The experiment consists of 'n' identical trials.
2. Each trial results in one of the two outcomes, called success and failure.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 9 Issue 10, October-2020

3. The probability of success, denoted ‘p’, remains the same from trial to trial.
4. The ‘n’ trials are independent.

If the above four conditions are satisfied, then the random variable X is a number of successes in ‘n’ trials is a binomial distribution with basic characteristics such as, Mean, Standard Deviation and Variance.

$$E(X) = np$$
$$Var(X) = np(1-q)$$
$$SD(X) = \sqrt{np(1-p)}$$

Develop and Design the Plan of Single Sampling Plan for Attributes Based on Prior Binomial Distribution for Average Acceptance Cost

The Linear Cost Model

The linear cost function is defined based on the study of Hald (1981) for acceptance sampling by attributes for a single sampling plan, the product of quality $\bar{p}_i$ i.e., the product coming from product i (i=1, 2,3,…..k).

$$\left.\begin{array}{l} n_i(S_{1i}+S_{2i}p_i)+(N_i-n_i)(A_{1i}+A_{2i}p_i) - \text{ lot is accepted with probability} P(p_i) \\ n_i(S_{1i}+S_{2i}p_i)+(N_i-n_i)(R_{1i}+R_{2i}p_i) - \text{ lot is rejected with probability } 1-P(p_i) \end{array}\right\} \quad (2)$$

The values of the constant can be found using the suggestions given by Hald.A (1960)

S_{1i}	-	Cost per item of sampling and testing.
S_{2i}	-	Repair Cost for a defective item found in sampling.
A_{1i}	-	Cost per item associated with handling the (N_i-n_i) items not inspected in an accepted lot (frequently is zero).
A_{2i}	-	Cost associated with a defective item which is accepted (may be quite large).
R_{1i}	-	Cost per item of inspecting the remaining (N_i-n_i) items in a rejected lot.
R_{2i}	-	Repair cost associated with a defective item in the remaining (N-n) items of a rejected lot.

Logically expect that, $S_1 \geq R_1$ and $S_2 \geq R_2$ (with equality frequently holding) since it should be no more expensive to sample or repair on a large scale on a small scale.

The main aim of this research insist on finding optimum single sampling plans (n,c) based on prior binomial distribution by minimizing the average acceptance cost K(N,n,c,p), subject to the condition that A_2, the cost associated with a defective items is minimum.

$$P_a(p_1) \geq 1-\alpha \quad (3)$$

$$\left.\begin{array}{c} P_a(p_2) \leq \beta \\ \text{(or)} \\ P_a(p_2) \geq 1-\beta \end{array}\right\} \quad (4)$$

Where p_1 = Quality level corresponding to the producer’s risk which is called Acceptance Quality Level (AQL)

p_2 = Quality level corresponding to the consumer’s risk called Limiting Quality Level (LQL)

$$K(N,n,c,p) = n + (N-n)\{[(A_1-R_1)+(A_2-R_2)p]P(p)+(R_1+R_2p)\} \quad (5)$$

$$K_s(p) = S_1 + S_2p, K_a(p) = A_1 + A_2p, K_r(p) = R_1 + R_2p \quad (6)$$

Operating Procedure

The following procedure is used to determine the optimum values of Single Sampling Plan (n, c) for the various values of the constant and is constructed with the following steps:

Step -1 : Assume the acceptance number c=0.

Step -2 : Define the lot size N and other constants S_1, S_2, A_1, A_2, R_1, R_2.

Step-3: Let A_2=31, minimum cost and initial stage is obtained with the minimum average acceptance cost K(N,n,c) subject to the condition A_2 is small.

Step-4: Identify a Single Sampling Plan (n, c) corresponding to the minimum average acceptance cost K(N, n, c)

Step-5: Repeat the step 1 by increasing the value of c (= 1, 2,…,8) and obtain SSP(n, c) based on minimum average acceptance cost for various values of N

Two Point Prior Binomial Distribution

Two Point Prior Binomial Distribution was given by Hald [6], [7], [8], [9] having the probability density function defined as:

$$f(p) = \left.\begin{aligned} &= w_{1,} \quad p = p_1 \\ &= w_{2,} \quad p = p_1 \end{aligned}\right\} \tag{7}$$

Where $w_2 = 1 - w_1$ and w's and p's are to be known.

Properties of Two Point Prior Binomial Distribution

1. If X is a random variable then $P_r(X=1) = p = 1\text{-}P_r(X=1) = 1$
2. The probability mass function of this distribution is
   $$\left.\begin{aligned} f(k;p) &= p, & \text{if } k = 1 \\ &= q = (1-p) & \text{if} k = 0 \end{aligned}\right\}$$
   This can also be written as
   $$f(k;p) = p^k (1-p)^{1-k} \text{ for } k \in \{0,1\} \tag{8}$$
3. The mean of the distribution is $E(x) = p$
4. The variance of the distribution is $v(x) = pq$
5. The skewness of the distribution is $\gamma_1 = \frac{q-p}{\sqrt{pq}}$
6. The kurtosis of the distribution is $\gamma_2 = \frac{6p^2-6p+1}{p(1-p)}$
7. The central moment of order k is given by $\mu_k = (1-p)(1-p)^k + p(1-p)^k$

Developing and Designing the Plan of Single Sampling Plan for Attributes Based on Two Point Prior Binomial Distribution on Acceptance Cost K(N,n,c,p)

