International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 2, February- 2013

A Replacement Policy for the Repair Facility in a Two-Unit Cold Standby Redundant Repairable System

Shankar Bhat K.* and Miriam K. Simon
Department of Statistics, Madras Christian College, Chennai – 600 059, India.

Abstract

The paper deals with a cold standby redundant system with finite repair capacity. The expected repair time of units increases after each repair. In order to increase the system reliability, a replacement policy based on MTSF is suggested. The analysis of the system is carried out identifying suitable regeneration points. The replacement policy is illustrated through an example.

Keywords: *Reliability engineering –maintainable systems – regenerative stochastic processes - MTSF.*

1. Introduction

Standby redundancy repair maintenance and replacements are important methods through which the system reliability is enhanced [1, 2, 3]. Cold standby redundancy optimization through optimal design configurations is studied by many reliability analysts for non-repairable series-parallel system with cold standby redundancy [4]. In the analysis of redundant repairable systems it is generally assumed that the repair facility is as-good-as-new after each repair[1, 2]. In other words, the expected repair time of each unit is assumed to remain unchanged repair after repair. In many practical situations we observe that in the process of making a unit good-as-new (that is, to restore the properties of a unit), considerable damage will be done to the operational ability of the repair facility, which may reflect upon the repair rates of the units in subsequent repairs. Intuitively, we expect the mean repair time of a unit to increase after each repair. Thus, it would be more realistic to assume different repair time distributions for the units, upon each failure. Moreover, it is clear that the repair facility may mot perform the desired operation satisfactorily after it has undergone a 'sufficient' number of repair completions. This indicates that the system reliability will reach the state of equilibrium once the repair facility undergoes sufficient number of repair completions. In order to increase the system reliability a replacement of repair facility based on mean time to system failure (MTSF) is suggested. The policy includes the computation of MTSF, the stabilization of which identifies the time epoch of replacement of repair facility.

2. System Description

(1) The system consists of two dissimilar units. We label them as U_1 and U_2. Their functional behaviour is same.

(2) Initially U_1 is put online and U_2 is kept as a cold standby.

(3) There is one repair facility RF. Online failed units are repaired using RF on first come first serve basis.

(4) The repair time distribution for the units are different upon each failure. Furthermore, it is assumed that the repair rate of each unit increases as the number of repair increases.

(5) The failure times and repair times of units are independent random variables

(6) The operational ability of the repair facility is considered not satisfactory once it completes 2k number of repairs.

(7) There are no switchover delays and the switch is perfect.

$f_i(.), F_i(.), \overline{F}_{ij}(.)$ p.d.f, c.d.f, s.f of failure time of unit i, $i = 1,2$

$g_{ij}(.), G_{ij}(.)$ p.d.f, c.d.f. of unit i while undertaking repair for the j-th time, $i = 1,2$; $j = 1,2,..k$

3. Reliability Analysis

We define the following events to characterize the system:

E_{i0} **:**event that unit i, which has not gone through any repair till then, just begins to operate online.

E_{ij} **:**event that j^{th} repair of unit i just begins; at this instant an operable standby unit is put online, $i = 1,2 \quad j = 1,2,..k$

Observing that these events are regenerative events exhibiting Kingman regenerative phenomenon, the following auxiliary system-down avoiding functions are defined to obtain p.d.f. of time intervals between E_{ij} events.

$$P_r(j,t)\delta t = \Pr\{E_{1j+1} \text{ occurs between } t \text{ and } t+\delta t \text{ and the system is operable in } (0,t) / E_{2j} \text{ at } t=0\}$$
$$j = 1,2,..k-1 \qquad [3.1]$$

$$Q_r(j,t)\delta t = \Pr\{E_{2j+1} \text{ occurs between } t \text{ and } t+\delta t \text{ and the system is operable in } (0,t) / E_{1j+1} \text{ at } t=0\}$$
$$j = 1,2,..k-1 \qquad [3.2]$$

$$H_r(j,t)\delta t = \Pr\{E_{2j} \text{ occurs between } t \text{ and } t+\delta t \text{ and the system is operable in } (0,t) / E_{2j-1} \text{ at } t=0\}$$
$$j = 1,2,..k \qquad [3.3]$$

$$\phi_r(j,t)\delta t = \Pr\{E_{2j} \text{ occurs between } t \text{ and } t+\delta t \text{ and the system is operable in } (0,t) / E_{20} \text{ at } t=0\}$$
$$j = 1,2,..k \qquad [3.4]$$

We observe that the functions [3.1] and [3.2] are system down avoiding functions in the sense that a system down is not permitted between the two events. These functions can be easily evaluated with the help of regenerative events E_{ij}.

