A Review note on Compensator Design for Control Education and Engineering

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***Abstract*— The compensators are one of the most important aspects in any undergraduate control engineering courses owing to their widespread industrial applications. The degree of convergence of the output waveform for any compensator depends upon proper selection and tuning of the compensator. In this paper the compensator parameters (α, β, τ) of the lead, lag and lag-lead compensators are tuned to their optimum values using the conventional root locus as well as frequency response approach. The compensator algorithm is studied using MATLAB and usefulness of these compensators for controlling process variables are demonstrated using proper tuning. The comparative studies showing promising results are discussed with suitable examples.**

Keywords*— *Lead compensator, Lag compensator, Lag-Lead compensator, root locus approach, frequency response approach.

I. INTRODUCTION

Feedback control system with compensator is an important technique that is widely used in the process industries. Their main advantage is that corrective action occurs as soon as the controlled variable deviates from the set point, regardless of the source and type of disturbance.

Compensators are corrective sub-systems introduced into the system to compensate for the deficiency in the performance of the plant. So given a plant and a set of specifications, suitable compensators are to be designed so that the overall system will meet the given specification. Due to its simplicity and effectiveness, phase lag and lead compensation is essential for various frequency-based design methods, especially for design based on the Bode plots [9]. Another alternative design in terms of root locus was given in [10]. A non-trial-and-error design technique for lag-lead compensator is developed in [7] and [8]. In [4] a new lead compensator design algorithm is presented by using computation technology to facilitate the course teaching and implementation. In [5] a new method for the design of compensators for linear time-invariant dual-input/single-output systems in continuous time or discrete time is presented to reduce the problem to two single-input/single-output design problems. The development procedure of a fully automated software based One-Touch-and-Go PID controller and Lead/Lag compensator design tool is presented in [6] to help industrial process control engineers. Also A graphical procedure for the compensator design on the Nyquist plane is presented in [11] with some numerical examples.

However, the nature of traditional trial-and-error graphical techniques currently used by almost all available textbooks [1-3] makes the learning and use of lag-lead compensation.

In this paper, the specifications based on time domain analysis such as settling time, rise time, peak time, maximum overshoot etc. are measured from conventional root locus plots of the uncompensated as well as compensated system. In another way the frequency domain specifications like phase margin, gain margin, bandwidth are measured from bode plot of uncompensated as well as compensated system. Finally all the specifications results obtained from root locus approach and frequency response approach are compared and which technique is the best is discussed. And hopefully this technique will facilitate the control system engineer to select the precise way to design the compensators.

II. LEAD COMPENSATOR

The lead compensator is designed to satisfy the transient response of any system. The compensator having a transfer function of the following form is known as lead compensator [1], [2].

$$G_c(s) = K_c \alpha \frac{\tau s+1}{\alpha \tau s+1} = K_c \frac{s+\frac{1}{\tau}}{s+\frac{1}{\alpha\tau}} \qquad (1)$$

Where $(0 < \alpha < 1)$ and $(\tau > 0)$

A. *Using Root Locus Approach*

At first from the performance specifications, the desired locations for the dominant closed loop poles are determined. And secondly by drawing root-locus plot of the uncompensated system ascertain whether or not the gain adjustment alone can yield the desired closed loop poles. If not, the angel of deficiency ϕ is calculated. This angel must be contributed by the lead compensator if the new root locus is to pass through the desired locations for the dominant closed -loop poles. If static error constants are not specified, the location of pole and zero of the lead compensator is determined so that the lead compensator will contribute the necessary angle ϕ. If not other requirements are imposed on the system to make the value of α as large as possible. A large value of α generally results in larger value of K_v,(velocity constant) which is desirable . If a particular static error constant is specified, it is generally simpler to use the frequency-response approach. Then finally the open-loop gain of the compensated system from the magnitude condition is determined.