The main aim of this research insist on finding optimum single sampling plans (n,c) based on two point prior binomial distribution by minimizing the average acceptance cost k(N,n,c,p), subject to the condition that A_2, the cost associated with a defective items is minimum.

$$P_a(p_1) \geq 1 - \alpha \tag{9}$$

$$\left.\begin{aligned} &P_a(p_2) \leq \beta \\ &\quad (\text{or}) \\ &P_a(p_2) \geq 1 - \beta \end{aligned}\right\} \tag{10}$$

Where p_1 = Quality level corresponding to the producer's risk which is called Acceptance Quality Level (AQL)

p_2 = Quality level corresponding to the consumer's risk which is called Limiting Quality Level (LQL)

$$K(N,n,c,p) = n + [N-n][\gamma_1 w_1 Q(p_1) + \gamma_2 w_2 Q(p_2)] \tag{11}$$

$$\gamma_1 = W_1[K_r(p_1) - K_a(p_1)/K_s - K_m]$$

$$\gamma_2 = W_2[K_a(p_2) - K_r(p_2)/K_s - K_m]$$

Where $k_S = w_1 k_s(p_1) + w_2 k_s(p_2)$, $\quad k_m = w_1 k_a(p_1) + w_2 k_r(p_2)$

In designing and modeling sampling plans under Bayesian theory, the two point prior binomial distribution is considered. Tables from 4.2.1 to 4.2.20 are constructed to obtain the optimum single sampling plan with two point prior distribution. The main objective is to minimize the cost function cost function $K(N_i,n_i,c_i)$ with respect to A_i (i = 1,2,3,…k). The cost such as replacement, cost of handling defective items in assembling and reassembling in the damage parts and costs of renewed testing and inspection procedures are considered in the determination of the average cost regarding both acceptance and rejection cost for the AQL and LQL.

Gamma – Poisson Prior distribution

The Gamma – Poisson distribution with parameter $\bar{p}$ and the shape parameter m.

$$p(d{:}\, n\bar{p},\, m) = \frac{(m+d-1)!}{d!\ (m-1)!}\left(\frac{n\bar{p}}{n\bar{p}+m}\right)^d \left(\frac{m}{n\bar{p}+m}\right)^m, d = 0,1, \ldots\ldots\ldots \tag{12}$$

Where $\bar{p}$ is process average; m = scale parameter; d = average number of defects.

The OC function of SSP under the conditions of Gamma-Poisson distribution is then given by

$$P_a(\bar{p}) = \sum_{d=0}^{c} p(d; n\bar{p}, m) \tag{13}$$

Conditions for the Gamma – Poisson Prior distribution

Gamma-Poisson distribution is also called the negative binomial distribution and the conditions are

1. The number of trials, 'n' is not fixed.

2. Each trial is independent.
3. Only two outcomes are possible.
4. Probability of success for each trial is constant.
5. A random variable is equal to the number of trials needed to make ‘r’ successes.

Develop and Design the Plan of Single Sampling Plan for Attributes Based on Gamma - Poisson distribution for Average Acceptance Cost

Bayesian inference concept is applied to design an optimum single sampling plans (n,c) based on Gamma - Poisson distribution in quality control environments. To optimize the cost based on acceptance criteria known as average acceptance cost K(N,n,c,p), satisfying the condition that A_2, the cost related with a defective time is minimum. Hence we have Employed a combination of costs and risk functions and tried to minimize the average acceptance cost K(N,n,c,p), subject to the condition that A_2, the cost associated with a defective items is minimum and satisfy the inequalities

$$P_a(p_1) \geq 1-\alpha \qquad (14)$$

$$\left.\begin{array}{c} P_a(p_2) \leq \beta \\ \text{(or)} \\ P_a(p_2) \geq 1-\beta \end{array}\right\} \qquad (15)$$

Where p_1 = Quality level corresponding to the producer’s risk which is called Acceptance Quality Level (AQL)
p_2 = Quality level corresponding to the consumer’s risk which is called Limiting Quality Level (LQL)
‘c’ is chosen as small as possible and $p_1 < p_2$ and $(1-\alpha) > \beta$

This leads to a uniquely determined value of ‘c’ and an interval for values of ‘n’, and all satisfying the conditions, we get

$$K(N,n,c,p) = n(S_1 + S_2 p) + (N-n)\{[(A_1 - R_1) + (A_2 - R_2)p]P(p) + (R_1 + R_2 p)\} \qquad (16)$$

$$K_s(p) = S_1 + S_2 p\,, K_a(p) = A_1 + A_2 p\,, K_r(p) = R_1 + R_2 p \qquad (17)$$

Where A_1 - Cost per item associated with the handling the (N-n) items not inspected in an acceptance lot. (frequently it is zero)
A_2 - Cost associated with a defective item which is accepted (may be quite large)
R_1 - Cost per item of inspecting the remaining (N-n) items in a rejected lot.
R_2 - Repair cost associated with a defective item in the (N-n) items in a lot.
S_1 - Cost per item of sampling and testing
S_2 - Repair cost for a defective item found in sampling

It presents a complete comparative study of the Bayesian Optimum Single Sampling Plan with the Prior Binomial, Two Point Prior Binomial and Gamma Poisson distributions based on Acceptance Cost and Rejection Cost. Corresponding Operating Curves are presented graphically for ease comparison.

