A Related Fixed Point Theorem of Integral Type on Two Fuzzy 2-Metric Spaces

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Abstract

In this paper, a related fixed point theorem is obtained. It extends a result proved by R.K. Namdeo, N.K. Tiwari, B. Fisher and K. Tas [9]. The notion of fuzzy 2-metric spaces satisfying integral type inequalities is used.

Keywords : Fuzzy 2-metric space, fixed point, related fixed point, integral type inequality.

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Intoduction

The concept of fuzzy sets was introduced by L. Zadeh [14] in 1965. Fuzzy metric space was introduced by Kramosil and Michalek [7] in 1975. Then, it was modified by George and Veeramani [4] in 1994. Fuzzy has been studied and developed by many mathematicians for many years. Introduction of fuzzy 2-metric space is one of such developments. Gahler [10, 11] investigated 2-metric spaces in a series of his papers. Fuzzy 2-metric space is studied in [6, 8, 12, 13] and many others. Related fixed point is studied in [1, 2, 3, 5, 9] and many more.

Some definitions are stated as follows:

Definition 1.1 : A binary operation $*: [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a t - norm in ([0, 1], $*$) if following conditions are satisfied:
For all $a, b, c, d \in [0, 1]$,
i. $a * 1 = a$,
ii. $a * b = b * a$,
iii. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$,
iv. $a * (b * c) = (a * b) * c$.

Definition 1.2. The 3-tuple $(X, \mu, *)$ is called a fuzzy 2- metric space if X is an arbitrary set, $*$ is a continuous t - norm and μ is a fuzzy set in $X^3 \times [0, \infty)$ satisfying the following conditions:
For all $x, y, z, u \in X$ and $t_1, t_2, t_3 > 0$,
i. $\mu(x, y, z, 0) = 0$,
ii. $\mu(x, y, z, t) = 1$, $t > 0$ and when at least two of the three points are equal,
iii. $\mu(x, y, z, t) = \mu(y, x, z, t) = \mu(z, x, y, t)$ (symmetry about three variables),
iv. $\mu(x, y, z, t_1+t_2+t_3) \geq \mu(x, y, u, t_1) * \mu(x, u, z, t_2) * \mu(u, y, z, t_3)$
v. $\mu(x, y, z, \cdot): [0, \infty) \rightarrow [0, 1]$ is left continuous,

vi. $\lim_{t\to\infty} \mu(x, y, z, t) = 1$.

Definition 1.3 : Let $(X, \mu, *)$ be a fuzzy 2-metric space. A sequence $\{x_n\}$ X is said to:

i. converge to x in X if and only if $\lim_{t\to\infty} \mu(x_n, x, a, t) = 1$ $\forall a \in X$ and $t > 0$.

ii. be a Cauchy sequence if and only if $\lim_{t\to\infty} \mu(x_{n+p}, x_n, a, t) = 1$ $\forall a \in X$, $p > 0$ and $t > 0$.

Definition 1.4 : A fuzzy 2-metric space $(X, \mu, *)$ is aid to be complete if and only if every Cauchy sequence in X is convergent in X.

The following was proved in [9].

Theorem 1.1 : Let (X, d) and (Y, ρ) be complete metric spaces. Let T be a mapping of X into Y and S be a mapping of Y into X satisfying the inequalities

$$d(Sy, Sy')\, d(STx, STx') \leq c \max\{ d(Sy, Sy')\, \rho(Tx, Tx'),\ d(x', Sy)\, \rho(y', Tx),\ d(x, x')\, d(Sy, Sy'),\ d(Sy, STx)\, d(Sy', STx')\}$$

$$\rho(Tx, Tx')\, \rho(TSy, TSy') \leq c \max\{ d(Sy, Sy')\, \rho(Tx, Tx'),\ d(x', Sy)\, \rho(y', Tx),\ \rho(y, y')\, \rho(Tx, Tx'),\ \rho(Tx, TSy)\, \rho(Tx', TSy')\}$$

for all x, x' in X and y, y' in Y, where $0 \leq c < 1$. If either S or T is continuous, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further, $Tz = w$ and $Sw = z$.