A.1. Example

The uncompensated system is taken as $G(s) = \frac{K}{s(s+1)}$.
The root locus plot is given in Fig.1. After considering above stated procedure the designed compensator is $G_c(s) = \frac{31.56(s+2)}{(s+9)}$ and the overall transfer function is $G(s)G_s(s) = \frac{31.56(s+2)}{s(s+1)(s+9)}$.
The root locus plot of this transfer function is given in Fig.2
All the Specifications regarding uncompensated and compensated are provide in TABLE I. The unit step response is also shown in Fig.3.

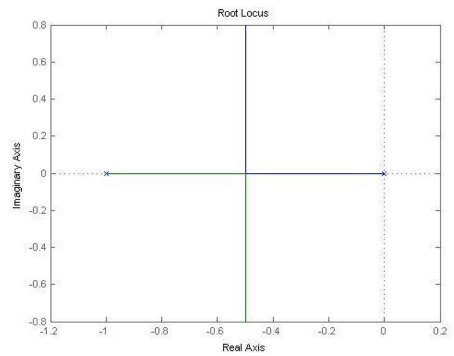

Fig.1. Root locus plot of uncompensated system [G(s)= K/s(s+1)]

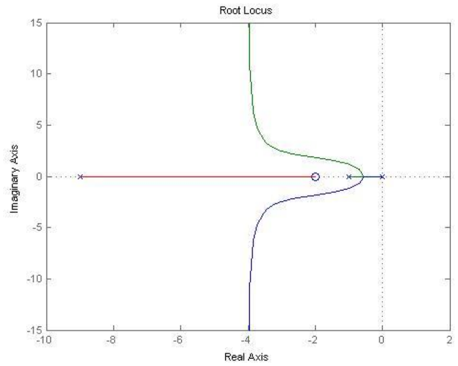

Fig.2. Root locus plot of the product of compensated system [G(s)= K/s(s+1)] and [G_c(s)=31.56(s+2)/(s+9)]

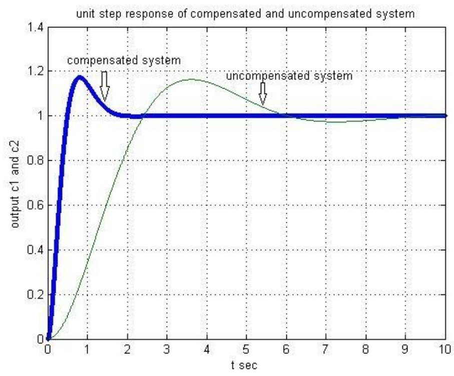

Fig.3.Output of the compensated and uncompensated system for lead compensator using root locus analysis

TABLE I
SPECIFICATION PARAMETERS FOR LEAD COMPENSATOR USING ROOT LOCUS ANALYSIS

System	Settling Time, T_s(sec)	Rise Time, T_r(sec)	Pick Time, T_p(sec)	Maximum Overshoot, %M_p
Uncompensated	8	2.42	3.63	16.30
Compensated	1.57	0.5	0.805	17.16

B. *Frequency Response Approach*

For designing the lead compensator firstly assumed $K_C\alpha = K$ and therefore $G_c(s) = K\frac{\tau s+1}{\alpha\tau s+1}$ according [1], [2]. The open-loop transfer function of the compensated system is now

$$G_c(s)G(s) = K\frac{\tau s+1}{\alpha\tau s+1}G(s) = \frac{\tau s+1}{\alpha\tau s+1}G_1(s) \qquad (2)$$

Where $G_1(s) = KG(s)$
Gain K is determined to satisfy the requirement on the given static error constant. Using the gain K, a Bode diagram of $G_1(j\omega)$ is drawn. The phase margin is evaluated from the plot. Then the necessary phase-lead angle to be added to the system is determined. An additional 5° to 12° to the phase-lead angle is required, because the addition of the lead compensator shifts the gain crossover frequency to the right and decreases the phase margin.
The attenuation factor α is determined by use of equation $\sin\Phi m = \frac{1-\alpha}{1+\alpha}$. The frequency where the magnitude of the uncompensated system $G_1(j\omega)$ is equal to -20log (1/√α) is determined and selected as the new gain crossover frequency. This frequency corresponds to $\omega_m = 1/\tau\sqrt{\alpha}$ and the maximum phase shift Φm occurs at this frequency.
The corner frequencies of the lead compensator are determined as follows:

