A Review on High Order Shear Deformation Theory for Orthotropic Composite Laminates

Ms. Komal U. Navale
Department of Civil Engineering
SKN Sinhgad College of Engineering Korti Pandharpur
Maharashtra, India

Dr. C. P. Pise
Department of Civil Engineering
SKN Sinhgad College of Engineering Korti Pandharpur
Maharashtra, India

Abstract— **The new higher-order shear deformation theory is developed and accessed for the bending, buckling and free vibration response of the thick and thin laminated composite and sandwich plates. The proposed theories assume nonlinear variation of transverse shear strain through the plate thickness and give transverse shear stress-free top and the bottom surface of the plate. The governing differentials equation of the laminated composite and sandwich plates derived through Hamilton's principle are solved employing analytical solution technique i.e. navier solution methodology for the cross-ply plates with simply supported boundary conditions over all the edges. In order to demonstrate the accuracy and applicability of the proposed higher-order shear deformation theories, a wide range of the numerical examples for the bending, buckling and free vibration response are considered. The present results are found to be in good agreement with those published earlier in the literature using equivalent single layer theories and three-dimensional exact solutions.**

Keywords— *Shear deformation theory, Orthotropic laminates, Static and Buckling analysis*

I.   INTRODUCTION

In materials science, a composite laminate is an assembly of layers of fibrous composite materials which can be joined to provide required engineering properties, including in-plane stiffness, bending stiffness, strength, and coefficient of thermal expansion. The individual layers consist of high-modulus, high-strength fibers in a polymeric, metallic, or ceramic matrix material. Typical fibers used include cellulose, graphite, glass, boron, and silicon carbide, and some matrix materials are epoxies, polyimides, aluminum, titanium, and alumina. Layers of different materials may be used, resulting in a hybrid laminate. The individual layers generally are orthotropic (that is, with principal properties in orthogonal directions) or transversely isotropic (with isotropic properties in the transverse plane) with the laminate then exhibiting anisotropic (with variable direction of principal properties), orthotropic, or quasi-isotropic properties. Quasi-isotropic laminates exhibit isotropic (that is, independent of direction) inplane response but are not restricted to isotropic out-of-plane (bending) response. Depending upon the stacking sequence of the individual layers, the laminate may exhibit coupling between in plane and out-of-plane response. An example of bending-stretching coupling is the presence of curvature developing as a result of in-plane loading.

In the orthotropic materials deformation is, in general, direction dependent. An exception occurs when loads are applied in natural (material) coordinates. These are by definition coordinates in the plane of the lamina, wherein the longitudinal coordinate is aligned with the fiber reinforcement and the transverse coordinate is aligned normal to the fiber reinforcement. Longitudinal and transverse directions are material axes of symmetry in a unidirectional reinforced composite. When loads are applied in these natural coordinates the material response is similar to that of isotropic materials, i.e., normal stresses produce normal strains only and shear stresses produce shear strains only. Here the longitudinal and transverse axes are respectively. Unidirectionally reinforced composites are often referred to as especially orthotropic. Furthermore, unidirectionally reinforced laminas are isotropic in the out-of-plane (normal to the plane of the lamina) direction
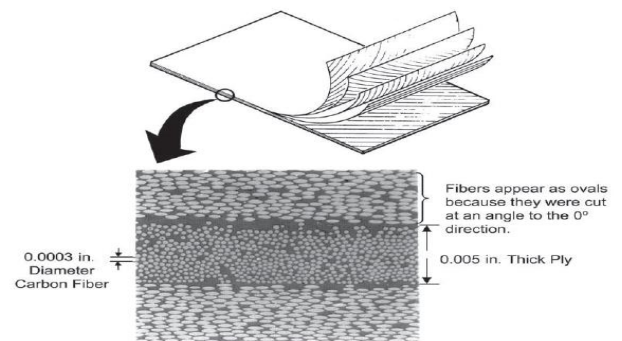

Fig. 1 Cross-section of cross-ply laminate

II.   CLASSIFICATION OF STRUCTURAL THEORIES

A. Equivalent Single Layer (ESL) Theory
    i. Classical Laminated Plate Theory
    ii. Shear Deformation Plate Theory
B. Three Dimensional Elasticity theory
    i. Layer-wise or Zig-Zag theory

A. Classical Laminated Plate Theory

The classical laminate theory is a direct extension of the classical plate theory for isotropic and homogeneous material as proposed by Kirchhoff. However, the extension of this theory to laminates requires some modifications to take into account the inhomogeneity in thickness direction. In the following, the assumptions made in this theory along with the assumptions made for classical plate theory are given.

