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A Review of Research Progress on Friction Pendulum Isolation Bearings: Theory, Experiment, and Numerical Simulation

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***Abstract*—The friction pendulum system (FPS), as a typical recentering–energy dissipation isolation device, has significant application value in structural seismic engineering. This paper systematically reviews the research progress in theoretical modeling, experimental investigations, and numerical simulations of FPS. Starting from the fundamental mechanical mechanism of the single friction pendulum (SFP), the decoupling characteristic between isolation period and structural mass is analyzed, and the development of double friction pendulum (DFP) and triple friction pendulum (TFP) systems is outlined, highlighting their staged control capability achieved through multiple sliding interfaces. Based on material- and component-level experiments, the time-dependent behavior of the friction coefficient under the coupled effects of velocity, normal pressure, and temperature is elucidated. Furthermore, refined finite element models and macroscopic simplified models are compared in terms of modeling strategies and applicability. The current limitations in multi-physics coupling mechanisms, three-dimensional dynamic response characterization, and life-cycle performance evaluation are identified, and future research directions are discussed. It is shown that establishing a unified analytical framework incorporating multi-field coupling and three-dimensional nonlinear analysis is essential for improving the accuracy and reliability of FPS-based seismic isolation systems.**

***Index Terms*—Friction Pendulum System (FPS),Multi-stage friction pendulum,Multi-physics coupling,Numerical simulation,Seismic performance**

I. Introduction

As a natural disaster characterized by high randomness and destructive power, earthquakes have long posed a central challenge in the field of civil engineering. Traditional seismic design relies on structural members entering a non-elastic phase to dissipate energy; this strategy of trading damage for safety is prone to causing significant residual deformation and structural damage during strong earthquakes, resulting in high post-earthquake repair costs and, in some cases, forced demolition due to loss of functionality [1–3]. To meet the demands of resilient urban development, the philosophy of seismic engineering is shifting from ensuring life safety to achieving functional recovery [4]. China's 2021 "Regulations on Seismic Management of Construction Projects" [5] explicitly require that important public buildings in high-seismic-intensity zones prioritize the use of seismic isolation and vibration-damping technologies. Against this backdrop, the Friction Pendulum System (FPS) has become a focal point of research and application due to its unique mechanical properties [6].

Compared to traditional lead-rubber bearings (LRB), as shown in Figure 1, the FPS offers significant advantages: The natural period depends solely on the radius of curvature of the sliding surface, completely decoupling it from the mass of the superstructure; gravitational forces can be utilized to provide restoring forces, enabling excellent self-resetting capability and significantly reducing residual displacement; simultaneously, energy dissipation through dry friction produces a full and stable hysteresis curve, demonstrating superior energy dissipation performance [7,8]. However, as application scenarios extend into complex environments, traditional single-stage FPS struggles to meet the demands of extreme conditions. Under the action of near-fault velocity pulses, the displacement of the seismic isolation layer is prone to exceeding limits; in high-rise structures, the problem of tensile detachment caused by overturning moments is prominent; and the thermal degradation of sliding plate materials under high-speed reciprocating friction severely affects long-term service performance [9]. Therefore, the development of multi-stage friction pendulum bearings and models of their performance degradation mechanisms has become an important research

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direction in the field of seismic isolation.

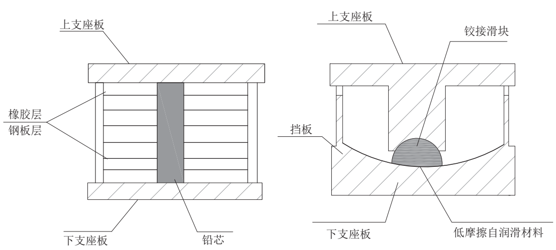

Fig. 1. Comparison between traditional Lead rubber bearing and traditional friction pendulum bearing

This paper systematically summarizes the latest advancements in friction pendulum bearings regarding the evolution of mechanical theories, multiscale experimental validation, and high-performance numerical simulation. It provides an in-depth analysis of the dynamic response mechanisms of multi-stage friction pendulums under complex loads, identifies research bottlenecks and breakthrough opportunities, and offers guidance for the design and promotion of high-performance, resilient seismic isolation systems.

Friction pendulum supports provide restoring force through their curved geometry and dissipate seismic energy via interfacial friction, making them a typical "restoring-damping" synergistic seismic isolation system [1,2,10]. Since Zayas et al. proposed the single-stage model in the 1980s, its form has evolved from single-stage to multi-stage configurations, essentially to meet performance control requirements under different seismic levels [8,11]. Based on this, we first analyze the fundamental mechanism of the single-stage friction pendulum.