Zero of lead compensator: $\omega = \frac{1}{\tau}$
Pole of lead compensator: $\omega = \frac{1}{\alpha\tau}$

Using the value of K and α ($K_c = \frac{K}{\alpha}$) K_c is determined. The gain margin is checked to be sure for satisfactory. If not, repeat the design process by modifying the pole–zero location of the compensator until a satisfactory result is obtained.

B.1.Example

Uncompensated system is taken as $G(s) = \frac{K}{s(s+1)}$
Designed lead compensator following the above procedure is $G_c(s) = \frac{0.377s+1}{0.125s+1}$. Hence the overall transfer functions $G(s)G_s(s) = \frac{12(0.377s+1)}{s(s+1)(0.125s+1)}$. All the specifications regarding the systems are listed in TABLE II. The bode plots of the compensated and uncompensated system are depicted in Fig.4. Also the unit step response is given in the Fig.5.

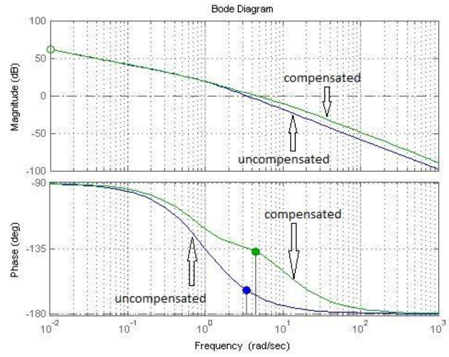

Fig.4. Bode plot of uncompensated [G(s)= K/s(s+1)] and compensated system

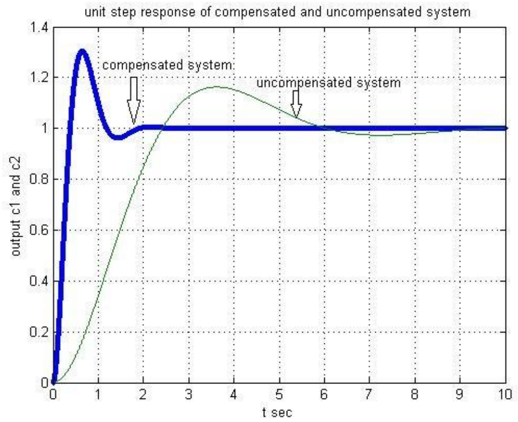

Fig.5. Output of the compensated and uncompensated system for lead compensator using frequency response analysis

TABLE II
SPECIFICATION PARAMETERS FOR LEAD COMPENSATOR USING FREQUENCY RESPONSE ANALYSIS

System	Phase Margin, PM (deg)	Gain Margin, GM (db)	T_s (sec)	T_r (sec)	T_p (sec)	$\%M_p$
Uncompensated	16.4	∞	8	2.42	3.63	16.3
Compensated	42.9	∞	1.67	0.38	0.645	30.39

C. *Comparative Study on the Specifications*

Depending upon the unit step response given in Fig.3 the specifications TABLE I is prepared. Similarly on the basis of step response in Fig.5 the TABLE II is listed. After comparing both the table it is observed that in the lead compensator using the root locus technique the maximum peak overshoot is quiet less than that of using Bode plot technique. It means that root locus technique is much more accurate for this design. While the changes of the rise time, the settling time and the peak time are very small amount comparing in both the techniques. Hence they are not so important aspects of consideration.

III. LAG COMPENSATOR

The lag compensator is used to satisfy steady-state characteristics. The transfer function of lag compensator is in the following form [1], [2].

$$G_c(s) = K_{c'}\beta\frac{\tau s+1}{\beta\tau s+1} = K_{c'}\frac{s+\frac{1}{\tau}}{s+\frac{1}{\beta\tau}} \qquad (3)$$

Where $(\beta > 1)$ and $(\tau > 0)$

A. *Using Root Locus Approach*

The procedure for designing the lag compensator using root locus approach is briefly discussed here [1].

