

A Review on Vibrational Analysis of a Functionally Graded Beams

C. VenkataSiva Murali¹

¹P.G student, Department of Mechanical Engineering, MITS, Madanapalle.

P. Suryanarayana Raju²

²Professor, Department of Mechanical Engineering, MITS, Madanapalle.

Abstract: This article presents an overview of vibrational analysis of functionally graded beams with an emphasis on current available published research papers. Diverse areas relevant to various aspects of vibration analysis of FGMs are reflected in this paper, including an introduction to FG materials, their applications, beam deformation theories, property gradation laws, derivation of governing equations and numerical methods for solving the equations.

Key words: FGMs, vibrational analysis.

I. INTRODUCTION

In today's world, the persistent requirement for optimum performance from every product while at the same time keeping the designs simple has given rise to research and development into special materials for customized applications wherein composite materials have been receiving a lot of attention. One of the benefits of the composite materials is that their properties could be functionally oriented to suit to a specific application. Such composites are called Functionally Graded Materials (FGM) the FGMs which are at first proposed by researchers in Japan, in 1984 during the development of thermal barriers to withstand high temperatures under the space plane project.

A. Functionally graded materials:

Functionally graded materials are combinations two or more materials whose volume fractions change gradually along desired special directions, resulting in a smooth and continuous change in their effective properties. The combination of different materials with specific physical properties allows a tailored material design. The versatility of the resulting materials broadens the structural design spaces by implementing the desired multi-functional response with a minimal weight increase.

The FGMs are mainly manufactured by combining metals and ceramics. Since ceramics withstand the high temperatures and provide the thermal shielding to the given system while metals can give the structural support to the given system, such materials are useful for high temperature applications with specified physical properties.

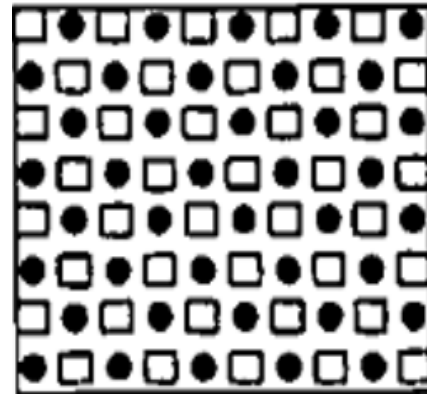


Fig. (a) Composites with equally deposition of metal and ceramics

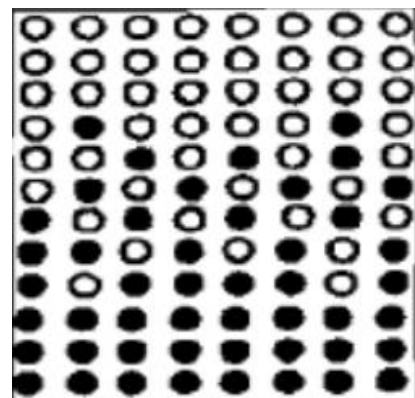


Fig (b) metal and ceramics deposition in FGMs.

B. FGM beams:

The FGM materials are used in many beam type structures in aerospace skins and turbine blades, shafts and biomedical components. Clearly, FGM Beams will be subjected to vibrations in their typical applications thus necessitating a detailed study is on vibration analysis of an FGM beams.

C. FGMs properties and applications:

In the FGMs the properties of material change with volume fractions of its constituent phases along the spatial directions. So it is possible to spatially modulate properties like toughness, strength and thermal baring capability. So FGMs are being increasingly used in many engineering sectors like space vehicles, aerospace, automotive, biomedical, optics, electronics, and military applications. Lately, FGMs are being preferred over the traditional composites owing to the properties of FGMs which enable

smooth transition of stress distributions, minimization or reducing the stress concentration, and improvement of fracture toughness. As the FGMs are being used more and more in engineering applications as mentioned above, vibration and instability analysis of a FGMs is a worthy topic for research. The present review article attempts to present the current status of the research in that area.

II. PROPERTY DISTRIBUTION LAWS:

There are many models that are developed to determine the effective properties in FGMs, and their best performance in applications. Due to the heterogeneous nature of these materials, the models used in their analysis are distinctive. The models can be classified into:

- Rule mixtures
- Variational approach
- Micro mechanical approaches

In the rule mixtures are developed from the continuum mechanics and they are designed for the bulk essential properties assuming no interface between phases. In the variational approach, variational principles of thermo-mechanics are used to define the bounds for effective thermo physical properties. The concept of Representative Volume Element (RVE) or unit cell is used in the regular micro mechanical approach to represent the micro structure of the FGM composite and thus deduce its local properties. It accounts for the detailed information about the spatial distribution of the component materials.