At the instant of failure of unit 1, unit 2 has completed its j^{th} repair and is found in operable condition in its standby state.

$$P_r(j,t) = f_1(t)G_{2j}(t) \qquad j = 1,2,..k \qquad [3.5]$$

At the instant of failure of unit 2, unit 1 has completed its $(j+1)^{\text{th}}$ repair and is found in operable condition in its standby state.

$$Q_r(j,t) = f_2(t)G_{1,j+1}(t) \qquad j = 1,2,..k-1 \qquad [3.6]$$

We observe that

$$H_r(j,t)\delta t = \Pr\{E_{1j} \text{ occurs between } t \text{ and } t+\delta t \text{ and the system is operable in } (0,t) / E_{2j-1} \text{ at } t=0\} \text{ © } \Pr\{E_{2j} \text{ occurs between } t \text{ and } t+\delta t \text{ and the system is operable in } (0,t) / E_{1j} \text{ at } t=0\}$$
$$j = 2,3,..k \qquad [3.7]$$

Using the expressions [3.1] and [3.2] we get

$$H_r(j,t) = P_r(j-1,t) \text{ © } Q_r(j-1,t) \qquad j = 2,3,..k \qquad [3.8]$$

We also observe that

$$\phi_r(j,t)\delta t = \Pr\{E_{2j-1} \text{ occurs between } t \text{ and } t+\delta t \text{ and the system is operable in } (0,t) / E_{20} \text{ at } t=0\} \text{ © } \Pr\{E_{2j} \text{ occurs between } t \text{ and } t+\delta t \text{ and the system is operable in } (0,t) / E_{2j-1} \text{ at } t=0\}$$
$$j = 2,3,..k \qquad [3.9]$$

Using [3.3] and [3.4] we obtain the following recurrence relation

$$\phi_r(j.t) = \phi_r(j-1,t) \text{ © } H_r(j,t) \quad j = 2,3,..k \qquad [3.10]$$

$$\phi_r(1,t)\delta t = \Pr\{E_{21} \text{ occurs between } t \text{ and } t+\delta t \text{ and the system is operable in } (0,t) / E_{20} \text{ at } t=0\} \qquad [3.11]$$

$$\phi_r(1,t) = H_r(1,t) \qquad [3.12]$$

$$H_r(1,t) = f_2(t)G_{1,1}(t) \qquad [3.13]$$

The reliability function R(t) of the system is

$$R(t) = \overline{F}_1(t) + f_1(t) \text{ © } \overline{F}_2(t) + f_1(t) \text{ © } \sum_{j=1}^{k} \phi_r(j,t) \text{ © } [\overline{F}_1(t) + P_r(j,t) \text{ © } \overline{F}_2(t)] \qquad [3.14]$$

The expression [3.14] is obtained by considering the following mutually exclusive and

exhaustive cases:
(i) unit 1, which has not gone through any repair till then, does not fail before t.
(ii) unit 2, which is instantaneously switched over from standby and has not gone through any repair till then, does not fail before t.
(iii) unit $i,(i=1,2)$ while operating online after j th repair $(j=1,2,..k)$ does not fail before t.

4. A Replacement Policy based on MTSF

We have assumed that each unit can make use of the repair facility only k times. In other words, the repair facility will be scrapped when it completes 2k repairs. However, this assumption can be modified so that the system might be available in the long run. When a repair facility completes 2k repairs. It is replaced by a similar new repair facility. **The policy of replacement is as follows:-**

"After nk-th repair completion of unit 1, the old repair facility is scrapped and a new repair facility is introduced, where n denotes the number of such replacements, n ≥ 1. We suggest replacement of repair facility only and not operable units. When a unit, while operating online after k-th repair, fails, it is switched over to the new repair facility; at this epoch an operable standby is instantaneously switched online."