Firstly by drawing the root-locus plot for the uncompensated system based on the transient-response specifications. The dominant closed-loop poles on the root locus plot are located. The particular static error constant specified in the problem is evaluated and to satisfy the specifications the static error constant is increased and the amount is determined. The pole and zero of the lag compensator that produce the necessary increment in the particular static error constant without appreciably altering the original root loci is determined. A new root-locus plot for the compensated system is drawn then. And the desired dominant closed-loop poles on the root locus must located on that. If the angle contribution of the lag network is very small (a few degrees) then the original and new root loci are almost identical. Otherwise, there will be a slight discrepancy between them. The gain $K_{c'}$ of the compensator from the magnitude condition is adjusted so that the dominant closed-loop poles lie at the desired location.

A.1.Example

The uncompensated system is taken as $G(s) = \frac{1.06}{s(s+1)(s+2)}$. The root locus plot is given in Fig.6. After considering above stated procedure the designed compensator is

$G_c(s) = \frac{0.966(s+0.05)}{(s+0.005)}$ and the overall transfer function is

$$G(s)G_s(s) = \frac{1.02396(s+0.05)}{s(s+2)(s+1)(s+0.005)}$$

The root locus for overall transfer function is given in Fig.7. Also the step responses are depicted in Fig.8. All the specifications are listed in TABLE III.

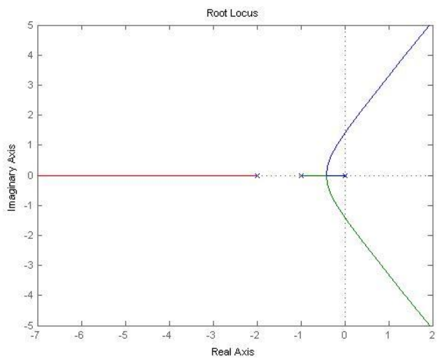

Fig.6. Root locus plot of uncompensated system [G(s)= 1.06/s(s+1)(s+2)]

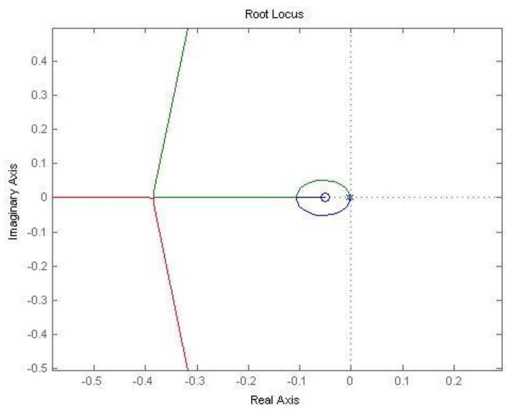

Fig.7. Root locus plot of the product of uncompensated system [G(s)= 1.06/s(s+1)(s+2)] and compensator [G_c(s)=0.966(s+0.05)/(s+0.005)]

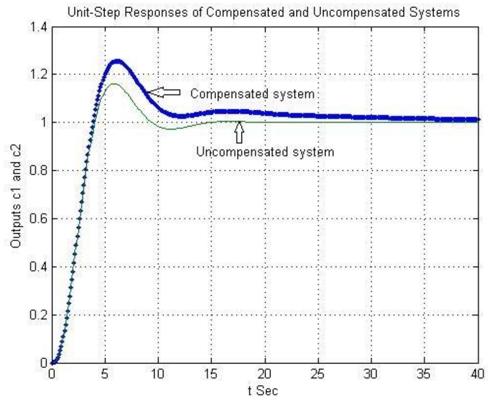

Fig.8. Output of the compensated and uncompensated system for lag compensator using root locus analysis

TABLE III
SPECIFICATION PARAMETERS FOR LAG COMPENSATOR USING ROOT LOCUS ANALYSIS

System	K_v (1/sec)	T_r (sec)	T_p (sec)	T_s (sec)	%M_p
Uncompensated	0.53	4.04	6	14	13.13
Compensated	5	3.88	6.5	25.8	20.06

