

A Proposed Compressive Sensing Based LFMCW Radar Signal Processor

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Abstract:- Application of Compressive Sensing (CS) in Linear Frequency Modulation Continuous Wave (LFMCW) radar had been investigated and proved by the authors in [8]. In this paper, a new approach for applying CS in LFMCW radar signal processing is introduced. The proposed approach depends on apply CS in range processing direction by acquiring the base band radar signal with a sampling rate, according to a Pseudo Random (PN) sequence, less than that of the well known Nyquist rate. The information of the received radar signal (target range and speed) are reconstructed by the use of Complex Approximate Message Passing (CAMP) reconstruction algorithm. The superiority of the proposed LFMCW radar signal processor (Based on CS) compared to that of the traditional one (based on Fast Fourier Transform (FFT)) from points of view of detection performance through Receiver Operating Characteristics (ROC) curves, resolution performance and hardware complexity is validated.

Keywords: LFMCW Radar, Compressive Sensing, CAMP, FFT.

1. INTRODUCTION

The small size, simplicity and economy of LFMCW radar systems were the basic reasons of wide applications in many areas such as aviation, military, security, navigation, and automotive. Signal processing plays an important role in such radars. The early work of Nyquist, Shannon [1] and Moore's law [2] on sampling and representation of continuous signals, signal processing are moved from the analog to the digital domain. The general block diagram of digital LFMCW radar receiver is shown in the Fig.(1) [3].

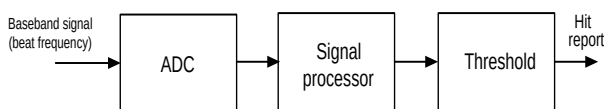


Figure 1 Block diagram of digital LFMCW radar receiver

There are many requirements of advanced LFMCW radar such as: small size, low cost, weight and complexity. But the main major factors are accuracy and resolution. For most modern high-resolution multi-channel radar systems, one of the major problems to deal with is the huge amount of data to be acquired, processed and/or stored. But why all these data are needed? According to the well known Nyquist-Shannon sampling theorem, natural signals have to be sampled at least twice the signal bandwidth to prevent aliasing. Many

challenges are met by the radar system to acquire and process wideband signals;

- 1) Acquiring signals at very high sampling rates,
- 2) Storing the resulting large amounts of data, and
- 3) Processing/analyzing large amounts of data.

Sampling of very wide bandwidth signals may require ADC hardware that is unavailable or very expensive. So, an efficient way to deal with these wide bandwidths and high data rates as well as the large amount of data is the emerging field of Compressive Sensing (CS) [4]. CS theory states that, given certain circumstances, it is possible to reconstruct a signal which is sampled at a rate below the Nyquist rate [5,6]. Applying CS theory in LFMCW radar signal processing may lead to a reduction in sampling rate, complexity, power consumption, and cost. On the other hand, performance is a critical point to be considered.

In this paper, a new approach is introduced for LFMCW radar signal processing based on the concept of compressive sensing (CS). However, while the traditional LFMCW radars use well established processing algorithms and detection schemes, such as Fast Fourier Transform (FFT) and Constant False Alarm Rate (CFAR) detectors to extract target range and velocity, the reconstruction of the target scene from the CS measurements involves the use of highly nonlinear algorithms such as L1-minimization and Complex Approximate Message passing (CAMP). These algorithms have several free parameters that must be tuned properly in order to achieve good performance [6-8].

The rest of this paper is organized as follows; section 2 gives a survey on the principle of CS theory. Section 3 illustrates the application of LFMCW radar based on CS. Section 4 describes the proposed CS-based LFMCW radar signal processor. Performance evaluation of LFMCW radar signal processing using CS compared to the traditional one is provided in section 5. Finally, conclusion comes in section 6.

2. PRINCIPLES OF COMPRESSIVE SENSING

Considering a discrete time (or space) signal having N samples. This signal can be represented as a complex N -length vector $x \in \mathbb{C}^N$. Known by the Matrix theory, any signal in the space $\mathbb{C}^N$ can be represented by orthonormal basis $\{x_i\}$ where i from 1 to N with linear form. Orthonormal basis $\{x_i\}$

can be rewritten as $x=[x_1, x_2 \dots x_N]$ in the form of Matrix. Any signal $S \in \mathbb{C}^N$ can be represented in terms of x as [4-6]:

$$S = \sum_{i=1}^N \xi_i x_i \quad (1)$$

Where, ξ_i is the coordinate of x . The specific form of ξ_i is $\xi_i = \langle S, x_i \rangle$.