The strain-displacement field is derived using two approaches. In the first approach the deformation of the laminate according to the Kirchhoff - Law assumptions for bending and stretching is used. The undeformed and deformed geometries of laminate are used to develop the displacement field. In the second approach the transverse strain components resulting from the above assumptions are used. Further, using mathematical definitions of these strain components the displacement field is obtained. Thus, from this displacement field all strain components are obtained the complete displacement field for a generic point in the laminate according to the classical laminate theory is given below:

$$u(x, y, z) = u_0(x, y) - z \frac{\partial w}{\partial x}$$
$$v(x, y, z) = v_0(x, y) - z \frac{\partial w}{\partial y}$$
$$w(x, y, z) = w_0(x, y)$$

The strain displacement relations for infinitesimal strains using the displacement field as in Equation can be given as

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w}{\partial x^2}$$
$$\epsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z \frac{\partial^2 w}{\partial y^2}$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z \frac{\partial^2 w}{\partial x \partial y}$$

The above equation can be written as

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_{xx}^{(0)} \\ \epsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix}$$

or

$$\{\epsilon\}_{xy} = \{\varepsilon^{(0)}\}_{xy} + z\{\kappa\}_{xy}$$

B.Shear Deformation Plate Theory

In the Reissner-Mindlin theory, also called First-order Shear Deformation Theory (FSDT), the third part of Kirchhoff hypothesis is removed, so the transverse normals do not remain perpendicular to the midsurface after deformation. In this way, transverse shear strains γxz and γyz are included in the theory. However, the inextensibility of transverse normal remains
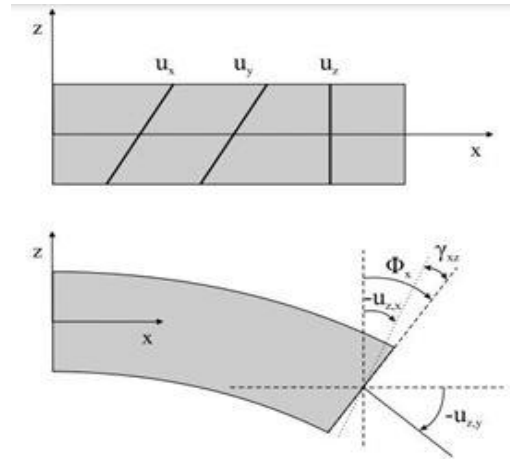

Fig.2 Distribution of displacements in FSDT

So displacement uz is constant in the thickness direction z. The displacement field in the case of FSDT is

ux(x, y, z) = ux0 (x, y) + φx(x, y)z
uy(x, y, z) = uy0 (x, y) + φy(x, y)z
uz(x, y, z) = uz0 (x, y)

The quantities (ux0 , uy0 , uz0 ,φx,φy) will be the unknowns. For thin plates, i.e., when the plate in-plane characteristic dimension to thickness ratio is on the order 50 or greater, the rotation functions φx and φy should approach the respective slopes of the transverse deflection −∂uz0 ∂x and −∂uz0 ∂y . Figure shows the typical distribution of displacement components according to FSDT: linear for ux and uy and constant for uz. Also the physical meaning of the rotations, φx and φy, is represented. The strain components are obtained by substituting the displacement field in the geometrical relations. Only strain εzz is zero, so the not-null strains are

εxx = ∂ux ∂x = ux0,x + φx,xz
εyy = ∂uy ∂y = uy0,y + φy,yz
γxy = ∂ux ∂y + ∂uy ∂x = ux0,y + uy0,x + φx,yz + φy,xz
γxz = ∂ux ∂z + ∂uz ∂x = φx + uz0,x
γxy = ∂ux ∂y + ∂uy ∂x = φy + uz0,y