The single-stage friction pendulum (SFP) consists of a concave spherical slideway and a spherical slider; the contact surfaces are typically coated with self-lubricating materials such as modified PTFE [12], as shown in Figure 2. Its operating mechanism is as follows: as the slider moves along the curved surface, the tangential component of gravity provides geometric self-resetting capability. Its equivalent lateral stiffness is determined solely by the vertical load and the radius of curvature, exhibiting geometric control characteristics that are proportional to the vertical load and inversely proportional to the radius of curvature [11,13]. Based on the motion equations of a simple pendulum, its seismic isolation period is:

$$T = 2\pi\sqrt{\frac{R}{g}} \tag{1}$$

This equation indicates that the seismic isolation period is determined solely by the radius of curvature and is independent of the structural mass, exhibiting a "period-mass decoupling" characteristic. However, because its displacement capacity and mechanical parameters are not adjustable, SFP systems struggle to simultaneously meet the requirements for both small-to-medium and large earthquakes; therefore, multi-stage friction pendulum systems have emerged as a direction for further development [8, 14].

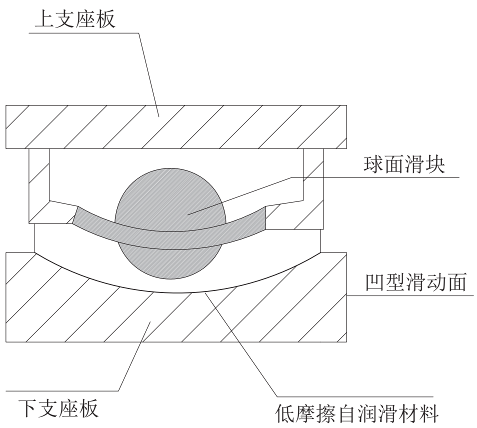

Fig. 2. Single friction pendulum bearing

To address the limitations of single-stage friction pendulums regarding displacement capacity and parameter flexibility, Daniel M. Fenz and Michael C. Constantinou proposed the Double Friction Pendulum (DFP) [8]. This device consists of two concave sliding surfaces—one upper and one lower—and a double-sided slider in between; it can essentially be viewed as a series configuration of two SFPs. Compared to the SFP, the DFP significantly extends the effective sliding path through coordinated sliding at both interfaces, thereby enhancing the displacement capacity of the seismic isolation layer. At the same time, by setting different friction coefficients, it achieves a staged sliding mechanism: under low shear forces, only the low-friction interface initiates sliding; as the load increases, both interfaces slide simultaneously, exhibiting a two-stage hysteresis characteristic [8,14]. This mechanism enables the structure to respond differentially to seismic motions of varying intensities, thereby achieving preliminary performance-based control; however, it also imposes higher design requirements for the stability of interface coordination.

Building upon the DFP, Daniel M. Fenz and Michael C. Constantinou proposed the Triple Friction Pendulum (TFP) and its multi-stage extensions [8] to further meet safety requirements under extremely rare earthquakes. Compared to the two-stage response of the DFP, the TFP achieves more refined hierarchical control through multiple slip interfaces

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TABLE I
IN-DEPTH COMPARISON OF KEY MECHANICAL PROPERTIES AND DESIGN CHARACTERISTICS OF FRICTION PENDULUM BEARINGS (FPS) WITH DIFFERENT ORDERS

Comparison Dimension	**Single Friction Pendulum (SFP)**	**Double Friction Pendulum (DFP)**	**Triple Friction Pendulum (TFP)**
Structural Composition	Single spherical slider + Single concave slider	Double-concave slider + Intermediate double-sided slider	Nested sliders + Four sliding interfaces + Double internal concave surfaces
Effective Radius (R_{eff})	$R_1 - h_1$	$(R_1 - h_1) + (R_2 - h_2)$	Involves complex combinations of 4 sets of curvature radii
Number of Friction Interfaces (μ)	1 set of fixed friction coefficients	2 sets (typically $\mu_1 = \mu_2$ or asymmetric)	4 sets ($\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$)
Hysteretic Characteristic Curves	Bilinear: Single stiffness	Trilinear: Variable stiffness inflection points	Multi-linear: Five-stage stiffness switching
Displacement Capacity (d_{max})	$2(R - h)\sin\theta$ (Limited by slider diameter)	Theoretically 2 times that of SFP with the same diameter	Extensive geometric redundancy, reaching 3-4 times that of SFP
Self-Centering Capability	Provided by single curvature, direct restoring path	Collaborative resetting by double surfaces, slip sequence exists	Complex progressive resetting, affected by multi-level friction gradients
Performance Adjustment Methods	Can only adjust single R and μ	Can adjust combinations of 2 sets of R and μ	Multi-degree-of-freedom adjustment: Can precisely tune responses for different earthquake levels
Main Mechanical States	Sliding / Stopping	1-surface sliding / 2-surface full sliding / Stopper engagement	Internal sliding / Coordinated sliding / External sliding / Stiffening

and parameter combinations. Its typical feature is a multi-stage, sequential activation mechanism: during low-intensity earthquakes, the inner interface slips first to isolate micro-vibrations; as seismic intensity increases, the remaining interfaces engage sequentially, allowing the system stiffness to adjust gradually and enhancing energy dissipation capacity; under extreme conditions, the displacement-constraint mechanism provides additional recovery and stabilization functions [8,15]. Through the combined design of geometric parameters and friction characteristics, the TFP enables adjustable control of hysteretic behavior and mechanical response, embodying a design philosophy of multi-level energy dissipation and hierarchical protection, thereby significantly enhancing the comprehensive seismic performance of seismic isolation structures [10,15].