Eq. (1) can be rewritten as [9]:

$$S = x\Psi \quad (2)$$

Where, $\Psi = [\xi_1, \xi_2, \dots, \xi_N]^T$.

It is clear that Ψ and S are equivalent representations of the same signal, X . In other words, S is the time domain signal, and Ψ is the representation in the transformation domain. Suppose K be a non-zero number of elements of Ψ . If K is smaller than N , it can be shown that the signal S is sparse or compressible. Meanwhile, basis vector $\{x_i\}$ is the sparse basis of the signal S [8,9].

The information contained in many natural signals can be represented more concisely if the signals are looked at in a proper transform domain. Then the complex signal of interest $x \in \mathbb{C}^N$ can be recovered from an under sampled set of linear projections which is called measurement vector $y \in \mathbb{C}^M$, where $M < N$, using one of a reconstruction algorithm. The sensing model can be mathematically represented as [5]:

$$y = Ax \quad (3)$$

where the $M \times N$ matrix A is called the sensing matrix. The sensing mechanism of acquiring M measurements via linear projection through the operator is mathematically represented as in Eq.(3). Using the sparse representation, then the compressed vector, y can be written as [5]:

$$y = Ax = A\Psi \quad (4)$$

Where Ψ is $N \times N$ measurement CS matrix. For the special class of matrices with Gaussian independent and identically distributed (i.i.d.) random entries, it has been demonstrated that the Restricted Isometry Property (RIP) condition is satisfied with high probability to the following equation [3]

$$M \geq CK \log(N/K) \quad (5)$$

Where N is the total number of the original signal samples, M is the number of measurement vector such that $M < N$ and C is a constant dependent on the total number of samples and the number of useful signals samples (K) [3].

3. APPLICATION OF CS IN LFM CW RADAR

Application of CS in LFM CW radar signal processing had been investigated and proved by the authors in [8] as shown

in Fig.(2). Sparse discrete signal of LFM CW radar is generated after multiplying the received base band radar signal (after ADC) with a transformation matrix Φ of dimension $N \times N$. The transformation matrix (Φ) should be Fourier matrix to satisfy the sparsity condition. The generated sparse signal is N samples. This sparse signal is used to generate a compressed or measurement vector, y , with length M where $M < N$ after multiplication with measurement matrix, Ψ ($N \times M$).

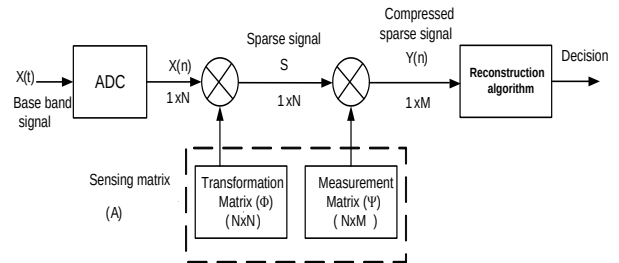


Figure 2 Application of CS in LFM CW radar [8].

According to [8,9], The target information (range and speed) contained in the original signal can be reconstructed from the compressed measurement using different reconstruction algorithms. According to [8], CAMP algorithm [10-12] gives the best result. The Nyquist rate based CS (CAMP) approach has been found to give the highest detection performance and an extra amount of calculations. In the next section, a proposed approach is introduced to reduce the sampling rate below the Nyquist and reduce the extra calculations (complexity).

4. THE PROPOSED CS-BASED LFM CW RADAR SIGNAL PROCESSOR

The compressed samples (M) are then transformed into sparse (measurement vector, y) by applying FFT on azimuth direction. To get the required target parameters (range and speed), a reconstruction process based on CAMP algorithm is performed on the obtained measurement, y . To achieve certain false alarm probability using the internal adaptive thresholding function in the CAMP algorithm, a gain factor (G) is used. The idea behind the proposed approach, shown in Fig.(3), is how to get the compressed sparse signal $y(n)$, directly from the base band signal without performing multiplication with a sensing matrix and without acquiring the base band signal with Nyquist rate.