Comparison

In practice that the prior distribution is only vaguely known, it is important to get some information, about how strongly the costs of the acceptance procedure are influenced by a change of prior distributions. For this purpose, the prior distributions with the same value are considered, namely

In the manufacturing environment the choosing the best distribution to apply depends not only on the effectiveness of lot size, sample size but also the average acceptance cost and rejection cost for both the consumer and producer. The following table gives a comparison of minimum average acceptance cost obtained by optimum singe sampling plan using Prior Binomial, Two-point Prior and Gamma- Poisson distributions.

Table 1: Average Acceptance Cost based on Average Lot Quality ‘p’

N	Prior Binomial Distribution				Two Point Prior Distribution					Gamma – Poisson Prior Distribution				C
	n	p	Average Acceptance Cost K(N,n,c,p) based on p	A_2	n	Average lot quality p_1	Average lot quality p_2	Average Acceptance Cost K(N,n,c,p) based on p	A_2	n	P	Average Acceptance Cost K(N,n,c,p) based on p	A_2	
1000	130	0.13	543.3623	37	120	0.01	0.1	195.65	18	150	0.16	681.9907	38	7
2000	170	0.09	791.0084	39	140	0.01	0.1	331.68	20	230	0.1	974.2938	42	7
3000	230	0.08	982.1391	42	140	0.01	0.1	434.73	20	230	0.11	1247.854	42	7
4000	270	0.07	1150.536	44	150	0.01	0.1	575.42	21	270	0.1	1466.887	44	7
5000	290	0.058	1266.214	45	160	0.01	0.1	728.87	22	350	0.07	1592.833	48	7
6000	310	0.05	1424.133	46	160	0.01	0.1	846.41	22	330	0.08	1809.144	47	7
7000	350	0.05	1517.455	48	160	0.01	0.1	963.95	22	370	0.07	1943.712	49	7
8000	370	0.05	1647.144	49	160	0.01	0.1	1081.48	22	390	0.07	2113.329	50	7
9000	390	0.04	1773.968	50	170	0.01	0.1	1267.81	23	390	0.06	2255.109	50	7
10000	430	0.04	1835.806	52	170	0.01	0.1	1392.14	23	450	0.06	2362.465	53	7

From the above table clearly indicates that, irrespective of the lot sizes ‘N’ the average acceptance cost are minimum at c=7, the optimum sample size ‘n’ and the value of A_2, the cost of handling the defective items in assembling and disassembling is also very small in two point prior binomial distribution when compared to the prior binomial and gamma Poisson prior distribution.

Table 2: Average Rejection Cost in Average Lot Quality ‘p’

N	Prior Binomial Distribution				Two Point Prior Distribution					Gamma – Poisson Prior Distribution				C
	N	P	Average rejection Cost R(N,n,c,p) based on p	A_2	N	Average lot quality		Average rejection Cost R(N,n,c,p) based on p	A_2	n	P	Average rejection Cost R(N,n,c,p) based on p	A_2	
						p_1	p_2							
1000	130	0.13	137.368	37	120	0.01	0.1	136.32	18	150	0.16	176.8797	38	7
2000	170	0.09	206.5369	39	140	0.01	0.1	155.14	20	230	0.1	278.5396	42	7
3000	230	0.08	237.2558	42	140	0.01	0.1	163.29	20	230	0.11	280.3289	42	7
4000	270	0.07	276.6515	44	150	0.01	0.1	168.12	21	270	0.1	315.0186	44	7
5000	290	0.058	319.7054	45	160	0.01	0.1	172.77	22	350	0.07	423.5343	48	7
6000	310	0.05	383.9864	46	160	0.01	0.1	175.40	22	330	0.08	396.555	47	7
7000	350	0.05	375.6056	48	160	0.01	0.1	178.04	22	370	0.07	449.1015	49	7
8000	370	0.05	385.4996	49	160	0.01	0.1	180.68	22	390	0.07	458.9592	50	7
9000	390	0.04	484.4058	50	170	0.01	0.1	182.62	23	390	0.06	545.2811	50	7
10000	430	0.04	470.2541	52	170	0.01	0.1	184.05	23	450	0.06	533.9019	53	7

From the above table clearly indicates that, irrespective of the lot sizes ‘N’ the average rejection cost are minimum at c=7, the optimum sample size ‘n’ and the value of A_2, the cost of handling the defective items in assembling and disassembling is also very small in two point prior binomial distribution when compared to the prior binomial and gamma poisson prior distribution.

Based on the chosen optimum sampling plan at c= 7, the average acceptance cost is compared for the lot size ranging from 1000 to 10000 and its corresponding sample sizes with its ’p’ value is given in the table 2. From this one can conclude that the two-point prior binomial based on the average lot quality has the minimum average acceptance and average rejection cost for all the lot sizes with sample size less than that of the Prior Binomial and Gamma-Poisson distributions. Hence one can utilize the Two-point prior distribution as per the need of the shop floor situation.

Comparison of prior binomial, two point prior and gamma poisson prior distribution based on average lot quality ‘p’.

