A Review on Control System Design Techniques

Dept. of Electrical and Electronics Engg.,
RVS College of Engineering and Technology,
Jamshedpur-831012, INDIA

Abstract— This paper is a review of the techniques for designing of a control system. The design of control systems is one of the most important aspects of the practical implementation of engineering systems. Several approaches have been presented in literature for designing of stable control systems. Bode plot; Nyquist plot, root locus, etc. are some of the techniques used for analysis of SISO control systems. These techniques are classified as Classical Control techniques. These techniques were succeeded by the state space design approach to control. Optimal control and multivariable control are added features of this approach which are known by the term modern control techniques. Over the past few decades, robust control theory has been developed to include the effects of model errors on the system. Several others techniques such as nonlinear control, adaptive control, intelligent control, etc. were developed for control of systems. In this paper, we review two of these control techniques: PID controller and Internal Model Control.

Keywords— Control System Design; PID Controller; Internal Model Control.

I. INTRODUCTION

Design of control systems plays an important role in modern systems. Improved control can result in immense benefits in the industry such as reduced energy consumption, improved quality of the product, minimization of wastage etc. Design of stable control systems is one of the most challenging and interesting topic in the field of research and has attracted the attention of design engineers over the past few decades. Proper design of a control system requires the knowledge of many disciplines such as mathematical modeling, knowledge of sensors, communication engineering, etc. Significant development in the field of control theory began during the period of the Second World War. Pioneering work of Bode [1], Evans [2], Nyquist [3] etc. appeared at this time. These techniques are usually termed as classical control techniques. State space approach for design of control systems succeeded the classical control techniques. Wiener, Kalman [4-5] etc. published significant work in this field generally known as modern control techniques. Early 1980’s saw the development of robust control techniques [3] for design of robust control systems. Several other techniques such as nonlinear control, auto-tuning controllers, etc. have also been developed. To provide a review of all these techniques would significantly increase the length of the manuscript and so we limit our discussion to two important techniques: the PID control [6] and IMC [7] technique with suitable examples.

The organization of the paper is as follows: next section discusses the PID controller followed by the discussion of IMC controller in the third section. The last section presents the conclusion of the work.

II. PID CONTROLLER

PID controller is the most widely used controller in the industry. A PID controller has three parameters- proportional constant ‘$K_p$’, integral constant ‘$K_i$’ and the derivative constant ‘$K_d$’. These three parameters are meant to take care of the present, future and the past errors. A PID controlled process having system transfer function ‘$G_s$’ and unity feedback is shown in Fig. 1.

Fig.1. PID Controller

\[ G_c = K_p + \frac{K_i}{s} + K_ds \]  

\[ G = \frac{G_cG_s}{1 + G_cG_s} \]  

\[ G = \frac{g_p + \frac{K_i}{s} + K_ds}{s + \left( \frac{K_p + \frac{K_i}{s} + K_ds}{G_s} \right)} \]  

Proportional action is meant to minimize the instantaneous errors. However, by itself it cannot make the error zero and provides a limited performance. The integral action forces the steady state error to zero, but has two disadvantages: due to the presence of a pole at the origin, it may result in system instability and the integral action may create an undesirable effect known as wind-up in the presence of actuator saturation. The derivative action acts on the rate of change of error and it may result in large control signals when the error signal is of high frequency.

Example: Consider a plant with the model given by:

\[ G_s = \frac{1}{(s+1)^4} \]

A PID controller is designed for this plant model using the PIDTOOL of MATLAB®. The parameters of the PID
controller are $K_P = 1.03$, $K_I = 0.34$ and $K_D = 0.783$. The transfer function of the PID controller is given below and the step response of unity feedback closed loop system with and without the presence of PID controller is shown in Fig. 2. The parameters of the unit step response are shown in Table 1.

$$G_c = 1.03 + \frac{0.34}{s} + 0.783s$$

The transfer function of the PID controller is given below and the step response of unity feedback closed loop system with and without the presence of PID controller is shown in Fig. 2. The parameters of the unit step response are shown in Table 1.

![Step Response of the closed loop system with and without PID controller](image)

**Table 1. Step response parameters**

<table>
<thead>
<tr>
<th></th>
<th>Setting Time (sec)</th>
<th>Rise Time (sec)</th>
<th>Overshoot (%)</th>
<th>Steady state error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without PID</td>
<td>12</td>
<td>2.21</td>
<td>23.74</td>
<td>50</td>
</tr>
<tr>
<td>With PID</td>
<td>6.11</td>
<td>3.64</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

From the step response parameters it can be clearly seen that the PID controller plays an important role in improving the response of the plant.