Now, theorem 1.1 is extended to two pairs of mappings in integral and fuzzy 2-metric space settings as follows.

Main result

Theorem 2.1 : Let (X, μ, a, t) and (Y, υ, a, t) be two complete fuzzy 2-metric spaces. Let A, B be mappings of X into Y and S, T be mappings of Y into X satisfying the inequalities

$$\int_1^{k\mu(Sy, Ty', a, t)\, \mu(SAx, TBx', a, t)} \varphi(s)\, ds \geq \int_1^{\min\{\mu(Sy, Ty', a, t)\upsilon(Ax, Bx', a, t),\ \mu(x', Sy, a, t)\upsilon(y', Ax, a, t),\ \mu(x, x', a, t)\, \mu(Sy, Ty', a, t),\ \mu(Sy, SAx, a, t)\, \mu(Ty', TBx', a, t)\}} \varphi(s)\, ds \quad (1)$$

$$\int_1^{k\upsilon(Ax, Bx', a, t)\, \upsilon(BSy, ATy', a, t)} \varphi(s)\, ds \geq \int_1^{\min\{\mu(Sy, Ty', a, t)\upsilon(Ax, Bx', a, t),\ \mu(x', Sy, a, t)\upsilon(y', Ax, a, t),\ \upsilon(y, y', a, t)\upsilon(Ax, Bx', a, t),\ \upsilon(Ax, BSy, a, t)\upsilon(Bx', ATy', a, t)\}} \varphi(s)\, ds \quad (2)$$

for all x, x' in X and y, y' in Y, where $k \in (0, 1)$. If A and S or B and T are continuous, then SA and TB have a unique common fixed point z in X and BS and AT have unique common fixed point w in Y. Further, $Az = Bz = w$ and $Sw = Tw = z$.

Proof : Let x be any arbitrary point in X. We define sequences $\{x_n\}$ and $\{y_n\}$ in X and Y respectively as:

$$Sy_{2n-1} = x_{2n-1},\quad Bx_{2n-1} = y_{2n},\quad Ty_{2n} = x_{2n},\quad Ax_{2n} = y_{2n-1},\quad \text{for } n = 1, 2, 3, \ldots$$

Applying inequality (1), we get

$$\int_1^{k\mu(Sy_{2n-1}, Ty_{2n}, a,t)\,\mu(SAx_{2n}, TBx_{2n-1}, a,t)} \varphi(s)\,ds$$

$$= \int_1^{k\mu^2(x_{2n-1}, x_{2n}, a,t)} \varphi(s)\,ds$$

$$\geq \int_1^{\substack{\min\{\mu(Sy_{2n-1}, Ty_{2n}, a,t)\,\upsilon(Ax_{2n}, Bx_{2n-1}, a,t),\ \mu(x_{2n-1}, Sy_{2n-1}, a,t)\upsilon(y_{2n}, Ax_{2n}, a,t), \\ \mu(x_{2n}, x_{2n-1}, a,t)\,\mu(Sy_{2n-1}, Ty_{2n}, a,t), \mu(Sy_{2n-1}, SAx_{2n}, a,t)\mu(Ty_{2n}, TBx_{2n-1}, a,t)\}}} \varphi(s)\,ds$$

$$= \int_1^{\substack{\min\{\mu(x_{2n-1}, x_{2n}, a,t)\,\upsilon(y_{2n-1}, y_{2n}, a,t),\ \mu(x_{2n-1}, x_{2n-1}, a,t)\,\upsilon(y_{2n}, y_{2n-1}, a,t), \\ \mu(x_{2n}, x_{2n-1}, a,t)\,\mu(x_{2n-1}, x_{2n}, a,t),\ \mu(x_{2n-1}, x_{2n-1}, a,t)\,\mu(x_{2n}, x_{2n}, a,t)\}}} \varphi(s)\,ds$$

from which it follows that

$$\int_1^{k\mu(x_{2n-1}, x_{2n}, a,t)} \varphi(s)\,ds \geq \int_1^{\min\{\upsilon(y_{2n-1}, y_{2n}, a,t), \mu(x_{2n-1}, x_{2n}, a,t)\}} \varphi(s)\,ds \qquad (3)$$