N		1000
C		7
P	Prior Binomial	0.13
	Two Point Prior Binomial	0.01,0.1)
	Gamma Poisson Prior Distribution	0.16
N	Prior Binomial	130
	Two Point Prior Binomial	120
	Gamma Poisson Prior Distribution	150
A_2	Prior Binomial	37
	Two Point Prior Binomial	18
	Gamma Poisson Prior Distribution	38
K(N,n,c,p	Prior Binomial	543.36
	Two Point Prior Binomial	195.65
	Gamma Poisson Prior Distribution	681.99
R(N,n,c,p)	Prior Binomial	137.36
	Two Point Prior Binomial	136.32
	Gamma Poisson Prior Distribution	176.87

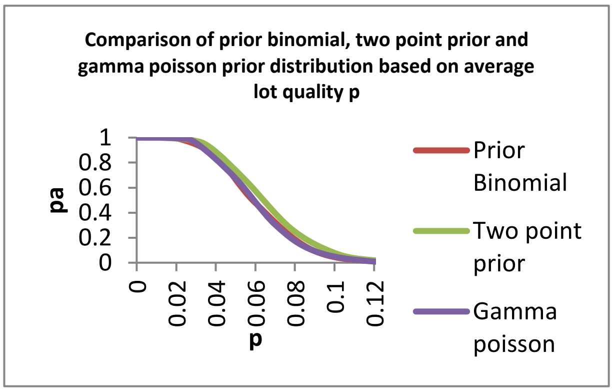

Fig:1

N		4000
C		7
P	Prior Binomial	0.07
	Two Point Prior Binomial	(0.01,0.1)
	Gamma Poisson Distribution	0.1
N	Prior Binomial	270
	Two Point Prior Binomial	150
	Gamma Poisson Prior Distribution	270
A_2	Prior Binomial	44
	Two Point Prior Binomial	21
	Gamma Poisson Prior Distribution	44
K(N,n,c,p	Prior Binomial	1150.53
	Two Point Prior Binomial	575.42
	Gamma Poisson Prior Distribution	1466.88
R(N,n,c,p)	Prior Binomial	276.65
	Two Point Prior Binomial	168.12
	Gamma Poisson Prior Distribution	315.01

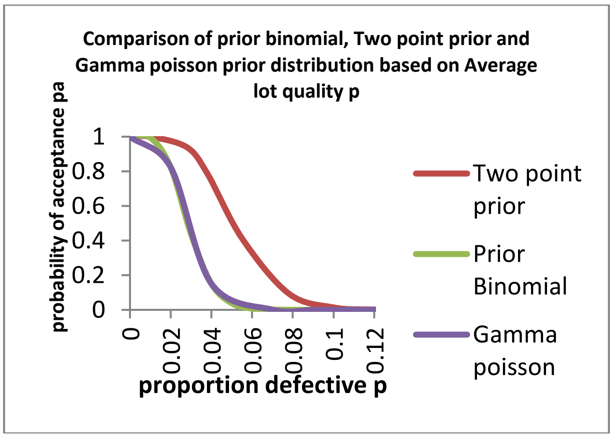

Fig : 2

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 9 Issue 10, October-2020

In this OC curves of the measures such as Average lot Quality, Acceptance Quality level and limiting quality level are drawn for the fixed lot size N=1000 & 4000 Based on the OC curves drawn when the lot size small the OC curves seems to be identical for all the distributions with reduced sample size and cost. When the lot size increases two point prior binomial and gamma Poisson are identical in nature and the sample size of two point prior binomial is minimum than that of Gamma Poisson. The details are given in the table adjacent to the OC curves having the sample size and its associated cost.

Several values are considered for the lot sizes, while lot size increases the discrimination of the OC curve leads to Ideal OC curve. The reader can visualize how producer's risk decreases without altering the consumer's risk. The decrease of the producer's risk by using such a scheme can be observed for each distribution considered in this study.

Table 3: Comparison of the average acceptance cost based on AQL

N	Prior Binomial Distribution				Two Point Prior Distribution					Gamma – Poisson Prior Distribution				C
	N	AQL	Average Acceptance Cost K(N,n,c,p) based on AQL	A_2	n	AQL		Average Acceptance Cost K(N,n,c,p) based on AQL	A_2	n	AQL	Average Acceptance Cost K(N,n,c,p) based on AQL	A_2	
						p_1	p_2							
1000	130	0.030	1203.4	37	120	0.0333	0.0334	140.2649	18	150	0.0234	1027.689	38	7
2000	170	0.024	1974.5	39	140	0.0287	0.0288	184.2478	20	230	0.015	1533.605	42	7
3000	230	0.017	2367.4	42	140	0.0288	0.0289	209.189	20	230	0.0149	2127.116	42	7
4000	270	0.014	2839.4	44	150	0.0269	0.027	243.6871	21	270	0.0127	2542.929	44	7
5000	290	0.013	3324.8	45	160	0.0252	0.0253	277.5927	22	350	0.0098	2801.452	48	7
6000	310	0.012	3791.5	46	160	0.0252	0.0253	301.8887	22	330	0.01038	3316.357	47	7
7000	350	0.011	4129.8	48	160	0.0252	0.0253	326.1847	22	370	0.009257	3627.434	49	7
8000	370	0.010	4540.6	49	160	0.0252	0.0253	350.4807	22	390	0.00879	3989.898	50	7
9000	390	0.01	4912.5	50	170	0.0237	0.0238	384.1473	23	390	0.00879	4410.797	50	7
10000	430	0.009	5203.2	52	170	0.0237	0.0238	408.3995	23	450	0.00762	4600.267	53	7

From the above table clearly indicates that, irrespective of the lot sizes 'N' the average acceptance cost are minimum at c=7, the optimum sample size 'n' and the value of A_2, the cost of handling the defective items in assembling and disassembling is also very small in two point prior binomial distribution when compared to the prior binomial and gamma poisson prior distribution.