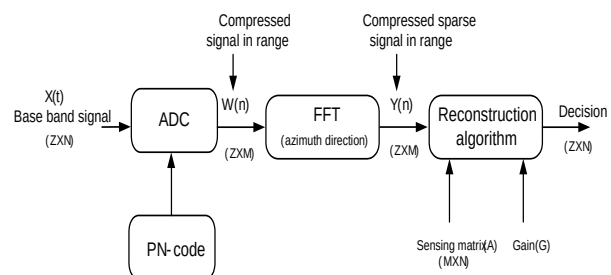


Figure 3 General block diagram of the proposed CS-based LFM CW radar signal processor

This is achieved by sampling the base band signal with a PN sequence with length $M < N$ achieving the required reduction ratio (δ) in range samples where $\delta = M/N$ according to Eq.(5).

A simple description for the block diagram of the proposed approach is shown in Fig.(4) where the reconstruction algorithm is initialized by the sensing matrix (A), the compressed sparse signal (y), and the threshold gain (G).

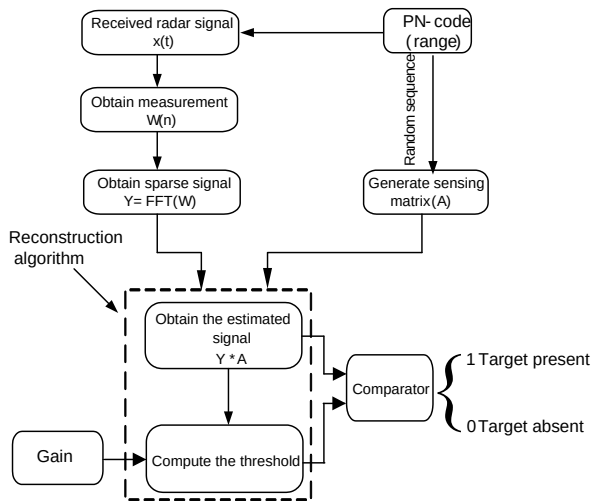


Figure 4 Function description of the proposed approach

The sensing matrix, A, is initially generated using $N \times N$ DFT matrix where N is the total number of range cells. PN-code generates a random sequence of length (M) according to a required reduction factor (δ).

The multiplication of $M \times N$ sensing matrix (A) by the sparse measurement signal, y, where $y = \text{FFT}(W)$, is performed to generate the output estimated signal, X_{rec} as:

$$X_{\text{rec } Z \times N} = Y_{Z \times M} * A_{M \times N} \quad (6)$$

As discussed before, the original signal reconstruction is not required in LFM CW radar system, but information extraction such as target range and speed is the main task. Therefore, the output detection from the reconstruction algorithm is a binary logic which is logic "1" when the target is present and logic "0" when the target is absent. The adaptive threshold is calculated according to [12]:

$$\text{Threshold} = \text{Gain} * \text{median}[A * \text{FFT}(y)] \quad (7)$$

It is expected that the proposed approach achieves a reduction in sampling rate, memory size, and processing time. All these factors are discussed in the next section along with the detection and resolution performance and compared to that of the traditional LFM CW radar.

5. PERFORMANCE EVALUATION OF THE PROPOSED LFM CW RADAR SIGNAL PROCESSOR

The performance of the proposed LFM CW radar signal processor based on CS is evaluated using Matlab. The base band radar signal is generated using the simulation model outlined in [13]. The number of total range-Doppler samples to be processed is assumed to be 128 range samples and 32 Doppler samples. It simulates a composite radar environment

with the receiver thermal noise and known target returns. The thermal noise simulated is Additive White Gaussian with zero mean and unity variance. The setup procedure for evaluating the performance of the proposed LFM CW radar signal processor based on CS compared to the traditional LFM CW radar signal processor based on FFT is shown in Fig.(5). The comparison aspects are the detection performance, resolution performance and hardware complexity.