Applying inequality (2), we get

$$\int_1^{k\upsilon(Ax_{2n}, Bx_{2n-1}, a,t)\,\upsilon(BSy_{2n-1}, ATy_{2n}, a,t)} \varphi(s)\,ds$$

$$= \int_1^{k\upsilon^2(y_{2n-1}, y_{2n}, a,t)} \varphi(s)\,ds$$

$$\geq \int_1^{\substack{\min\{\mu(Sy_{2n-1}, Ty_{2n}, a,t)\,\upsilon(Ax_{2n}, Bx_{2n-1}, a,t),\ \mu(x_{2n-1}, Sy_{2n-1}, a,t)\upsilon(y_{2n}, Ax_{2n}, a,t), \\ \upsilon(y_{2n-1}, y_{2n}, a,t)\,\upsilon(Ax_{2n}, Bx_{2n-1}, a,t),\ \upsilon(Ax_{2n}, BSy_{2n-1}, a,t)\,\upsilon(Bx_{2n-1}, ATy_{2n}, a,t)\}}} \varphi(s)\,ds$$

$$= \int_1^{\substack{\max\{\mu(x_{2n-1}, x_{2n}, a,t)\,\upsilon(y_{2n-1}, y_{2n}, a,t),\ \mu(x_{2n-1}, x_{2n-1}, a,t)\,\upsilon(y_{2n}, y_{2n-1}, a,t), \\ \upsilon(y_{2n-1}, y_{2n}, a,t)\,\upsilon(y_{2n-1}, y_{2n}, a,t), \upsilon(y_{2n-1}, y_{2n}, a,t)\,\upsilon(y_{2n}, y_{2n-1}, a,t)\}}} \varphi(s)\,ds$$

from which it follows that

$$\int_1^{k\upsilon(y_{2n-1}, y_{2n}, a,t)} \varphi(s)\,ds \geq \int_1^{\min\{\upsilon(y_{2n-1}, y_{2n}, a,t), \mu(x_{2n-1}, x_{2n}, a,t)\}} \varphi(s)\,ds \qquad (4)$$

(3) and (4) can be written as

$$\int_1^{k\mu(x_{n-1}, x_n, a,t)} \varphi(s)\,ds \geq \int_1^{\min\{\upsilon(y_{n-1}, y_n, a,t), \mu(x_{n-1}, x_n, a,t)\}} \varphi(s)\,ds$$

$$\int_1^{k\upsilon(y_{n-1},\, y_n,\, a,t)} \varphi(s)\,ds \ \geq \int_1^{\min\{\upsilon(y_{n-1},\, y_n,\, a,t),\, \mu(x_{n-1},\, x_n,\, a,t)\}} \varphi(s)\,ds$$

which can be again written as

$$\int_1^{k\mu(x_{n+1},\, x_n,\, a,t)} \varphi(s)\,ds \geq \int_1^{\min\{\upsilon(y_{n+1},\, y_n,\, a,t),\, \mu(x_{n+1},\, x_n,\, a,t)\}} \varphi(s)\,ds \qquad (5)$$

$$\int_1^{k\upsilon(y_{n+1},\, y_n,\, a,t)} \varphi(s)\,ds \geq \int_1^{\min\{\upsilon(y_{n+1},\, y_n,\, a,t),\, \mu(x_{n+1},\, x_n,\, a,t)\}} \varphi(s)\,ds \qquad (6)$$