Table 4: Comparison of the average rejection cost based on AQL

N	Prior Binomial Distribution				Two Point Prior Distribution					Gamma – Poisson Prior Distribution				C
	N	AQL	Average Rejection Cost R(N,n,c,p) based on AQL	A_2	n	AQL		Average Rejection Cost R(N,n,c,p) based on AQL	A_2	n	AQL	Average Rejection Cost R(N,n,c,p) based on AQL	A_2	
						p_1	p_2							
1000	130	0.030	491.2822	37	120	0.0333	0.0334	237.19	18	150	0.0234	487.8837	38	7
2000	170	0.024	933.9623	39	140	0.0287	0.0288	382.52	20	230	0.015	730.4549	42	7
3000	230	0.017	1077.455	42	140	0.0288	0.0289	515.43	20	230	0.0149	1008.903	42	7
4000	270	0.014	1305.662	44	150	0.0269	0.027	649.75	21	270	0.0127	1206.698	44	7
5000	290	0.013	1577.911	45	160	0.0252	0.0253	780.40	22	350	0.0098	1333.393	48	7
6000	310	0.012	1799.068	46	160	0.0252	0.0253	908.58	22	330	0.01038	1573.767	47	7
7000	350	0.011	1961.343	48	160	0.0252	0.0253	1036.764	22	370	0.009257	1722.529	49	7
8000	370	0.010	2156.286	49	160	0.0252	0.0253	1164.946	22	390	0.00879	1849.099	50	7
9000	390	0.01	2332.935	50	170	0.0237	0.0238	1289.09	23	390	0.00879	2091.747	50	7
10000	430	0.009	2473.002	52	170	0.0237	0.0238	1415.83	23	450	0.00762	2184.858	53	7

From the above table clearly indicates that, irrespective of the lot sizes 'N' the average acceptance cost are minimum at c=7, the optimum sample size 'n' and the value of A_2, the cost of handling the defective items in assembling and disassembling is also very small in two point prior binomial distribution when compared to the prior binomial and gamma poisson prior distribution.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 9 Issue 10, October-2020

Comparison of prior binomial, two point prior and Gamma Poisson prior distribution based on AQL.

N		1000
C		7
p	**Prior Binomial**	0.030
	Two Point Prior Binomial	(0.0333,0.0334)
	Gamma Poisson Distribution	0.0234
n	**Prior Binomial**	130
	Two Point Prior Binomial	120
	Gamma Poisson Prior Distribution	150
A_2	**Prior Binomial**	37
	Two Point Prior Binomial	18
	Gamma Poisson Prior Distribution	38
K	**Prior Binomial**	1203.4
	Two Point Prior Binomial	140.26
	Gamma Poisson Prior Distribution	1027.68
R	**Prior Binomial**	491.2
	Two Point Prior Binomial	237.19
	Gamma Poisson Prior Distribution	487.88

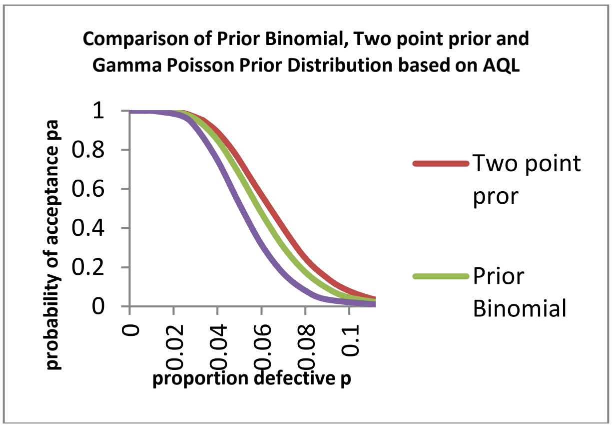

Fig : 3

N		5000
C		7
p	**Prior Binomial**	0.013
	Two Point Prior Binomial	(0.0252,0.0253)
	Gamma Poisson Distribution	0.0098
n	**Prior Binomial**	290
	Two Point Prior Binomial	160
	Gamma Poisson Prior Distribution	350
A_2	**Prior Binomial**	45
	Two Point Prior Binomial	22
	Gamma Poisson Prior Distribution	48
K	**Prior Binomial**	3324.8
	Two Point Prior Binomial	277.54
	Gamma Poisson Prior Distribution	2801.45
R	**Prior Binomial**	1577.91
	Two Point Prior Binomial	780.40
	Gamma Poisson Prior Distribution	1333.39

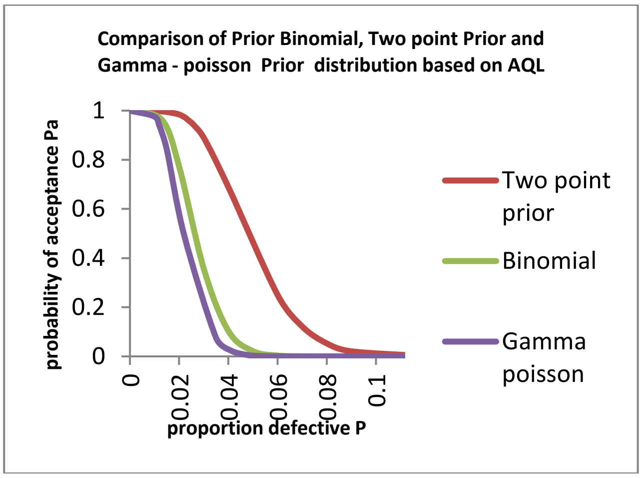

Fig : 4

Based on AQL, the OC curves are drawn for the optimum Single Sampling Plan at c=7, lot sizes N=1000,5000,8000 shown in figures(6.1.4,6.1.5,6.1.6) for the smaller lot sizes 1000, slighter variation occurs for both producer and consumer risk, where as for the larger lot sizes 8000, prior binomial distribution and Gamma Poisson are identical in nature and gives identical OC curves (Fig 6.1.6) two point prior Binomial gives a better result like smaller sample size and minimum acceptance and rejection costs with higher the probability of acceptance.