Reliability Analysis

Let

$\phi_r(0,t)\delta t = \Pr\{E_{11}$ occurs between t and $t+\delta t$ and the system is operable in $(0,t] / E_{11}$ at $t=0\}$ [4.1]

The function $\phi_r(0,t)$ is the p.d.f. of time interval between two successive E_{11} events, the system being operable between these two events. Thus

$\phi_r(0,t) = \phi_r(k,t) \copyright g_{2k}(t) f_1(t)$ [4.2]

The reliability function of the modified system is given by

$$R_1(t) = {}_1R_1(t) + \left[\sum_{n=1}^{\infty} \phi_r(0,t)^{(n)}\right] \copyright \left\{\overline{F}_2(t) + \sum_{j=1}^{k} \phi_r(j,t)\, \overline{F}_1(t) + g_{2k}(t) f_1(t) \copyright \overline{F}_2(t)\right\} \quad [4.3]$$

where ${}_1R_1(t)$ is given by the expression on the right hand side of [3.14].

The equation [4.3] is derived by considering the following mutually exclusive and exhaustive possibilities :

(i) the interval $(0,t]$ is not intercepted by an E_{11} event.

(ii) the interval $(0,t]$ is intercepted by at least one E_{11} event.

The mean-time-to-system failure of the system is given by

MTSF = $R_1^*(0)$

5. Illustration

For the purpose of illustration we consider a model in which both the units are identical by virtue of their statistical properties.
Furthermore,

$f_i(t) = \lambda e^{-\lambda t}$ $\quad \lambda > 0, i = 1,2$

$g_{ij}(t) = \mu_j^2 t e^{-\mu_j t}$ $\quad \mu_j > 0, i = 1,2$

$j = 1,2,..k$

The integral equations given in [3.14] can be solved using Laplace transform technique. The Laplace transform of $R(t)$ is given by

$$R^*(s) = \frac{1}{s+\lambda} + \frac{\lambda}{(s+\lambda)^2} + \frac{\lambda}{s+\lambda} \sum_{j=1}^{k} L_1^R(s) \prod_{n=2}^{j} L_n^R(s) L_{n-1}^R(s) \left\{\frac{1}{s+\lambda}\left(1 + L_j^R(s)\right)\right\} \quad [5.1]$$

where $L_j^R(s) = \dfrac{\lambda \mu_j^2}{(s+\lambda)(s+\lambda+\mu_j)^2}$ $\quad j = 1,2,..n$

We observe that $R^*(s)$ is a rational function of its arguments and can be easily inverted for small values. Thus, the reliability can be explicitly computed.

The Mean-Time-to-System Failure (MTSF) after simplification is

$$R^*(0) = \frac{2}{\lambda} + \sum_{j=1}^{k} L_1^R(0) \prod_{n=2}^{j} L_n^R(0) L_{n-1}^R(0) \left\{\frac{1}{\lambda}\left(1 + L_j^R(0)\right)\right\} \quad [5.2]$$

$$L_j^R(0) = \frac{\mu_j^2}{(\lambda+\mu_j)^2} \quad j = 1,2,..n$$

The MTSF for various values of k where λ = 0.065, μ = 10, computed using Visual Basic is listed in Table 1.

Table 1: MTSF of the system for specified values of the parameters λ and μ

k	R*(0)
1	60.9468
2	89.7917
3	116.1626
4	138.2602
5	153.9272
6	162.0818
7	**164.5604**
8	164.8793
9	164.8911
10	**164.8912**
11	164.8912
12	164.8912
13	164.8912
14	164.8912
15	164.8912

We observe that for a system that contains identical units, U_1 is identical to U_2, MTSF stabilizes at k = 10 and conclude that system improvement is not attained beyond k = 10. This clearly indicates that it is not worthwhile to retain the repair facility once it completes 10 repairs. This means each unit can be repaired 5 times efficiently using the present RF. Therefore, we suggest that at this stage the repair facility may be replaced by a new one in order to increase the system performance. When a unit fails for the 6^{th} time, it may be sent to the new repair facility for repair.

REFERENCES

[1] Birolini, A. [1985]. *Stochastic Processes Used in Modeling Reliability Problems.* Lectures notes in Economic and Mathematical systems, **252,** Springer Verlag: Berlin.

[2] Srinivasan, S.K. and R. Subramanian. [1980]. *Probabilistic Analysis of Redundant Systems,* Lecture Notes in Economics and Mathematical Systems, 175, Springer Verlag : Berlin.

[3] Popova, E. and Wilson, J.G. [1999]. *Group replacement policies for parallel system whose components have phase distributed failure times,* Annals of Operations Research, Vol. 91, 163-198.

[4] Coit**,** D. W. **(**2001**),** *Cold standby redundancy optimization for nonrepairable system,*. IIE. Transactions, Vol. 33, No.6, 471-478.