From (5) and (6) , by induction, we get

$$\int_1^{\mu(x_{n+1},\, x_n,\, a,t)} \varphi(s)\,ds \geq \int_1^{\frac{1}{k^n}\min\{\upsilon(y_1,\, y_2,\, a,t),\, \mu(x_1,\, x_2,\, a,t)\}} \varphi(s)\,ds$$

$$\int_1^{\upsilon(y_{n+1},\, y_n,\, a,t)} \varphi(s)\,ds \geq \int_1^{\frac{1}{k^n}\min\{\upsilon(y_1,\, y_2,\, a,t),\, \mu(x_1,\, x_2,\, a,t)\}} \varphi(s)\,ds$$

Let $t_1 = \frac{t}{p}$. Now,

$$\int_1^{\mu(x_n,\, x_{n+p},\, a,t)} \varphi(s)ds = \int_1^{\mu(x_n,\, x_{n+p},\, a,\, t_1 + t_1 + \ldots p \text{ times})} \varphi(s)ds$$

$$\geq \int_1^{\mu(x_n,\, x_{n+1},\, a,\, t_1)} \varphi(s)ds * \int_1^{\mu(x_{n+1},\, x_{n+2},\, a,\, t_1)} \varphi(s)ds * \ldots * \int_1^{\mu(x_{n+p-1},\, x_{n+p},\, a,\, t_1)} \varphi(s)ds$$

$$\geq \int_1^{\frac{1}{k^n}\min\{\upsilon(y_1,\, y_2,\, a,t),\, \mu(x_1,\, x_2,\, a,t)\}} \varphi(s)\,ds * \ldots * \int_1^{\frac{1}{k^{n+p-1}}\min\{\upsilon(y_1,\, y_2,\, a,t),\, \mu(x_1,\, x_2,\, a,t)\}} \varphi(s)\,ds$$

which implies that

$$\lim \int_1^{\mu(x_n,\, x_{n+p},\, a,t)} \varphi(s)ds \geq 1$$

$$\Rightarrow \mu(x_n,\, x_{n+p},\, a,t) \geq 1$$

$\Rightarrow$ $\{x_n\}$ is a Cauchy sequence with a limit z in X.
Similarly, $\{y_n\}$ is a Cauchy sequence with a limit w in Y.

Now, on using the continuity of A and S respectively, we get

$$w = \lim y_{2n-1} = \lim Ax_{2n} = Az \quad \text{and} \quad z = \lim x_{2n} = \lim Sy_{2n} = Sw$$

so that we get

$$Az = w \tag{7}$$

$$Sw = z \tag{8}$$

From (7) and (8), we get

$$SAz = z \tag{9}$$

Again applying inequality (1), we get

$$\int_1^{k\mu(SAx_{2n}, TBx_{2n-1}, a, t)} \varphi(s)\, ds$$

$$\geq \int_1^{\min\{\upsilon(Ax_{2n}, Bx_{2n-1}, a, t), \upsilon(y_{2n}, Ax_{2n}, a, t), \mu(x_{2n-1}, x_{2n}, a, t)\}} \varphi(s)\, ds \tag{10}$$

On letting $n \to \infty$, we have

$$\int_1^{k\mu(Sw, TBz, a, t)} \varphi(s)\, ds \geq \int_1^{\upsilon(Az, w, a, t)} \varphi(s)\, ds$$

By (7), we have

$$\int_1^{k\mu(Sw, TBz, a, t)} \varphi(s)\, ds \geq 0$$

$$\Rightarrow k\mu(Sw, TBz, a, t) \geq 1$$

which implies that

$$Sw = TBz$$

and from (8) , we get

$$z = TBz \tag{11}$$

From (9) and (11), we get

$$SAz = z = TBz \tag{12}$$

Now, (10) gives

$$\int_1^{k\mu(x_{2n-1}, Ty_{2n}, a, t)} \varphi(s)\, ds$$

$$\geq \int_1^{\min\{\upsilon(Ax_{2n}, Bx_{2n-1}, a, t),\ \upsilon(y_{2n}, Ax_{2n}, a, t),\ \mu(x_{2n-1}, x_{2n}, a, t)\}} \varphi(s)\, ds$$