The constitutive relations are used to obtain the in-plane stresses and the shear stress components

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$\tau_{xz} = \kappa \, G \gamma_{xz} \quad \tau_{yz} = \kappa \, G \gamma_{yz}$$

Where κ is the shear correction factor

C.Layer-wise or Zig-Zag theory

Displacement Field

In the layerwise theory of Reddy the displacement of the k the layer is given by,

$$u^k(x,y,z,t) = \sum_{j=1}^{m} u_j^k(x,y,t)\phi_j^k(z)$$

$$v^k(x,y,z,t) = \sum_{j=1}^{m} v_j^k(x,y,t)\phi_j^k(z)$$

$$w^k(x,y,z,t) = \sum_{j=1}^{n} w_j^k(x,y,t)\psi_j^k(z)$$

Stresses and Strains

The Strains associated with the displacement field

$$\varepsilon_{xx} = \sum_{I=1}^{N} \frac{\partial U_I}{\partial x}\Phi^I + \frac{1}{2}\left(\sum_{I=1}^{M}\frac{\partial W_I}{\partial x}\Psi^I\right)\left(\sum_{J=1}^{M}\frac{\partial W_J}{\partial x}\Psi^J\right)$$

$$\varepsilon_{yy} = \sum_{I=1}^{N} \frac{\partial V_I}{\partial y}\Phi^I + \frac{1}{2}\left(\sum_{I=1}^{M}\frac{\partial W_I}{\partial y}\Psi^I\right)\left(\sum_{J=1}^{M}\frac{\partial W_J}{\partial y}\Psi^J\right)$$

$$\gamma_{xy} = \sum_{I=1}^{N}\left(\frac{\partial U_I}{\partial y} + \frac{\partial V_I}{\partial x}\right)\Phi^I + \left(\sum_{I=1}^{M}\frac{\partial W_I}{\partial x}\Psi^I\right)\left(\sum_{J=1}^{M}\frac{\partial W_J}{\partial y}\Psi^J\right)$$

$$\varepsilon_{zz} = \sum_{I=1}^{M} W_I \frac{d\Psi^I}{dz}$$

$$\gamma_{yz} = \sum_{I=1}^{N} V_I \frac{d\Phi^I}{dz} + \sum_{I=1}^{M}\frac{\partial W_I}{\partial y}\Psi^I$$

$$\gamma_{xz} = \sum_{I=1}^{N} U_I \frac{d\Phi^I}{dz} + \sum_{I=1}^{M}\frac{\partial W_I}{\partial x}\Psi^I$$

III PRESENT THEORIES

M. Narwariya et al., [1] presented the vibration and harmonic analysis of orthotropic laminated composite plate. The response of plate is determined using Finite Element Method. The eight noded shell 281 elements are used to analyze the orthotropic plates and results are obtained so that the right choice can be made in applications such as aircrafts, rockets, missiles, etc. to reduce the vibration amplitudes.

M. Nasihatgozara et al., [2] in this study the effect of different boundary conditions on the free vibration analysis response of a sandwich plate is presented using the higher order shear deformation theory. The face sheets are orthotropic laminated composites that follow the first order shear deformation theory (FSDT) based on the Rissners-Mindlin (RM) kinematics field. The motion equations are derived considering the continuity boundary conditions between the layers based on the energy method and Hamilton's principle.

Durgesh Bahadur Singh et al., [3] Proposed three dimensional braided composites exhibit several distinct advantages which are not realized in traditional laminates. This is due to the out of plane orientation of some fibers & fully integrated nature of fiber arrangements of three dimensional braided composites that enhance stiffness/strength in the thickness direction and eliminate the inter-laminar surface characteristics. A recently developed Inverse Hyperbolic Shear Deformation Theory (IHSDT) has proved its accuracy and efficiency for static and buckling analysis of laminated composite and sandwich structures.

Neeraj Grover et al., [4] studied; an inverse trigonometric shear deformation theory developed by the authors is extended to assess the flutter behavior of multilayered composite plates subjected to yawed supersonic flow. The shear deformation is considered in terms of an inverse cotangent function which yields non-linear distribution of shear stresses. A generalized finite element formulation is presented to consider the shear strain function based theories.