B. *Frequency Response Approach*

The open-loop transfer function of the compensated system is

$$G_c(s)G(s) = K\,\frac{\tau s+1}{\beta\tau s+1}G(s) \;=\; \frac{\tau s+1}{\beta\tau s+1}G_1(s) \qquad (4)$$

Where $G_1(s) = KG(s)$ and $K_c\beta = K$

The required gain K to satisfy static velocity error constant is determined. If the gain-adjusted but uncompensated system $G_1(j\omega) = KG(j\omega)$ does not satisfy the specifications on the phase and gain margins, then the frequency point where the phase angle of the open-loop transfer function is equal to (-180°) plus the required phase margin is measured. The required phase margin is the specified phase margin plus 5° to 12°. This addition of 5° to 12° compensates the phase lag of the lag compensator. This frequency is chosen as the new gain crossover frequency. To prevent detrimental effects of phase lag due to the lag compensator, the pole and zero of the lag compensator must be located substantially lower than the new gain crossover frequency. Therefore, the corner frequency $\omega = \frac{1}{\tau}$ which is corresponding to the zero of the lag compensator is selected 1 octave to 1 decade below the new gain crossover frequency. The attenuation necessary to bring the magnitude curve down to 0 dB at the new gain crossover frequency is determined. Determining the value of β from this as the attenuation is -20logβ the other corner frequency (corresponding to the pole of the lag compensator) is determined from $\omega = \frac{1}{\beta\tau}$.

B.1.Example

Uncompensated system is taken as $G(s) = \frac{0.53}{s(s+1)(0.5s+1)}$

Designed lag compensator following the above procedure is $G_c(s) = \frac{0.943(0.377s+0.1)}{(s+0.01)}$. Hence the overall transfer functions

$$G(s)G_s(s) = \frac{5(10s+1)}{s(100s+1)(s+1)(0.5s+1)}.$$

All the specifications regarding the systems are listed in TABLE IV. The bode plots of the compensated and uncompensated system are depicted in Fig.9. Also the step response is given in the Fig.10.

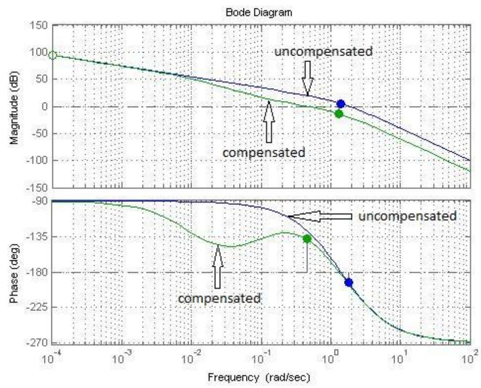

Fig.9. Bode plot of uncompensated [G(s)= 0.53/s(s+1)(0.5s+1)] and compensated system

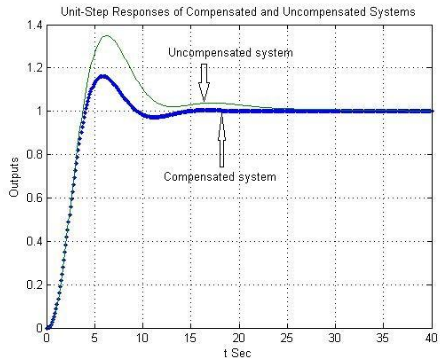

Fig.10. Output of the compensated and uncompensated system for lag compensator using frequency response analysis

TABLE IV
SPECIFICATION PARAMETERS FOR LAG COMPENSATOR USING FREQUENCY RESPONSE ANALYSIS

System	PM (deg)	GM (db)	T_s (sec)	T_r (sec)	T_p (sec)	$\%M_p$
Uncompensated	-13	-4.44	14	4.04	6.1	13.13
Compensated	41.6	14.3	13	4.17	6.9	11.92

C. *Comparative Study on the Specifications*

Considering the TABLE III and TABLE IV and the step responses of the lag compensator there is considerable changes in settling time occurred. In the frequency response technique the system settles down in a much lesser time. Hence the frequency response technique is the better choice than root locus technique with respect to T_s. Although the step response depending upon root locus approach is more smooth than other since no undershoot is present here. So with respect to that point root locus approach may also be adopted.