On letting $n \to \infty$, we get

$$\int_1^{k\mu(z, Tw, a, t)} \varphi(t)\, dt \geq 0$$

$$\Rightarrow k\mu(z, Tw, a, t) \geq 1$$

which implies that

$$z = Tw \tag{13}$$

Again, applying inequality (2), we get

$$\int_1^{k\upsilon(BSy_{2n-1}, ATy_{2n}, a, t)} \varphi(s)\, ds$$

$$\geq \int_1^{\min\{\mu(Sy_{2n-1}, Ty_{2n}, a, t),\ \mu(x_{2n-1}, Sy_{2n-1}, a, t),\ \upsilon(y_{2n-1}, y_{2n}, a, t),\ \upsilon(Ax_{2n}, Bx_{2n-1}, a, t)\}} \varphi(s)\, ds \tag{14}$$

On letting $n \to \infty$, we get

$$\int_1^{k\upsilon(BSw, ATw, a, t)} \varphi(s)\, ds \geq 0$$

$$\Rightarrow k\upsilon(BSw, ATw, a, t) \geq 1$$

which implies that

$$BSw = ATw \tag{15}$$

Now, (14) gives

$$\int_1^{k\upsilon(y_{2n}, ATy_{2n}, a, t)} \varphi(s)\, ds$$

$$\geq \int_1^{\min\{\mu(Sy_{2n-1}, Ty_{2n}, a, t),\ \mu(x_{2n-1}, Sy_{2n-1}, a, t),\ \upsilon(y_{2n-1}, y_{2n}, a, t),\ \upsilon(Ax_{2n}, Bx_{2n-1}, a, t)\}} \varphi(s)\, ds$$

On letting $n \to \infty$, we get

$$\int_1^{k\upsilon(w, ATw, a, t)} \varphi(s)\, ds \geq 0$$

$$\Rightarrow\ k\upsilon(w, ATw, a, t) \ \geq\ 1$$

which implies that

$$w \ = \ ATw \tag{16}$$

From (15) and (16), we get

$$BSw \ = \ w \ = \ ATw \tag{17}$$

From (8) and (17) , we get

$$Bz \ = \ w \tag{18}$$

From (7) and (18), we get

$$Az \ = \ Bz \ = \ w \tag{19}$$

From (8) and (13) , we get

$$Sw \ = \ Tw \ = \ z \tag{20}$$

Similarly, on using the continuity of B and T, the above results hold.

To prove the uniqueness, let SA and TB have a second distinct common fixed point z' in X and BS and AT have a second distinct common fixed point w' in Y.

Applying inequality (1), we have

$$\int_1^{k\mu^2(z\,,\, z',\,,a,t)} \varphi(s)\,ds \geq \int_1^{\min\{\mu(z\,,\, z',a,t)\,\upsilon(Az\,,\, Bz',a,t),\ \mu(z',\, z',a,t)\,\upsilon(Bz',\, Az,a,t),\ \mu(z,z',a,t)\,\mu(z\,,\, z',a,t),\ \mu(z',\, z',a,t)\,\mu(z,z,a,t)\}} \varphi(s)\,ds$$

$$\Rightarrow \int_1^{k\mu(z\,,\, z',a,t)} \varphi(s)\,ds \ \geq \int_1^{\min\{\upsilon(Az\,,\, Bz',a,t),\ (Bz',\, Az,a,t)\}} \varphi(s)\,ds$$

$$\Rightarrow \int_1^{k\mu(z\,,\, z',a,t)} \varphi(s)\,ds \ \geq \int_1^{\upsilon(Az\,,\, Bz',a,t)} \varphi(s)\,ds \tag{21}$$