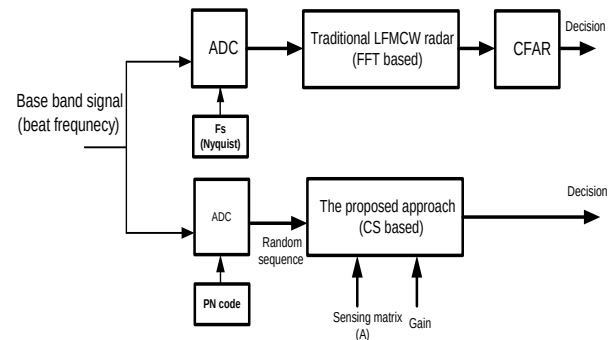


Figure 5 Setup procedure for evaluating the proposed approach compared to the traditional one

5.1 Detection Performance

Detection performance is evaluated for both the proposed algorithm and the traditional one through ROC by calculating both probability of false alarm (P_{fa}) and probability of detection (P_d). The probability of false alarm (P_{fa}) is calculated using the CFAR processing in the traditional LFM CW radar. But in the proposed approach, it is computed after the reconstruction algorithm where the internal threshold is used to control the output false alarm probability. The reconstruction algorithm output can directly indicate the target existence or not. Fig.(6) shows the comparison between the probabilities of false alarm (P_{fa}) and the threshold gains for both the proposed approach and the traditional LFM CW radar. This figure is used to set the correct threshold gains in both the proposed approach and the traditional one which achieve the same probability of false alarm insuring fair detection comparison.

The detection performance of the proposed approach is evaluated using different reduction percentages in range samples; 25%, 50% and 75%. This detection performance is compared with that of the traditional one at different SNRs and at P_{fa} of 10^{-5} .

The detection performance through ROC of both the proposed approach and the traditional one is evaluated for three cases representing the existence of different number of targets and at different reduction ratios.

- Case I: assumes that a single target is located in one Coherent Processing Interval (CPI) such that its expected range cell is 64 and its normalized Doppler frequency is 0.5 and at different reduction ratios (δ) in range samples. The detection performance of this case is shown in Fig.(7) and its range-Doppler representation is shown in Fig.(8).

- Case II: assumes that nine targets are located such that their expected range cells are 16,32, 64 ,48,52,70,80,96 and 105 with the same normalized Doppler frequency as in case I. The detection performance in this case is shown in Fig.(9) and its range-Doppler representation is shown in Fig.(10).

- Case III: assumes that ten targets are located such that their expected range cells are 16, 32, 64, 48, 52, 70, 80, 96, 105 and 114 with the same normalized Doppler frequency of 0.5. The detection performance of this case is shown in Fig.(11) and its range-Doppler representation is shown in Fig.(12).

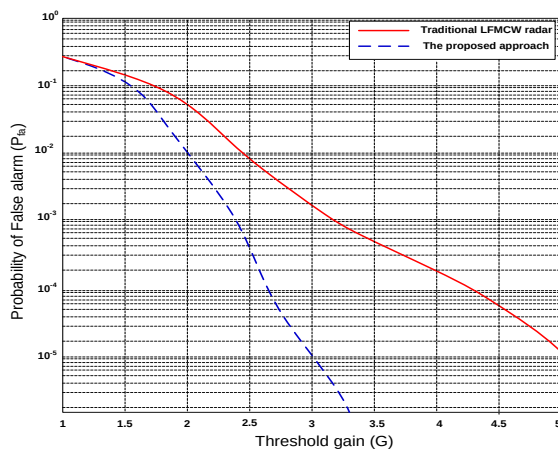


Figure 6 The probability of false alarm, P_{fa} , against the threshold gain for the proposed approach and the traditional LFM CW radar