The Gamma Poisson is better because of lower values of ‘p’ (when quality is maintained) at given higher probability of acceptance. Whereas for higher values of ‘p’ (when quality is stated or low), it gives a much low probability of acceptance compared to two point prior or Prior Binomial distribution. Thus steeper is the OC curve, better is the plan.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 9 Issue 10, October-2020

Table 5: Comparison of the average acceptance cost based on LQL

N	Prior Binomial Distribution				Two Point Prior Distribution					Gamma – Poisson Prior Distribution				C
	N	LQL	Average Acceptance Cost K(N,n,c,p) based on LQL	A_2	n	LQL p_1	LQL p_2	Average Acceptance Cost K(N,n,c,p) based on LQL	A_2	n	LQL	Average Acceptance Cost K(N,n,c,p) based on LQL	A_2	
1000	130	0.09	702.2	37	120	0.0962	0.0963	611.2377	18	150	0.10056	785.8016	38	7
2000	170	0.069	1071.2	39	140	0.0827	0.0828	1159.166	20	230	0.0652	1161.515	42	7
3000	230	0.051	1333.6	42	140	0.0827	0.0828	1707.105	20	230	0.06558	1532.758	42	7
4000	270	0.043	1551.8	44	150	0.0784	0.0785	2279.658	21	270	0.0557	1813.682	44	7
5000	290	0.040	1786.5	45	160	0.0736	0.0737	2821.067	22	350	0.0429	2011.402	48	7
6000	310	0.038	2012.7	46	160	0.0736	0.0737	3370.874	22	330	0.0455	2320.428	47	7
7000	350	0.034	2174.0	48	160	0.0736	0.0737	3920.682	22	370	0.0406	2524.954	49	7
8000	370	0.032	2389.4	49	160	0.0736	0.0737	4470.489	22	390	0.0384	2760.283	50	7
9000	390	0.031	2551.4	50	170	0.0693	0.0694	4995.041	23	390	0.0386	2999.589	50	7
10000	430	0.028	2698.8	52	170	0.0693	0.0694	5541.479	23	450	0.0333	3148.749	53	7

From the above table clearly indicates that, irrespective of the lot sizes 'N' the average acceptance cost are minimum at c=7, in prior binomial distribution when compared to the two point prior binomial and gamma- poisson prior distribution. The optimum sample size 'n' and the value of A_2, the cost of handling the defective items in assembling and disassembling is also very small in prior binomial distribution when compared to the two point prior binomial and gamma poisson prior distribution.

Table 6: Comparison of the average Rejection cost based on LQL

N	Prior Binomial Distribution				Two Point Prior Distribution					Gamma – Poisson Prior Distribution				C
	N	LQL	Average Rejection Cost R(N,n,c,p) based on LQL	A_2	N	LQL p_1	LQL p_2	Average Rejection Cost R(N,n,c,p) based on LQL	A_2	n	LQL	Average Rejection Cost R(N,n,c,p) based on LQL	A_2	
1000	130	0.09	269.5926	37	120	0.0962	0.0963	694.26	18	150	0.10056	288.7847	38	7
2000	170	0.069	389.5838	39	140	0.0827	0.0828	1347.31	20	230	0.0652	442.5705	42	7
3000	230	0.051	501.3476	42	140	0.0827	0.0828	1996.575	20	230	0.06558	557.4262	42	7
4000	270	0.043	583.2616	44	150	0.0784	0.0785	2654.983	21	270	0.0557	667.3779	44	7
5000	290	0.040	667.1293	45	160	0.0736	0.0737	3304.16	22	350	0.0429	770.051	48	7
6000	310	0.038	750.9083	46	160	0.0736	0.0737	3953.78	22	330	0.0455	861.5217	47	7
7000	350	0.034	818.1374	48	160	0.0736	0.0737	4603.4	22	370	0.0406	948.0168	49	7
8000	370	0.032	903.2632	49	160	0.0736	0.0737	5253.02	22	390	0.0384	1038.102	50	7
9000	390	0.031	961.008	50	170	0.0693	0.0694	5897.00	23	390	0.0386	1112.959	50	7
10000	430	0.028	1027.54	52	170	0.0693	0.0694	6545.58	23	450	0.0333	1197.361	53	7

From the above table clearly indicates that, irrespective of the lot sizes 'N' the average rejection cost are minimum at c=7, in prior binomial distribution when compared to the two point prior binomial and gamma- poisson prior distribution. The optimum sample size 'n' and the value of A_2, the cost of handling the defective items in assembling and disassembling is also very small in prior binomial distribution when compared to the two point prior binomial and gamma poisson prior distribution.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 9 Issue 10, October-2020

Comparison of prior binomial, two point prior and gamma poisson prior distribution based on LQL.