The material properties of FGM materials are varied along with the thickness of FGM beams. This gradation can be designed to follow several possible models, but mainly two distribution laws are being studied by the majority of researchers whose work is being reviewed in the current literature survey. Those are:

- A. Power-law exponent gradation
- B. Exponential –law exponent gradation

A. Power-law gradation:

The effective properties of an FGM beams can be described as a function of the location along the thickness of the beam by power gradation law as:

$$p(z) = (p_m - p_c) \left(\frac{2z + h}{2z} \right)^k + p_c$$

In this power law equation p_m and p_c are the corresponding material properties of the metal and ceramic. The other parameters in the equation are:

K = power law index

h = thick ness of FGMs layer

z = position from the reference plane

B. Exponential –law gradation:

This is the another method for the property distribution in FGMs where after the power law gradation in this law is widely used.

$$p(z) = p_0 e^{k(z+\frac{t}{2})}$$

The above equation gives the local material properties of FGMs as a function of the location along the thickness of the beam according to the exponential law. Here:

p_0 = it denotes the material properties of the bottom surface of FGM

K = exponential gradation index

III. EFFECT OF SYSTEM PARAMETERS:

The system parameters are also important in this study because they may also contribute to the changes in the physical properties of the FGM beams and also the response of the beams to loading. The parameters of interest include temperature, effect of variable or constant poisons ratio, effect of variable or constant density, the effect of beam being pre twisted or rotated, axially loaded and influence other environmental effects. There are many factors that were considered by researchers and some are presented here.

IV. VARIOUS DEFORMATION BEAM THEORIES AVAILABLE FOR VIBRATION ANALYSIS OF AN FGM BEAM:

There are mainly two beam theories that are being used by researchers: the well-known classical beam theory (CBT) or Euler- Bernoulli beam theory and shear deformation beam theories which are developed to get a more accurate analysis. The shear deformation theories include:

- First order shear deformation beam theory (FSDBT) or Timoshenko beam theory
- Second order shear deformation beam theory (SSDBT)
- Third order shear deformation beam theory (TSDBT)

In this Classical beam theory (CBT) is assumed that a straight line perpendicular to the mid-plane before bending will remain straight and perpendicular to the mid-plane even after bending. The consequence of this assumption is that the transverse shear strain is neglected. This assumption is good for slender beams and plates as it does not produce significant errors. For thick plates and beams, however, it doesn't provide the accurate results.

The limitations of the Classical Beam Theory are overcome by the shear deformation theories like the Timoshenko beam theory. In this theory the transverse shear distribution is assumed to be constant along the thickness coordinate. In other words a straight line which is perpendicular to the mid plane before bending will no longer remain straight and perpendicular to the mid plane after bending.

The second order shear deformation plate theory is where it assumed that the straight line normal to the mid plane before deformation will be changed to the form of a cubic curve after deformation.

In the third order shear deformation beam theory it is assumed that the transverse shear stress and strain are distributed in parabolic manner with respect to the thickness of a beam with rectangular cross-section (Wang et al., 2000)

The main difference between the classical beam theory and shear deformation theories is that in the studies using shear deformation theories, the transverse shear stress is considered to find the natural frequency of beams and plates, to forecast the vibrational loads or buckling loads and deflections of a beam (Reddy, 1983) whereas in studies using the classical beam theory the transverse shear is neglected.

V. LITERATURE SURVEY:

A. Functionally graded materials:

Victor Birman et (2007) presented a good introduction on the Functionally Graded materials and structures. They give a clear idea about the manufacturing process of FGMs and its applications with examples. And discussed on the heat transfer, stress and stability issues of FGMs.

T.P.D. Rajan et al (2009) conducted a survey of the manufacturing process of FGM materials and developments in different manufacturing processes that are currently available and presented an analysis of the best method is for FGMs. The processing technologies that are discussed include diffusion bonding, liquid state process, casting process, and different deposition processes with specific focus on manufacturing aluminum based FGMs.

B. Kieback et al (2003) proposed novel processing techniques for manufacturing FGM materials and also he conducted a numerical test for predicting the properties of the materials and the results were used to deduce the optimum process that will give the more effective properties of an FGM type structure.

B. Functionally graded beams for free vibration analysis:

A. Based on Static and dynamic analysis:

Ravikiran Kadoli et al (2008) presented a static analysis of an FGM beams by using higher order shear deformation beam theory. Using different edge boundary conditions like simply supported (s-s), clamped at both ends (c-c) they conducted a parametric study in static analysis.

Raghuvir Mehta et al (2013) using a finite element model they developed an FGM beam for both static and dynamic vibration analysis. And compared the frequencies with other edge conditions for material property varying power law is used.

Kamran Asemi et al (2014) presented their article on both static and dynamic analysis of FGM skew plate with 3d

elasticity, using the power law for static analysis and exponential law for dynamic analysis with a finite element method in ANSYS.

A new unified approach was used to find the static and dynamic analysis of FGM beams considering the rotatory inertia and shear deformation in the Timoshenko and classical beam theory in the study of .X.-F. Li (2008).

A mixed Ritz-Differential quadrature(DQ) has developed by a S.M.R. Khalili et al (2010) to perform a dynamic analysis of a forced vibration on FGM beams with consisting of a moving mass.