Based on the aforementioned evolutionary process (see Table 1), the focus of research on friction pendulum bearings has shifted from simple displacement compensation to multi-objective performance tuning. SFP laid the foundation for pendulum-type seismic isolation, DFP addressed the geometric limitations on displacement capacity, and TFP achieved precise matching between mechanical response and seismic requirements through a complex matrix of interface parameters. This evolutionary trajectory provides the physical basis for the subsequent chapters to delve into the theory of nonlinear dynamics and refined numerical simulations.

As friction pendulum seismic isolation technology evolves from single-stage systems to multi-stage sliding structures, its mechanical behavior exhibits significant nonlinearity and multi-physics coupling characteristics, placing higher demands on traditional design theories and analysis methods. To systematically review research progress and key issues in this field, this paper provides a comprehensive overview across three dimensions—theoretical models, experimental studies, and numerical methods—and extends to engineering applications and future trends, thereby establishing a complete analytical framework spanning from mechanistic understanding to engineering implementation.

Chapter 2 focuses on the evolution of theoretical models, systematically analyzing the development of the constitutive model for the restoring force of friction pendulum bearings from an ideal bilinear model to a time-varying model that accounts for the coupling of pressure, velocity, and temperature, and elucidating the phased sliding mechanism and stiffness control principles of multi-stage friction pendulums (DFP/TFP). Chapter 3 centers on advances in experimental research, reviewing the results of friction interface material property tests and component-level dynamic loading tests. It focuses on the experimental verification of friction performance evolution and multi-stage sliding behavior under the coupled effects of multiple factors, and summarizes the limitations of existing experimental systems. Chapter 4 focuses on numerical

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simulation methods, comparing the modeling strategies and applicability of refined finite element models and macroscopic equivalent models, and analyzing trends in contact algorithms, multiphysics coupling, and efficient computational methods. Building on this foundation, subsequent chapters further integrate engineering applications to summarize the key issues and future directions of friction pendulum seismic isolation technology under complex operating conditions.

Overall, through a comprehensive multi-scale and multi-method analysis, this paper aims to provide a systematic reference for the theoretical research and engineering design of high-performance friction pendulum seismic isolation systems.

II. Advances in Theoretical Research on Friction Pendulum Seismic Isolation Systems

The mechanical constitutive model of a friction pendulum support serves as the theoretical foundation for evaluating the dynamic response, energy dissipation efficiency, and post-seismic recovery capacity of seismic isolation structures. This chapter begins with the fundamental force mechanisms of a single-stage friction pendulum to analyze its restoring force characteristics and period decoupling mechanisms, and further explores the evolution of time-varying friction characteristics under multiphysics coupling conditions.

A. Basic Mechanical Constitutive Model of a Friction Pendulum Bearing

The fundamental mechanism of a single-degree-of-freedom friction pendulum is to utilize the structural self-weight to provide a restoring force and to dissipate seismic energy through the work done by dry friction at the sliding interface [3, 16, 17]. Based on the large-radius-of-curvature and small-deformation assumptions ($\sin\theta \approx \tan\theta \approx u/R$), the horizontal output force of the support at any given instant can be expressed as [8, 11]:

$$F = \frac{W}{Reff}u + \mu W \operatorname{sgn}(\dot{u}) \tag{2}$$

In this model, the restoring force term $\frac{W}{Refff}u$ defines the system's restoring stiffness. $R_{eff} = R - h$ represents the effective radius of curvature, and h is the perpendicular distance from the center of the slider to the sliding surface. This term reveals the unique decoupling characteristic of the FPS system with respect to its period T, namely, that the system's seismic isolation period is primarily determined by the geometric curvature R and is independent of the mass of the superstructure [8,11], which significantly enhances the system's robustness under live load fluctuations. The friction term E defines the system's characteristic strength, where $\mu W \operatorname{sgn}(\dot{u})$ is the interfacial sliding friction coefficient, and $\operatorname{sgn}(\dot{u})$ ensures that the direction of the frictional force is always collinear with the velocity vector. In terms of macroscopic dynamic behavior, this constitutive model exhibits typical bilinear hysteresis characteristics. Its post-yield stiffness depends solely on the axial pressure and geometric radius; this proportionality between stiffness and load forms the physical basis for the FPS to achieve cooperative seismic isolation in multi-mass structures and effectively suppress torsional effects [8].

1) *V-Dependence:*Constantinou et al. proposed, based on extensive dynamic test data, that A asymptotically transitions from the initial sliding friction coefficient B to the high-slip friction coefficient C as the sliding speed increases. This evolution is typically described using an exponential decay model [17]:

$$\mu = \mu\max - (\mu\max - \mu\min)e^{-av} \tag{3}$$

where a represents the rate-sensitive hardening index.