### III. INTERNAL MODEL CONTROL

Consider an open loop system shown in Fig.3. The output of the system $Y(s)$ depends on the reference input $R(s)$ and is given by the equation given below:

$$Y(s) = G_c(s)G_r(s)R(s)$$

![Open loop control](image)

If $G_c$ is exactly the inverse of $G_r$, $Y(s)$ would become exactly equal to $R(s)$ i.e. it would result in perfect tracking. However the practical implementation of such a controller is not possible. If the plant model has Right Hand Plane (RHP) zeros, then the controller would have RHP poles and hence the output of the controller would not be bounded resulting in system instability. So, the idea behind the IMC technique is to make the controller transfer function as close as possible to $[G_r]^{-1}$. The general structure of IMC is shown in Fig. 4.

The transfer functions used in Figure 4 are listed below:

- $d(s)$ = disturbance
- $Q_r$ = process model
- $G_r$ = Actual process
- $G_c$ = IMC controller

The closed loop transfer function of the IMC controller is given by Equation (3)

$$Y(s) = \frac{G_c(s)G_r(s)}{1+G_c(s)(G_r(s)-Q_r(s))}R(s) + \frac{1-G_c(s)Q_r(s)}{1+G_c(s)(G_r(s)-Q_r(s))}d(s)$$

(3)

The steps involved in the design of an IMC controller are as follows:

1. The process model is separated into two factors consisting of invertible elements ($Q_r$) and non-invertible elements ($Q_{\text{non}}$) as shown below:

$$Q_r(s) = Q_{\text{inv}}(s)Q_{\text{non}}(s)$$

2. The form of the IMC controller is taken as the product of inverse of the invertible term of the process model and a shaping filter to make the controller proper. A transfer function is said to be proper if the degree of its denominator is at least equal to the degree of the numerator.

$$G_c(s) = [Q_{\text{inv}}(s)]^{-1}f(s)$$

In order to achieve proper tracking of the input, the filter transfer function is set as:

$$f(s) = \frac{1}{(\lambda s + 1)^n}$$

Where the factor ‘$n$’ is chosen to make the transfer function proper.

In order to track the changes in set point it would be desirable to choose the filter transfer function as:

$$f(s) = \frac{n\lambda s + 1}{(\lambda s + 1)^n}$$

3. Finally the parameter ‘$\lambda$’ is selected depending on the speed of response of the closed loop system. If the value of ‘$\lambda$’ is large the closed loop system is less sensitive to variation in parameters and for small values of ‘$\lambda$’ the response becomes fast. The output
of the closed loop system with IMC controller becomes:

\[ Y(s) = \left[ Q_c(s) \right]^{-1} f(s)G_c(s)R(s) \]

If the plant model is same as the actual process then the output can be written as:

\[ Y(s) = Q_c(s)f(s)R(s) \]

Example: Let us consider the plant model used in the previous example and let us assume that the plant model is same as the transfer function actual process:

\[ G_c(s) = \frac{1}{(s+1)^4} \]

IMC controller is designed for the above process using the SISOTOOL of MATLAB®. The transfer function of the IMC controller was obtained as:

\[ G_c(s) = \frac{(s+1)^4}{(0.454s+1)^5} \]

Here \( \lambda = 0.454 \). The step response of the closed loop system with and without the IMC controller is shown in Fig. 5 and the parameters of the step response are shown in Table II.

### Table 2. Step response parameters

<table>
<thead>
<tr>
<th></th>
<th>Without IMC</th>
<th>With IMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Settling Time (sec)</td>
<td>12</td>
<td>4.13</td>
</tr>
<tr>
<td>Rise Time (sec)</td>
<td>2.21</td>
<td>2.24</td>
</tr>
<tr>
<td>Overshoot (%)</td>
<td>23.74</td>
<td>0</td>
</tr>
<tr>
<td>Steady state error (%)</td>
<td>50</td>
<td>0</td>
</tr>
</tbody>
</table>

IV. CONCLUSION

A review of two important control system design techniques namely, PID control and IMC controller were presented in this paper. The steps involved in the design of these controllers were presented along with suitable examples. The design of these controllers was done using MATLAB® and the results were presented. It was observed that these techniques improved the closed loop response of the system.

REFERENCES