Nguyen et al., [5] presented unified structure for modeling and analysis of composite plate which is polynomial form of higher shear deformation theory in unified formulation with consideration of thickness stretching consideration. The various non-polynomial shear deformation theories are constructed in diagram which also called as Best Theory Diagrams for composite laminated plates. Best theory diagrams used to calculate number of variables required to reduce the fixed error based on axiomatic/asymptotic method which gives accurate results for composite plates.

A.H. Akbarzadeh et al., [6] examined the influence of various micromechanical models on the macroscopic behavior of a functionally graded plate based on classical and shear-deformation plate theories. Different micromechanical models are tested to obtain the effective material properties of a two-phase particle composite as a function of the volume fraction of particles which continuously varies through the thickness of a functionally graded plate. The static, buckling, and free- and forced-vibration analyses are conducted for a simply-supported functionally graded plate resting on a Pasternak-type elastic foundation. The volume fractions of particles are assumed to change according to the power law, Sigmoid, and exponential functions.

Neeraj Grover al., [7] presented a secant function based shear deformable finite element model is developed for the flexural behavior of laminated composite and sandwich plates with various conditions. The structural kinematics of the plate is expressed by means of secant function based shear deformation theory newly developed by the authors. The theory possesses non-linear shear deformation and also satisfies the zero transverse shear conditions on top and bottom surfaces of the plate. The field variables are elegantly utilized in order to ensure C0 continuity requirement. Penalty parameter is implemented to secure the constraints arising due to independent field variables. A biquadratic quadrilateral element with eight nodes and 56 degrees of freedom is employed to discretize the domain.

J.L. Mantari et al., [8] Presents the comparison of polynomial, trigonometric, hyperbolic trigonometric and Exponential shear deformation was carried out and used exponential shear stress parameter to develop higher shear deformation theory.Neeraj

Grover et al [9] the proposed theory based upon shear strain shape function yields non-linear distribution of transverse shear stresses and also satisfies traction free boundary conditions. Principle of virtual work is employed to develop the governing differential equations assuming the linear kinematics. A Navier type closed form solution methodology is also proposed for cross-ply simply supported plates which limits its applicability. However, it provides accurate solution which is free from any numerical/ computational error

Yiyo Kuo et al [10] proposed a grey-based Taguchi method to solve the multi-response simulation problem. The grey-based Taguchi method is based on the optimizing procedure of the Taguchi method, and adopts grey relational

analysis (GRA) to transfer multi-response problems into single-response problems. A practical case study from an integrated-circuit packaging company illustrates that differences in performance of the proposed grey-based Taguchi method and other methods found in the literature were not significant.

J.N. Reddy et al., [11] Presents a higher-order shear deformation theory is used to determine the natural frequencies and buckling loads of elastic plates. The theory accounts for parabolic distribution of the transverse shear strains through the thickness of the plate and rotary inertia. Exact solutions of simply supported plates are obtained and the results are compared with the exact solutions of three-dimensional elasticity theory, the first-order shear deformation theory, and the classical plate theory. The present theory predicts the frequencies and buckling loads more accurately when compared to the first-order and classical plate theories.

J.L. Mantari11, A.S. Oktem2, C. Guedes Soares3 [12] A new inverse hyperbolic shear deformation theory is developed and implemented in the present work for the static and buckling distribution of transverse shear stresses and also satisfies the zero tangential traction boundary conditions on the surface of the plate. Principle of virtual work is applied to derive the governing differential equations assuming the linear kinematics followed by a closed form solution methodology based upon Navier solution technique, which limits the applicability of solution technique to simply supported plates; however the solution free from any numerical/computational error is obtained. Various numerical examples are worked out to ensure the accuracy and efficacy of the proposed theory.

Yiyo Kuo1, Taho Yang2, [13] The grey-based Taguchi method to solve a multi-response simulation optimization problem. Following the procedure of the Taguchi method, GRA was used to transform a multi-response problem into a single-response problem. A practical case study illustrated that the difference in performance between the proposed method and other methods identified in the Literature was not significant. However, the results of this study illustrate that the GRA procedure is simple and straightforward in calculations and optimization; it is therefore a very suitable method for solving multi-response simulation problems.