2) *P-Dependence:*As the axial stress p increases, the increase in the shear strength of composite sliding plate materials is typically smaller than the increase in normal stress, resulting in a nonlinear decrease in the coefficient of friction [3,18]. This characteristic has a significant impact on the seismic isolation design of heavily loaded structures in high-intensity zones.
3) *T-Dependence:*The heat generated by high-frequency reciprocating friction under strong vibrations causes an instantaneous temperature rise at the interface, which in turn triggers thermal softening of the sliding plate material. The thermal softening effect has been verified through numerical analysis and experimental studies of frictional heat generation [19]. Current detailed theoretical studies have incorporated thermo-mechanical coupling algorithms to correct for energy dissipation attenuation under long-range dynamic responses.

Although traditional single-stage constitutive models offer extremely high computational efficiency, their single-stage stiffness behavior cannot capture the sequential initiation mechanism of multi-level sliding interfaces, making it difficult to meet the detailed requirements of multi-level seismic design.

B. Evolution of the Series Mechanical Constitutive Model for Multi-Stage Friction Pendulum Bearings

To address the limitations of SFP in terms of displacement capacity and performance control, multiple sliding interfaces were introduced to evolve the system from bilinear to multi-stage nonlinear behavior [20]. The key lies in the coordinated sliding and displacement distribution mechanisms among these multiple interfaces, thereby enabling hierarchical control of seismic isolation performance [15, 21].

The DFP can be regarded as a series configuration of two SFPs. The forces at each interface remain consistent, and the total displacement satisfies the superposition principle. Its mechanical response exhibits a phased evolution. During the initiation phase, the low-friction interface slips preferentially. In the cooperative sliding phase, as the load increases, both interfaces participate simultaneously, and the system stiffness undergoes a transition. This mechanism extends the equivalent radius of curvature, thereby significantly increasing the displacement capacity within a finite size and enhancing the

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TABLE II
PERFORMANCE DIFFERENCES OF THREE TYPES OF FRICTION PENDULUM SYSTEMS

Indicator	SFP	DFP	TFP
Stiffness Regulation Capability	Single	Dual-stage	Multi-stage
Energy Dissipation Capacity	Moderate	Strong	Strongest
Parameter Complexity	Low	Medium	High
Design Difficulty	Low	Medium	High
Engineering Applicability	Relatively Broad	Relatively Optimal	Limited

system's resilience to strong earthquakes (particularly those near faults) [22].

Building upon the DFP, the TFP constructs a multi-stage response system through multiple sliding interfaces, whose hysteresis behavior can be summarized as a continuous process of stage-by-stage activation—stiffness transition—and ultimate constraint. In the initial stage, the inner interface is activated to achieve micro-vibration isolation. During the transition stage, multiple interfaces slip in coordination, with stiffness evolving smoothly to reduce the acceleration response. In the ultimate stage, geometric confinement triggers stiffness hardening, providing displacement restraint [6,21]. Through the combined design of friction coefficients and geometric parameters, the TFP enables precise control of hysteretic performance, demonstrating multi-level energy dissipation and hierarchical protection characteristics.

For a generalized system comprising n sliding interfaces, its restoring force–displacement relationship can be uniformly described based on displacement superposition and force equilibrium, reflecting the coupled control of multiple interface parameters on the overall nonlinear response [15,21]. This framework reveals a theoretical shift in multi-order friction pendulums from a single passive response to multi-parameter coordinated control. However, existing models often neglect the effects of slip eccentricity and the thermal–wear coupling at interfaces. Future research should incorporate multi-field coupling mechanisms to improve prediction accuracy under extreme seismic loads.

C. Shortcomings and Challenges in Current Theoretical Research

Although the theory of friction pendulum bearings has evolved from single-order models to multi-order series systems, there are still shortcomings in its detailed description under complex operating conditions. The key issues and potential directions for future research can be summarized as follows:

1) *Inadequate Characterization of Multiphysics Coupling: Understanding Mechanisms and Approaches to Unified Modeling*

   Most existing P–V–T models are based on empirical fitting and struggle to systematically characterize the intrinsic coupling mechanisms among frictional heat generation, material thermal softening, and interfacial wear. This limitation stems from the lack of a unified theoretical framework for energy conversion and material degradation. Future research should focus on interfacial energy conservation and dissipation mechanisms to develop a unified multi-field constitutive model integrating thermal, mechanical, and wear processes, thereby enabling a time-dependent description of how the coefficient of friction and energy dissipation capacity evolve under varying operating conditions.

2) *Simplified Mechanism of Multi-Interface Cooperative Sliding: Asynchronous Evolution and Path Dependence Mechanisms*

   Traditional serial models are based on the assumptions of force continuity and superposition of displacements, implicitly assuming that all slip interfaces initiate simultaneously or in an ideal sequential order. As such, they struggle to capture the actual differences in initiation timing and path-dependent characteristics among interfaces. The fundamental issue lies in the lack of a description of the evolution of interface states. Future work should introduce state variables and slip criteria to develop higher-order constitutive models capable of describing asynchronous slip across multiple interfaces, energy distribution, and path-dependent evolution.

3) *Absence of Multidirectional Coupling Effects in Space: Three-Dimensional Dynamic Response and Eccentricity Mechanisms*

   Existing theories are largely based on unidirectional or planar analyses, neglecting the spatial motion trajectories of the sliding block under complex seismic actions and the resulting dynamic eccentricity effects. Consequently, it is difficult to accurately assess the spatial non-uniformity of contact pressure distribution and friction energy dissipation. In the future, it will be necessary to develop a three-dimensional nonlinear dynamic analysis framework that systematically accounts for multidirectional seismic inputs and geometric coupling effects.