Applying inequality (2), we get

$$\int_1^{k\upsilon^2(Az\,,Bz',a,t)} \varphi(s)\,ds$$

$$\geq \int_1^{\substack{\min\{\mu(z, z^/, a,t)\,\upsilon(Az, Bz^/, a,t),\ \mu(z^/, z^/, a,t)\,\upsilon(Bz^/, Az, a,t),\\ \upsilon(Az, Bz^/, a,t)\,\upsilon(Az, Bz^/, a,t),\ \upsilon(Az, Bz^/, a,t)\,\upsilon(Bz^/, Az, a,t)\}}} \varphi(s)\,ds$$

$$\Rightarrow \int_1^{k\upsilon(Az, Bz^/, a,t)} \varphi(s)\,ds \geq \int_1^{\mu(z, z^/, a,t)} \varphi(s)\,ds \qquad (22)$$

From (21) and (22), we get

$$\int_1^{k^2\mu(z, z^/, a,t)} \varphi(s)\,ds \geq \int_1^{\mu(z, z^/, a,t)} \varphi(s)\,ds$$

$$\Rightarrow \int_1^{\mu(z, z^/, a,t)} \varphi(s)\,ds \geq \int_1^{\frac{1}{k^2}\mu(z, z^/, a,t)} \varphi(s)\,ds$$

$$\Rightarrow \int_1^{\mu(z, z^/, a,t)} \varphi(s)\,ds \geq \int_1^{\frac{1}{k^2}\mu(z, z^/, a,t)} \varphi(s)\,ds \geq \ldots \geq \int_1^{\frac{1}{k^n}\mu(z, z^/, a,t)} \varphi(s)\,ds$$

$$\Rightarrow \int_1^{\mu(z, z^/, a,t)} \varphi(s)\,ds \geq \lim \int_1^{\frac{1}{k^n}\mu(z, z^/, a,t)} \varphi(s)\,ds \geq 1$$

$$\Rightarrow \mu(z, z^/, a,t) \geq 1$$

which implies that

$$z = z^/.$$

This proves the uniqueness of z. Similarly, the uniqueness of w can be proved.

The following corollary is a fuzzy 2-metric space version of theorem 1.1 in integral setting.

Corollary 2.2 : Let (X, μ, a, t) and (Y, υ, a, t) be two complete fuzzy 2-metric spaces. Let S be mappings of X into Y and T be mappings of Y into X satisfying the inequalities

$$\int_1^{k\mu(Ty, Ty^/, a,t)\,\mu(TSx, TSx^/, a,t)} \varphi(s)\,ds \geq \int_1^{\substack{\min\{\mu(Ty, Ty^/, a,t)\,\upsilon(Sx, Sx^/, a,t),\ \mu(x^/, Ty, a,t)\,\upsilon(y^/, Sx, a,t),\\ \mu(x, x^/, a,t)\,\mu(Ty, Ty^/, a,t),\ \mu(Ty, TSx, a,t)\,\mu(Ty^/, TSx^/, a,t)\}}} \varphi(s)\,ds$$

$$\int_1^{k\upsilon(Sx, Sx^/, a,t)\,\upsilon(STy, STy^/, a,t)} \varphi(s)\,ds \geq \int_1^{\substack{\min\{\mu(Ty, Ty^/, a,t)\,\upsilon(Sx, Sx^/, a,t),\ \mu(x^/, Ty, a,t)\,\upsilon(y^/, Sx, a,t),\\ \upsilon(y, y^/, a,t)\,\upsilon(Sx, Sx^/, a,t),\ \upsilon(Sx, STy, a,t)\,\upsilon(Sx^/, STy^/, a,t)\}}} \varphi(s)\,ds$$

for all x, x' in X and y, y' in Y, where $k \in (0, 1)$. If either S or T is continuous, then TS has a unique fixed point z in X and ST has a unique fixed point w in Y. Further, $Sz = w$ and $Tw = z$.

Proof : By putting $A = B = S$ and $S = T = T$ in theorem 2.1, the result easily follows.

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