IV. LAG –LEAD COMPENSATOR

When both the transient and steady-state responses required improvement, a lag-lead compensator is required. It is a combination of a lag compensator and a lead compensator connected in series. The lag-lead compensator has a transfer function of the form –

$$G_c(s) = K_c\,\frac{(\tau_1 s+1)(\tau_2 s+1)}{(\frac{\tau_1}{\beta}s+1)(\beta\tau_2 s+1)}\ ;\ \beta > 1 \qquad (4)$$

Here the phase-lead portion of the lag-lead compensator is the portion involving the factor $\tau_1$and the phase-lag portion is the portion having τ_2 factor.

A. *Using Root Locus Approach*

In equation (4) there are two parts are available. In the root locus technique at first the design of lead section is done to meet the specifications on transient response. From this a suitable value of β is found. The error constant is determined for the lead compensator system. If the error constant is satisfactory then the design is complete. If the specified error constant is much higher than that obtained by lead compensator then the design of lag section is proceeded such that the overall system meet the specification on steady state performance. Since β is already determined the error constant is increased by a factor β by the lag section.

A.1.Example

The uncompensated system is taken as $G(s) = \frac{K}{s(s+1)(s+2)}$. The root locus plot is given in Fig.11. After considering above stated procedure the designed compensator is $G_c(s) = \frac{(s+0.1)(s+0.24)}{(s+4.3)(s+0.005)}$ and the overall transfer function is

$$G(s)G_s(s) = \frac{13.2(s+0.1)(s+0.24)}{s(s+1)(s+2)(s+4.3)(s+0.005)}$$

The root locus for overall transfer function is given in Fig.11. Also the step responses are depicted in Fig.12. All the specifications are listed in TABLE V.

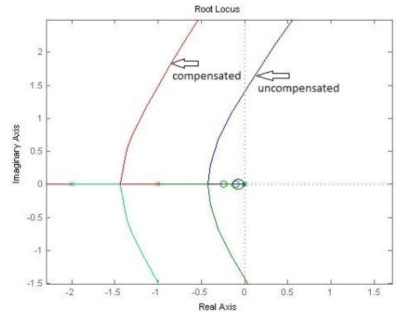

Fig.11. Root locus plot of uncompensated [G(s) = K/s(s+1)(s+2)] and compensated system

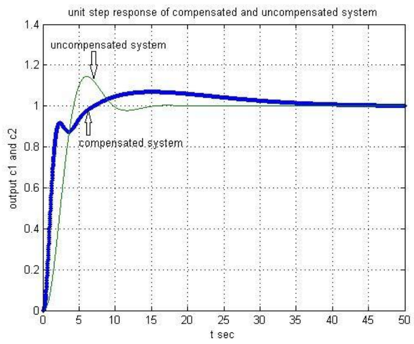

Fig.12. Output of the compensated and uncompensated system for lag- lead compensator using root locus analysis

TABLE V
SPECIFICATION PARAMETERS FOR LAG-LEAD COMPENSATOR USING ROOT LOCUS ANALYSIS

System	K_v (1/sec)	T_r (sec)	T_p (sec)	T_s (sec)	$\%M_p$
Uncompensated	0.5	4.25	6	14	9.99%
Compensated	7.36	6.04	2.5	30	-6.39%

B. *Frequency Response Approach*

According to equation (4) the phase-lead portion alters the frequency-response curve by adding phase-lead angle and increasing the phase margin at the gain crossover frequency. The phase-lag portion provides attenuation near and above the gain crossover frequency and thereby allows an increase of gain at the low-frequency range to improve the steady-state performance.