Fan Yell 1 , Lin Fangyong 2 [14] On the basis of the Reddy's higher-order theory of composites, this paper introduces a displacement function t~ into it and transforms its three differential equations for symmetric cross-ply composites into only one eight-order differential equation generated by the displacement-function. When a proper ~ is chosen, both solutions are obtained, namely, tire Navier-type solution of simply supported rectangular laminated plates and the Levy-type solution with the boundary condition where two opp6site edges are simply supported and remains are arbitrary.

K. Vijayakumar [15] presented shear deformation and higher order theories of plates in bending are (generally) based on plate element equilibrium equations derived either through variational principles or other methods. They involve coupling of flexure with torsion (torsion-type) problem and if applied vertical load is along one face of theplate, coupling even with extension problem. These coupled problems with reference to vertical deflection of plate in flexure result in artificial deflection due to torsion and increased deflection of faces of the plate due to extension.

G. A. MAUGIN [16] A new standard plate theory, that accounts for cosine shear stress distribution and free boundary conditions for shear stress upon the top and bottom surfaces of the plate, is presented. The theory is of the same order of complexity as the first order shear deformation theory, but does not use shear correction factors, and is more efficient than the first order shear deformation theory and some refined plate theories. The theory is based on the kinematical approach in which the shear is represented by a certain sinusoidal function. The boundary value problem is deduced from the virtual power principle.

Chien H. Thai1, A.J.M. Ferreira2, [17] presented a new inverse tangent shear deformation theory (ITSDT) for the static, free vibration and buckling analysis of laminated composite and sandwich plates. In the present theory, shear stresses are vanished at the top and bottom surfaces of the plates and shear correction factors are no longer required. A weak form of the static, free vibration and buckling models for laminated composite and sandwich plates based on ITSDT is then derived and is numerically solved using an isogeometric analysis (IGA).

J.L. Mantari1, A.S. Oktem2, [18] developed new higher order shear deformation theory for elastic composite/sandwich plates and shells. The new displacement field depends on a parameter ''m'', whose value is determined so as to give results closest to the 3D elasticity bending solutions. The present theory accounts for an approximately parabolic distribution of the transverse shear strains through the shell thickness and tangential stress-free boundary conditions on the shell boundary surface. The governing equations and boundary conditions are derived by employing the principle of virtual work.

Sandeep Singh1 , Jeeoot Singh2 [19] In this study, buckling analysis of isotropic, orthotropic, laminated composite and sandwich plates utilizing trigonometric shear deformation theory and mesh less method based on the finite point formulation using thin plate, polynomial and inverse multiquadric radial basis function is presented. The convergence of the present method is studied for isotropic and laminated composite plates for different radial basis functions with optimal value of shape parameter. Numerical examples of laminated and sandwich plates subjected to various types of in-plane loads are solved to demonstrate accuracy and applicability of present method.

IV. CONCLUSIONS

1) The newly developed higher-order shear deformation theory is accessed for the bending, buckling and free vibration response of the laminated composite and sandwich plates. The proposed plate theory assumes nonlinear variation of transverse shear strain through the plate thickness and thus delivers transverse shear stress-free top and bottom surface of the plate.
2) The governing equation of plate is solved using Navier solution technique with simply supported boundary conditions to investigate the accuracy of the proposed theories. From the proposed higher-order shear deformation theory predicted better deflection of laminated composite plates

3) The proposed higher-order shear deformation theory with combination of trigonometric and polynomial function as a transverse shear strain function predicted results close of those of predicted with TSDT of Reddy

4) The results predicted with different proposed higher-order shear deformation theory differ marginally for the thick plates and these differences disappear for the thin plates.

5) For the sandwich plates, the higher-order shear deformation theory a transverse shear strain function predicted results closed those of obtained thorough exact solution.

6) The present's results are found to be in good agreement when compared to results due to three dimensional elasticity solutions due to Refs.

V. ACKNOWLEDGMENT

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