N		3000
C		7
P	**Prior Binomial**	0.051
	Two Point Prior Binomial	(0.0827,0.0828)
	Gamma Poisson Distribution	0.06558
N	**Prior Binomial**	230
	Two Point Prior Binomial	140
	Gamma Poisson Prior Distribution	230
A_2	**Prior Binomial**	42
	Two Point Prior Binomial	22
	Gamma Poisson Prior Distribution	42
K	**Prior Binomial**	1333.6
	Two Point Prior Binomial	1707.105
	Gamma Poisson Prior Distribution	1532.758
R	**Prior Binomial**	501.347
	Two Point Prior Binomial	1996.575
	Gamma Poisson Prior Distribution	557.426

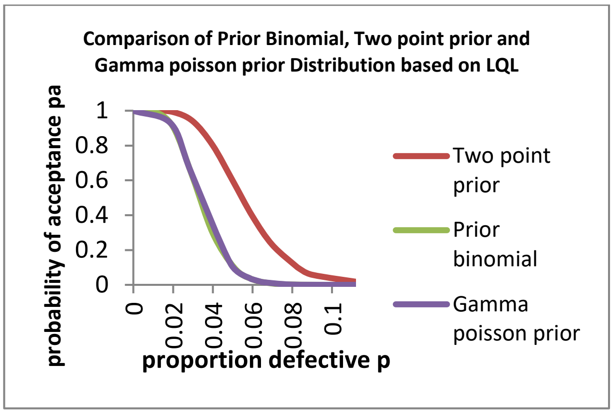

Fig : 5

N		7000
C		7
P	**Prior Binomial**	0.034
	Two Point Prior Binomial	(0.0736,0.0737)
	Gamma Poisson Distribution	0.040
N	**Prior Binomial**	350
	Two Point Prior Binomial	160
	Gamma Poisson Prior Distribution	370
A_2	**Prior Binomial**	48
	Two Point Prior Binomial	22
	Gamma Poisson Prior Distribution	49
K	**Prior Binomial**	2174
	Two Point Prior Binomial	3920.68
	Gamma Poisson Prior Distribution	2524.95
R	**Prior Binomial**	818.13
	Two Point Prior Binomial	4603.4
	Gamma Poisson Prior Distribution	948.016

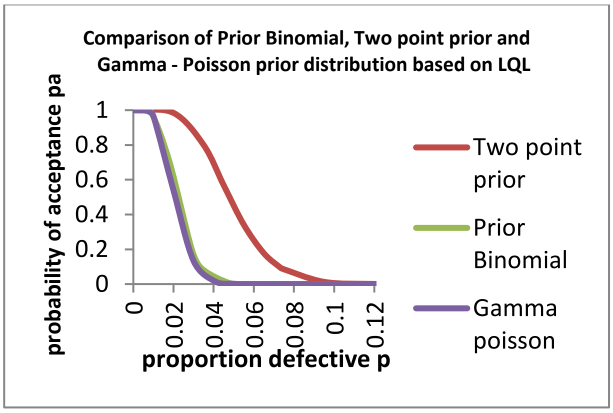

Fig : 6

OC curve displays the discriminatory power of the Single Sampling Plan i.e., the OC curve of the sampling plan with three distributions at N, n, c is shown in the figure 6.2.1 based on the cost aspect according to ALQ the different 'p' values. The OC curves of prior Binomial and Gamma Poisson distribution are very similar leading to ideal OC curve when lot size increases with increased sample sizes. Thus the precision for segregating good and bad lots increases with the size of the sample. The greater the slope of the OC curve, the greater the discriminatory power, thus Gamma Poisson distribution is more advantageous than other distributions.

CONCLUSION

For all the distributions, costs and lot sizes taken into account, it turns that the costs saved by using the Two –Point prior distribution are negligible. Accordingly the OC curves of the optimum single sampling plans using Prior Binomial and Gamma-Poisson distributions are identical, whereas Two-point prior gives minimum value for average acceptance cost. Therefore, results obtained above suggest a more critical examination of whether there exist situations in which the prior distributions offer any practical advantage compared with the existing procedures and distributions especially related with various costs faced in the inspection process.

REFERENCES

[1] **American National Standards Institute/ American Society for Quality Control (ANSI/ASQC) Standard A_2 (1987)** Terms, Symbols and Definition for Acceptance Sampling, American Society for Quality Control, Milwaukee, Wincontisin, USA.
[2] **Anscombe, F. J. (1949),** Tables of Sequential Inspection Schemes to Control Fraction Defective. Journal of the Royal Statistical Society. Series A, Vol.112, No.2, pp.180-206.
[3] **Anscombe, F. J. (1951),** The Cost of Inspection. Statistical Methods in Industrial Production, R. S. S., London.