4) *Insufficient Understanding of Response Mechanisms Under Extreme Conditions: Impact Behavior and Local Damage Evolution*

   Existing research on the response of third-order friction pendulums during the limit displacement phase has primarily focused on stiffness switching criteria, while in-depth analysis of the impact dynamics and local damage mechanisms caused by limit contact is lacking. The key issue is that the high-frequency response during the impact process and material failure have not been incorporated into a unified description. Future research should integrate contact impact theory with damage mechanics to refine dynamic response models under extreme seismic loads.

5) *Missing Long-Term Service Performance Evolution: Time-Varying Constitutive Models and Reliability Modeling*

Most existing resilience models are based on initial state parameters and fail to account for the ongoing effects of material aging, corrosion, and cumulative wear on friction properties. The fundamental issue lies in the absence of a parameter evolution mechanism that incorporates the time dimension. In the future, time-dependent constitutive models covering the entire life cycle should be developed, combined with probabilistic statistical methods to describe the uncertainty in parameter degradation, thereby supporting the long-term performance evaluation and reliability design of seismic isolation systems.

Upon further examination, although the single-stage friction pendulum (SFP) model is theoretically well-established, its fundamental assumptions still have certain limitations. Traditional models typically simplify the friction coefficient to a constant or consider only the velocity effect [23], making it difficult to reflect the coupled effects of multiple factors such as temperature, pressure, and interface wear; simultaneously, due to its structural characteristics based on a single radius of curvature, it struggles to characterize multi-stage dynamic responses under complex seismic loads. In contrast, multi-stage friction pendulums (DFP, TFP) achieve graded control of stiffness and energy dissipation capacity by introducing multiple slip interfaces and parameter combinations, offering significant advantages in terms of displacement capacity, response adaptability, and control under extreme conditions. Therefore, in the context of high-performance seismic design, friction pendulum isolation systems are evolving from single-stage to multi-stage configurations [24]. Based on existing research findings, the performance differences among the three types of friction pendulum systems can be summarized as follows (see Table 2).

Therefore, developing a refined mechanical model that accounts for multi-directional spatial coupling, multi-field evolution at interfaces, and long-term performance degradation remains a critical bottleneck in future theoretical research on friction pendulum seismic isolation.

III. Recent Advances in Research on Friction Pendulum Bearings

The accuracy of theoretical constitutive models depends on high-precision experimental data. The performance of friction pendulum bearings under seismic loading is influenced by the coupled effects of multiple factors, including material wear, heat generation at interfaces, and contact pressure distribution. Based on this, this paper systematically reviews recent research progress in China and internationally regarding material friction characteristics, mechanical properties of components, and structural shake table tests, with the aim of elucidating the relationship between theoretical predictions and actual dynamic responses.

A. Testing of Friction Interface Characteristics and Material Properties

The stability of the friction coefficient is a key factor determining the seismic isolation efficiency of FPS systems [8, 11]. Early theoretical studies often assumed the friction coefficient to be constant; however, extensive tribological experiments at the interface have shown that its value exhibits significant multi-parameter correlations and spatiotemporal evolution characteristics [25].

The stability of the friction coefficient is a key factor affecting the seismic isolation performance of friction pendulum supports (FPS) [26]. Early studies often simplified it as a constant, but extensive tribological experiments have shown that the friction coefficient exhibits significant multi-parameter coupling and spatiotemporal evolution characteristics [13]. First, under the coupled effects of velocity and pressure, the friction behavior at the interface exhibits typical nonlinear patterns: as sliding velocity increases, the friction coefficient rapidly transitions from static to kinetic friction and stabilizes, a process that can be described by an exponential model; conversely, as axial pressure increases, the increase in shear strength is smaller than the increase in normal stress, leading to an overall decrease in the friction coefficient [25]. This effect is particularly pronounced in heavy-load seismic isolation designs. Second, under long-term or large-displacement cyclic loading, frictional heat generation and material wear further alter interface properties. Experiments indicate that high-frequency reciprocating sliding can cause a significant rise in interface temperature and lead to thermal softening of polymeric materials [27], thereby triggering hysteretic strength degradation; simultaneously, the generation and migration of wear particles alter the contact morphology, causing random fluctuations in the friction coefficient of approximately $10\%-20\%$. Furthermore, environmental factors exert a significant influence on interface properties. Under low-temperature conditions, the viscoelasticity of materials increases, manifesting as rises in initial friction force and equivalent stiffness; corrosion and salt spray exposure, on the other hand, exacerbate sliding plate wear and roughen the sliding tracks, thereby increasing the variability of the coefficient of friction [28]. During long-term service, these factors interact to alter the microstructure of the interface, consequently affecting the stability and durability of the seismic isolation system.