B.1.Example

Uncompensated system is taken as $G(s) = \frac{K}{s(s+1)(s+2)}$

Designed lag-lead compensator following the above procedure is $G_c(s) = \frac{(s+0.7)(s+0.15)}{(s+7)(s+0.015)}$. Hence the overall transfer functions $G(s)G_s(s) = \frac{20(s+0.7)(s+0.15)}{s(s+1)(s+2)(s+7)(s+0.015)}$.

All the specifications regarding the systems are listed in TABLE V. The bode plots of the compensated and uncompensated system are depicted in Fig.13. Also the step response is given in the Fig.14.

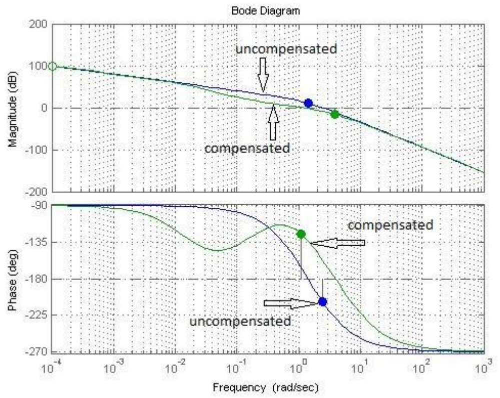

Fig.13. Bode plot of uncompensated and compensated system with lag-lead compensator

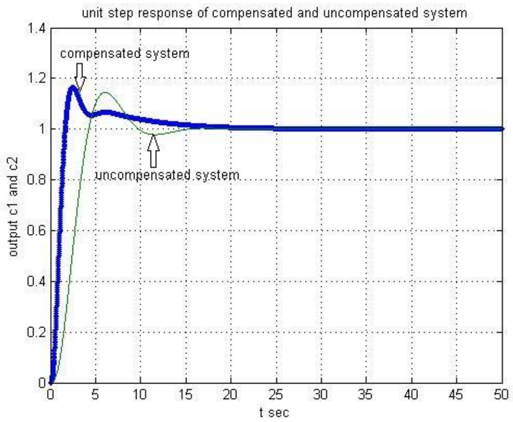

Fig.14. Output of the compensated and uncompensated system for lag-lead compensator using frequency response analysis

TABLE VI
SPECIFICATION PARAMETERS FOR LAG-LEAD COMPENSATOR USING FREQUENCY RESPONSE ANALYSIS

System	**PM**	**GM (db)**	**T_r (sec)**	**T_p (sec)**	**$\%M_p$**	**T_s (sec)**
Uncompensated	-28.1°	-10.5	4.25	6	9.99	15
Compensated	55.2°	16.9	1.655	2.485	16.66	15

C. *Comparative Study on the Specifications*

By comparing TABLE V and TABLE VI it can be noted that in root locus approach the transient response specifications like maximum overshoot, rise time etc. are not so satisfactory. Although the steady- state response is quite normal. But in the other hand in the frequency response technique both the transient as well as steady-state behaviour are quite satisfactory. Therefore the selection of frequency response technique will be more technically appropriate. Also in lag-lead compensator bode plot technique is more suitable for higher order system. Because in higher order system the angle contributed by lead portion of lag-lead compensator is too large.

V. CONCLUSION

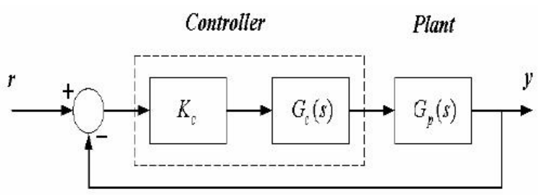

Fig.15. A typical Feed-forward Compensation System

Every process control system has a general block diagram like a feed back control system shown on the Fig.15. It contains a controller and a plant which have to control with the help of the controller. Such controller is known as compensator. The physical parameters of a control system which are generally controlled in the industries and laboratories are temperature, pressure, flow, level etc. The proper selection of a compensator is a very crucial issue. After selecting the proper compensator the control of these plant variables will become quite effortless.

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