[4] **Barnard, G.A. (1954),** Sampling Inspection and Statistical Decision**.** Journal of the Royal Statistical Society, Series B, Vol.16, pp.151-171.
[5] **Basler, H. (1968),** Calculation of Cost-Optimal Sampling Inspection Plans by means of Minimax Principle (in German). Metrika, Vol.12, pp.115-154.
[6] **Breakwell, J. V. (1956),** Economically Optimum Acceptance Tests. Journal of the American Statistical Association, Vol.51, pp.243-256
[7] **Cameron, J.M. (1952),** Tables for Constructing and for Computing the Operating Characteristics of Single Sampling Plans, Industrial Quality Control, Vol.9, No.1, pp.37-39.
[8] **Champernowne, D. G. (1953),** The Economics of Sequential Sampling Procedures for Defectives. Applied Statistics Vol.2, No.2, pp. 118-130.
[9] **Chiu, W.K. (1974),** A New Prior Distribution for Attribute Sampling, Technometrics, Vol.16, pp. 93-102.
[10] **Chiu, W.K, Wetherill, G.B. (1972),** A Review of Acceptance Sampling Practices. Roy. Statist, Soc. Conference, London
[11] **Chiu, W.K. Wetherill, G.B. (1973),** The Economic Design of Continuous Inspection Procedures; A Review Paper, International Statistical Review, Vol.41, pp. 357-373.
[12] **Calvin, T.W. (1984),** How and When to Perform Bayesian Acceptance Sampling, Vol.7, American Society for Quality Control, Milwaukee, Wisconsin, USA.
[13] **Case, K.E. and Keats, J.B. (1982),** On the Selection of a Prior Distribution in Bayesian Acceptance Sampling, Journal of Quality Technology, Vol.14, No 1, pp.10-18.
[14] **Dodge, H.F. and Romig, H.G., (1929),** A Method for Sampling Inspection. Bell Syst. Tech. J., 8, 613-631.
[15] **Dodge, H.F. and Romig, H.G., (1959),** Sampling Inspection Tables. John Wiley and Sons, New York.
[16] **U.S.Army Ordnance Corps, (1952),** Tables of Cumulative Binomial Probabilities, ORDP 20-1, Office of Technical Services, Washington, D.C.
[17] **Uhlmann, W. (1969),** Cost-Optimal Sampling Plans for the Acceptance Inspection by Attributes (in German). Metrika, Vol.13, pp.206-224.
[18] **Vagholkar, M. K. (1959),** The Process Curve and the Equivalent Mixed Binomial With Two Components. Journal of the royal statistical society, series B, Vol.21, 63-66.
[19] **Vagholkar, M. K. and Wetherill, G. B. (1960),** The Most Economical Binomial Sequential Probability Ratio Test. Biometrika, Vol.47, pp.103-109.
[20] **van der Waerden, B. L. (1960),** Sampling Inspection as a Minimum Loss Problem. Annals of Mathematical Statistics, Vol.31, pp.369-384.
[21] **van der Waerden, B. L. (1965),** Sequential Sampling Inspection as a Minimum Problem (in German). Zeit, Wahrscheinlichkeitsth., Vol.4, pp.187-202.
[22] **Vijayaraghavan.R, Loganathan.A & Rajagopal.K (2007),** Analysis of Operating Characteristic Curves of Gamma – Poisson Sampling Plans, American Journal of Mathematical and Management Sciences, Vol.27, Nos. 1&2, 163-177.
[23] **Vijayaraghavan.R, Loganathan.A & Rajagopal.K (2008),** A Procedure for Selection of a Gamma – Poisson Single Sampling Plan by Attributes, Journal of Applied Statistics, Vol.35, No. 2, 149-160.
[24] **Wetherill, G.B. (1960),** Some Remarks on the Bayesian Solution of Single Sample Inspection Scheme, Technometrics, Vol 2, No 3, 341-351.
[25] **WetherilL, G.B. and Campling, G.E.C. (1966),** The Decision Theory approach to Sampling Inspection, Journal of Royal Statistical Society Series B, Vol.28, pp.381-416,
[26] **Williamson, ERIC and Bretherton, Michael, (1963),** Tables of the Negative Binomial Probability Distribution, John Wiley and Sons, New York.
[27] **Weiler.H, (1965),** The Use of Incomplete Beta Functions for Prior Distributions in Binomial Sampling, Technometrics, Vol.7, No.3, pp.335-347.
[28] **Wetherill, G.B. (1969),** Sampling Inspection and Quality Control, London: Methuen.
[29] **Wortham, A.W. and Wilson, E. B. (1971),** A Backward Recursive Technique for Optimal Sequential Sampling Plans, Naval Research Logistics Quarterly, Vol 18, 203-213.
[30] Wetherill,G.B. and Chiu, W.K. (1974) A Simplified Attribute Sampling Scheme. Appl.Statist. Vol.23, No.2, pp.143-148.
[31] **Wetherill,G.B. and Chiu, W.K. (1975)** A Review of Acceptance Sampling Schemes with Emphasis on the Economic Aspect. Int. Stat. Rev., Vol.43, No.2, pp. 191-210.
[32] **Wadsworth, H.M, Stephens, K.S, Godfrey, A.B. (2002),** Modern Methods for Quality Control and Improvement, 2nd Ed., New York; John Wiley & Sons.
[33] **Yuvaraj, V and Subramaniam,R, (2011),** Bayesian Single Sampling Plan for Attributes Based Beta Prior distribution. International Journal of Statistics and Analysis, Vol.1, No. 2, pp.119-126.
[34] **Yuvaraj.V (2012**), Selection of Bayesian Single Sampling plans for attributes based on Binomial, Beta Binomial and Two Point Prior Distribution. Ph.D., Thesis, Department of Statistics, PSG College of Arts and Science, Coimbatore.
[35] **Yuvaraj, V (2016),** Selection of Bayesian Single Sampling Plan for Attributes Based on prior binomial distribution. International Journal of Applied Research, 2(2), pp.303-305.