In summary, the coefficient of friction is jointly governed by velocity, pressure, temperature, and environmental conditions, and its dynamic evolution determines the time-varying characteristics of the restoring force. Therefore, elucidating the mechanisms of interfacial friction is fundamental to accurately

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TABLE III
COMPARISON OF DOMINANT NONLINEAR FACTORS OF FRICTION PENDULUM BEARINGS IN DIFFERENT ENGINEERING SCENARIOS

Engineering scenario	**Core mechanical characteristics**	**Dominant nonlinear factors**	**Requirements for constitutive model**
High-rise buildings	Large axial pressure, large height-to-width ratio causing overturning moment	Pressure dependence	Need to introduce a variable friction coefficient model that considers pressure dependence
Large-span spatial structures	Large displacement span, long period, low-frequency response	Geometric nonlinearity	Large displacement switch must be turned on to correct restoring force characteristics
Alpine/Severe cold regions	Extremely low ambient temperature (-40°C)	Ambient temperature	Need to consider the initial sliding criterion modified by ambient temperature
Near-fault/Strong earthquake zones	Extremely fast reciprocating velocity, long sliding history	Velocity and heat generation	Need to introduce a velocity-temperature coupling subroutine to simulate strength degradation of hysteresis loops
Coastal/Heavy corrosion environments	Long-term salt spray erosion, high humidity	Surface roughness	Need to consider performance drift and parameter randomness over long service life

characterizing the multi-stage slippage and energy dissipation behavior of multi-stage friction pendulums (DFP/TFP).

B. Study on Mechanical Properties of Members (DFP/TFP) Under Loading Conditions

Building on the understanding at the material level, component-scale tests further validated the overall mechanical behavior of multi-stage friction pendulum bearings [29]. Extensive pseudo-dynamic and quasi-static loading tests on full-scale and large-scale models demonstrated that the synergistic interaction of multiple slip interfaces can generate a stable, multi-stage response mechanism at the macroscopic level [1, 15]. The test results show that as the external load increases, each sliding interface activates sequentially, causing a change in the stiffness of the hysteresis curve, which validates the predictive capability of the series constitutive model for multi-stage sliding sequences. At the same time, there is an uneven distribution of displacement between interfaces, and due to differences in initial friction, the actual sliding sequence may deviate slightly from the theoretical results.

For the third-order friction pendulum [30], tests conducted during the large-displacement phase further revealed its ultimate mechanical characteristics: when the slider contacts the geometric stop, the system stiffness increases significantly, which is consistent with theoretical predictions and demonstrates an effective displacement constraint mechanism [8]; however, under high-speed loading conditions, the stop collision generates an instantaneous impact effect, placing higher demands on material strength and connection design [31]. Shake table tests also indicate that the friction pendulum support exhibits excellent overall dynamic performance. On the one hand, its stiffness-axial force relationship causes the stiffness center of the isolation layer to approach the mass center, thereby effectively suppressing torsional response; on the other hand, although friction introduces some residual displacement, this value is small under conditions of reasonably designed curvature radius, and the system still possesses good self-resetting capability.

Overall, the component-level tests have verified the synergistic mechanism of “staged slippage—stiffness switching—energy dissipation” in multi-stage friction pendulum systems at the macro level, providing critical experimental evidence for their engineering applications. At the same time, the results indicate that impact and multi-field coupling effects still require further consideration under extreme operating conditions.

C. Limitations and Challenges of Current Experimental Studies

Based on the results of comprehensive material interface tests and component-level dynamic tests, it is evident that while existing research has validated the effectiveness of friction pendulum seismic isolation systems across multiple scales, there are still shortcomings in terms of high-precision performance evaluation and adaptability to complex operating conditions. These shortcomings are primarily reflected in the following aspects:

1) Insufficient characterization of multi-field coupling mechanisms:Although existing experiments have identified the patterns by which pressure, velocity, and temperature influence friction behavior, there has been a lack of systematic investigation into the coupling mechanisms underlying the evolution of frictional heating, material softening, and wear under long-term seismic loading, which has limited the ability to accurately characterize energy-dissipative degradation processes.

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2) There is insufficient data on long-term service in extreme environments:Most existing low-temperature and corrosion tests have been conducted under short-term conditions, and there is a lack of long-term test data that accounts for the coupled effects of aging, wear, and repeated seismic actions, making it difficult to support the evaluation of the performance of seismic isolation systems over their entire life cycle.
3) There is a lack of research on the effects of multi-directional coupling dynamics:Existing component tests primarily involve uniaxial or biaxial loading, with insufficient attention paid to fluctuations in vertical axial force, the spatial motion of the sliding block, and the coupled effects of triaxial seismic inputs; this limits the accuracy of mechanical model validation under complex loading conditions.
4) Scale effects and boundary conditions have a significant impact:Due to limitations in test equipment and cost, existing research largely relies on scaled models, which struggle to fully capture the effects of frictional heat generation, impact forces, and boundary constraints on mechanical responses in actual engineering applications. This limits the applicability of these findings to heavy-load and large-scale structures.
5) Existing general macro models often use a uniform friction coefficient, which can lead to significant errors when modeling pressure fluctuations in high-rise buildings or the initial dynamic characteristics in cold regions, as shown in Table 3.