Using a first order shear deformation beam theory Trung-Kien Nguyen et al., (2013) presented a research article on static analysis of an FGM beams with an axially loaded condition.

The work of Mohanty S.C. was work on static and dynamic analysis of FGM beams with both power law and exponential law variation of a Timoshenko beam theory with an edge condition of the Winklers elastic foundation, the analysis is carried out in the finite element method to achieve natural frequencies

B. Based on gradation laws:

X. Zho et al (2009) have proposed a power law for the vibrational analysis of an FGM plates using of a this law the properties will change the length to thickness ratio of a plate or beam in his the supporting equations were generated by using the kp- Ritz method for Timoshenko beam theory to conduct a vibration analysis on aluminum skew plates with different combinations to get frequencies.

Sanjay et al (2012) used finite element method for the investigation of free vibration analysis of an FGM beams and power law for properties gradation, principle of virtual work for the equation derivation and they used CBT and TBT theories for finding frequencies.

Amal E, et al. (2011) studied with power law for physical property variation and conducted a test on free vibration analysis by using finite element method to get natural frequencies. The equations are developed using the principle of virtual work.

Ying-An Kang et al (2009) have also applied the power law for a cantilever FGM beams with an adding an end force.

Erin demir et al (2013) estimated the frequencies for the sandwich thimoshenko FGM beam by using the material gradation laws both power law and exponential law and conducted finite element analysis using Ansys for a simply supported edge condition.

Nuttawit et al (2012) used power law variation index for vibration analysis of an FGM. By using the Differential Transformations method with elasticity edge conditions of S-s beam.

H.J. Xiang et al (2008) focused their research on free and forced vibration of an FGM beams under the heat conduction of laminated plates with Power law variation. By using the Timoshenko beam theory the natural frequencies were tabulated.

C. Depend on various deformation beam theories:

The different shear deformation beam theories were used in the literature to find the natural frequency of a functionally graded beams and plates. It has been widely reported that variations are obtained in the achieved frequency depending upon the applied beam theories. In this study mainly focuses on CBT (classical beam theory) and Timoshenko beam theory (TBT) as they are used by most researchers to find the natural frequency and mode shapes of an FGM beams.

Metin et al[1] presented a paper on simply supported FGM beam for free vibrational analysis by using CBT theory and various shear deformation beam theories like first order shear deformation beam theory, parabolic shear deformation beam theory and exponential shear deformation beam theory. The property gradation was presented with using the power law variation and the governing equations were generated by using the Hamilton's principle. The Navier type equations were solved and the frequencies and mode shapes were found. It is compared with other deformation theories. S.A sina et al. developed a new beam theory for laminated composite beam. This is used for free vibration analysis and he compared with normal first order shear deformation theory and mode shapes were drawn by using the same principle as above and differential equations were used to solve in the analytical method. Mesut Simsek concentrated his work on dynamic analysis of FGM simply supported beam subjected to a moving mass. For this he developed equations by using Lagrange multipliers, used Euler-Bernoulli, FSDBT and TSDBT theories to find natural frequencies. Huu-Tai Thai (2012) focused his work on bending and the free vibration of FGM beams with different higher order shear deformation beam theories. Hierarchical deformation theories were used (G. Giunta et al (2011)) for study of the free vibrations in FGM beams and used the power law variation and principal of virtual work for the numerical equation generation to get the fundamental frequencies.

VI. CONCLUSION:

In view of the increasing demand of FGM materials in many engineering applications and other sectors, the review is conducted on the topic of vibration analysis for the FGM beam type structures. These conclusions cannot encompass all significant directions, trends, and needs. Nevertheless, they reflect some of the observations of the authors based on the published research articles and their own analysis of the subject.

It is observed in this review that most of the researchers are using power law gradient for the physical property variation of the FGM beam analysis because of its efficient grading, and accuracy in frequencies results. For the static analysis

power law can be used and for the dynamic analysis exponential law is used in some literature reviewed here.

The higher order shear deformation beam theory is widely being used for considering the transverse shear in the analysis. Several scientists used them for the vibration analysis of a FGM beams and in some cases they also developed new deformation beam theories for their own system parameters and shape functions. The higher order shear deformation beam theories are especially suitable for the thick beams.

There are many investigators that used principle of virtual work and Hamilton's principle for generating the governing equations and some of the researchers used LaGrange multipliers, KP-Ritz method is used for this type of analysis. There are many computational methods available for these types of analyses, we can use those methods for the analysis and it could be compared with experimental results and other methods. Most researchers are using finite element method in ANSYS and some researchers are developing their own computational software for the analysis.

For future work the analysis may be extended for different types of structures like tapered and some complicated structures and these analyses are carried out by suitable system parameters, shape functions, deformation theories and methods.. Only limited amount of research has been done on turbine blades, pre twisted rotating blades made of FGM beams by using Rayleigh, Rayleigh-Ritz, methods so this can be taken up as further research on this topic.

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