Therefore, the development of refined numerical simulation methods that account for multi-field evolution, multi-body coupling, and scale-dependent effects has become a key approach to overcoming the limitations of experiments and gaining a deeper understanding of the complex mechanical behavior of friction pendulums.

IV. Recent Advances in Numerical Simulation of Friction Pendulum Bearings

With the advancement of high-performance computing, numerical simulation has become a key method for analyzing the complex mechanical behavior of friction pendulum bearings. Compared to physical testing, it allows for the simulation of extreme operating conditions at a lower cost and reveals microscopic field information, such as interface pressure distributions and heat generation dynamics. Based on this, this paper reviews recent research progress in three areas: refined finite element modeling, macroscopic constitutive algorithms, and multiphysics coupling simulations.

A. Recent Advances in Refined Finite Element Modeling

In general-purpose finite element platforms such as ABAQUS, the definition of contact constitutions and interface properties is critical to modeling [32]. Normal behavior is typically modeled using a hard-contact model, combined with penalty functions or the Lagrangian method to improve numerical stability under high-pressure contact; regarding tangential behavior, traditional Coulomb models struggle to capture the nonlinear evolution of friction. In recent years, coupled effects of velocity, pressure, and temperature have been introduced via user subroutines [33] to enable dynamic updates of the friction coefficient, thereby enhancing the accuracy of hysteresis response simulations.

Regarding discretization strategies, sliding blocks and tracks generally employ reduced-integral hexahedral elements to avoid numerical locking during large-deformation shear processes [34]; simultaneously, geometric nonlinearity options must be enabled to accurately describe path changes caused by spherical slippage [35]. Studies indicate that mesh density in the contact region significantly influences contact pressure and friction response, and local refinement is an effective means of improving computational accuracy [36]. For highly nonlinear contact problems, the choice of algorithm is equally critical. Implicit methods are suitable for conventional hysteresis analysis but may experience convergence difficulties during the initial stages of multi-interface contact; explicit methods, however, offer advantages in strong seismic impact and limited-deflection collision problems, and when combined with mass scaling and damping control, can effectively capture transient nonlinear responses.

Overall, refined models can accurately characterize mesoscale mechanisms such as contact stress distribution, local stress concentration, and frictional heat generation, serving as primary tools for structural optimization and extreme condition analysis. However, their high computational cost limits their application in large-scale structural analysis [34,35].

B. Recent Advances in Macro-Scale Models and Simplified Computational Units

To improve the efficiency of engineering analysis, macroscopic models based on the principle of mechanical equivalence have become an important branch of numerical simulation [3]. These methods achieve an equivalent representation of complex contact behavior by abstracting the restoring force and energy dissipation characteristics of friction pendulum supports.

In platforms such as ABAQUS, constructing macroscopic models using connection elements is a common strategy [34]. By combining axial constraints with slip constitutive relations, geometric restoring forces and frictional energy dissipation can be equivalently described; for multi-stage friction pendulums (DFP/TFP) [8,37], multiple elements are typically connected in series with differentiated slip thresholds to simulate multi-stage stiffness evolution. To improve numerical stability, idealized elastoplastic constitutions are often regularized. In platforms such as OpenSees [1, 38] and SAP2000, specialized friction pendulum elements further enhance analysis efficiency. Based on the Fenz–Constantinou theoretical framework, these elements achieve multi-stage slip responses through displacement accumulation and state determination; simultaneously, geometric nonlinearity corrections are introduced under large-displacement conditions to account for the effects of slider lifting and axial force variations on restoring forces, thereby

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mitigating the underestimation of displacement responses inherent in traditional linear models. From an applicability perspective, refined models and macroscopic models each possess distinct advantages and are complementary: the former, based on solid modeling, can accurately characterize contact stresses and interface evolution mechanisms, making them suitable for mechanism studies and ultimate performance analysis; the latter significantly reduces computational complexity through constitutive abstraction, making it suitable for dynamic time-history analysis and parameter sensitivity studies of long-span or high-rise structures.

Therefore, current research often adopts a collaborative strategy of "using detailed models to reveal mechanisms and macro models for engineering analysis" to strike a balance between computational accuracy and efficiency.

C. Summary of This Chapter

This chapter provides a systematic overview of the current state of research and technical approaches regarding friction pendulum bearings in the field of numerical simulation. A comparative analysis reveals that existing numerical studies have established a two-scale simulation framework that emphasizes both microscopic mechanisms and macroscopic responses.

1) Refined solid finite element models, by accurately characterizing contact, geometric nonlinearities, and material constitutive behavior, reveal the evolution of bearing stress distribution and frictional energy dissipation, providing support for refined design and ultimate limit assessment.
2) Macroscopic equivalent models, based on the abstraction of mechanical characteristics, employ series-connected nonlinear elements to efficiently simulate the variable stiffness behavior of multi-stage friction pendulums. They significantly reduce computational costs while ensuring accuracy, making them suitable for large-scale structural analysis.
3) Numerical methods have expanded from implicit static analysis to explicit dynamic and multi-physics coupled simulations, advancing research from single-mechanism responses to the simulation of service performance incorporating multi-factor coupling such as heat generation, wear, and corrosion.
4) To intuitively compare the applicability and performance differences of various modeling methods, Table 4 presents a systematic comparison between macroscopic models and refined finite element models.

Overall, numerical simulation studies not only serve as an effective complement to theoretical models and experimental research, but also play a crucial role in advancing friction pendulum technology from component development to performance design across the entire engineering lifecycle.

D. Shortcomings and Challenges in Current Numerical Simulation Studies

Although numerical simulations have made significant progress in elucidating the multiscale mechanical behavior of friction pendulum bearings, there remain critical shortcomings in terms of high-precision prediction and engineering applicability, primarily in the following areas:

1) Inadequate characterization of multiphysics coupling: Most existing models treat heat generation, wear, and corrosion effects separately, lacking a unified thermal-mechanical-wear coupling framework, which makes it difficult to accurately describe the synergistic evolution of interfacial temperature rise, material softening, and performance degradation.
2) Limited descriptive capability of macro-scale models: Macro-scale equivalent models are typically based on low-dimensional assumptions, making it difficult to capture the three-dimensional motion of the sliding block under multi-directional seismic loading, fluctuations in vertical axial force, and strong geometric nonlinear effects, which limits the accuracy of response predictions under complex conditions.
3) Insufficient synergy between computational accuracy and efficiency: Although refined finite element models can capture the microscopic mechanisms of contact, they are computationally expensive; existing methods still lack effective approaches to reduce computational complexity and improve efficiency while maintaining fidelity in key physical processes.
4) Inadequate full-lifecycle performance simulation: Current simulations are primarily based on initial parameters and do not fully account for the time-varying characteristics of parameters caused by material aging, corrosion, and cumulative damage, making it difficult to reliably assess the long-term performance of seismic isolation systems.

In summary, the development of a unified numerical framework that combines multi-field coupling capabilities, three-dimensional nonlinear response analysis, and high computational efficiency is a key direction for advancing friction pendulum bearings from mechanistic simulation to full-lifecycle performance prediction.

V. Summary and Outlook

A. Summary of the Entire Text

This paper systematically reviews research progress on friction pendulum bearings from three perspectives: theory, experimentation, and numerical simulation. Theoretically, a framework was established for the evolution from a first-order bilinear model to a time-varying constitutive model that accounts for the coupling of velocity, pressure, and temperature, and mechanisms for staged slippage and variable stiffness in multi-order friction pendulums were developed. Experimentally, the study reveals the nonlinear evolution of the friction coefficient under the coupled effects of multiple

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TABLE IV
COMPARISON OF APPLICABILITY BETWEEN MACRO-MODELS AND REFINED MODELS FOR FRICTION PENDULUM BEARINGS

Comparison Dimension	**Macro-model**	**Refined model**
Modeling Method	Constitutive model	Solid elements + contact
Computational Efficiency	High	Low
Computational Accuracy	Global response	Local behavior
Simulatable Content	Displacement, acceleration, hysteresis	Stress distribution, wear, heat generation
Geometric Nonlinearity	Considered equivalently	Considered explicitly
Applicable Scale	Global structural analysis	Component-interface mechanisms
Typical Applications	Time-history analysis, parameter analysis	Structural optimization, limit states

factors and verifies the synergistic mechanism of multi-stage slippage and energy dissipation. In terms of numerical simulation, a multi-scale analysis system has been established that integrates complementary fine-scale and macro-scale models. Overall, research on friction pendulums is shifting from single-mechanical analysis toward multi-field coupling and full-lifecycle performance evaluation.

B. Current Research Gaps and Future Directions

Although systematic progress has been made in friction pendulum seismic isolation technology at the theoretical, experimental, and numerical simulation levels, several key challenges remain to be addressed in order to meet complex engineering requirements and achieve high-performance seismic resistance:

First, the mechanisms of multi-field coupling need to be refined. Existing studies often treat thermal effects, wear, and environmental influences separately, lacking a unified framework for thermal–mechanical–wear coupling. Future research should focus on interfacial energy conversion and material degradation to develop constitutive models that reflect the time-dependent evolution of friction properties. Second, the characterization of spatial multi-directional dynamic responses remains inadequate. Existing models are often based on simplified assumptions and provide limited consideration of triaxial seismic inputs and the spatial motion of sliding blocks. Subsequent research should develop three-dimensional nonlinear analysis methods to improve prediction accuracy under complex operating conditions. Third, performance evaluation over the entire life cycle remains weak. Current research is largely based on initial parameters and does not fully account for the effects of aging, corrosion, and cumulative damage. Future efforts should combine long-term monitoring with accelerated testing to establish time-dependent models and conduct reliability analyses. Finally, the level of intelligence in seismic isolation systems needs to be improved. Traditional passive friction pendulums struggle to meet the demands of multi-level seismic events; future research could integrate smart materials to develop active-passive cooperative control technologies.

In summary, establishing an integrated theoretical and methodological framework that combines multi-field coupling capabilities, three-dimensional dynamic response analysis, and long-term performance prediction is a key direction for advancing friction pendulum seismic isolation technology toward higher precision and reliability.

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