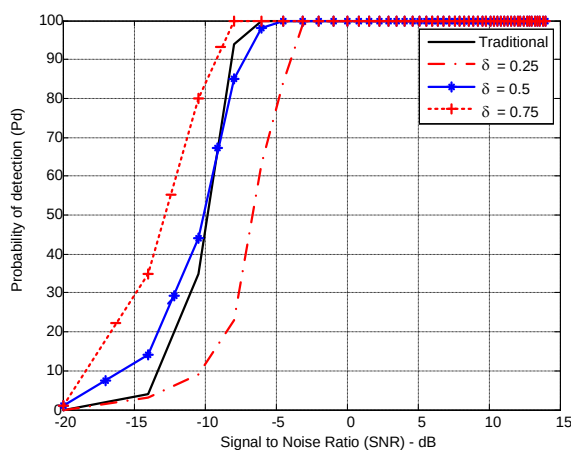


Figure 7 ROC of the proposed approach at different reduction ratios in range samples and the traditional LFM CW radar for case I for $P_{fa} = 10^{-5}$

The obtained results from Fig.(7) through Fig.(12) confirm that, as the reduction ratio in range samples decreases, the detection performance decreases. Also, as the number of targets increases, the detection performance decreases. So, Eq.(5) should be considered to determine the available reduction ratios for the required number of targets to be detected. When this equation is met and as the reduction ratio in range samples increases, the detection performance increases outperforming that of the traditional one.

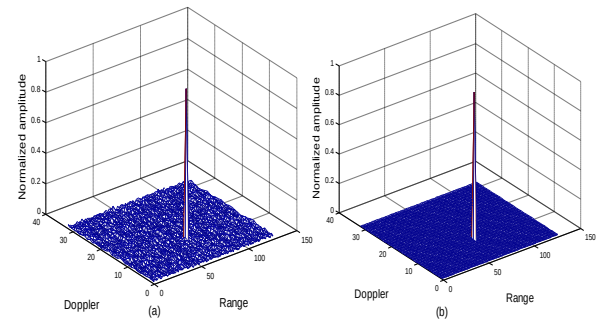


Figure 8 Range-Doppler processing representation for case I, SNR= -5dB and $P_{fa} = 10^{-5}$.

(a) Traditional LFM CW radar.
(b) The proposed approach.

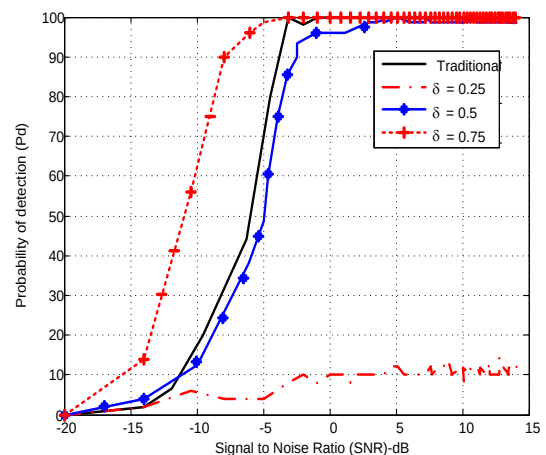


Figure 9 ROC of the proposed approach at different reduction ratios in range samples and the traditional LFM CW radar for case II for $P_{fa} = 10^{-5}$.

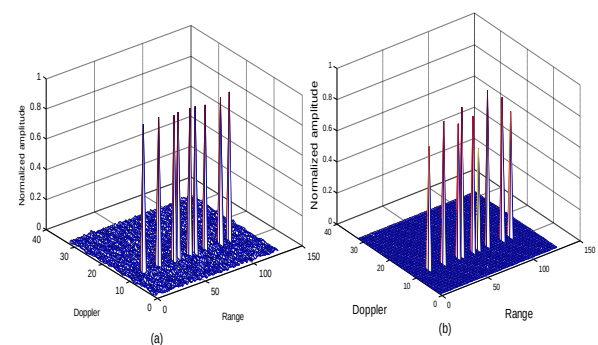


Figure 10 Range-Doppler processing representation for case II, SNR= -5dB and $P_{fa}=10^{-5}$.

(a) Traditional LFM CW radar.
(b) The proposed approach.

Fig.(13) illustrates the range-Doppler processing representation of twelve targets ($K=12$) for both the traditional LFM CW radar and the proposed approach at 50% reduction ratio in range samples and SNR=-5dB. The severe degradation is clear in this case. This is because $k=12$ does not satisfy Eq.(5) for $N=128$ and $\delta=50\%$. So, the relation between the reduction ratio in samples and the number of detected targets should be met.

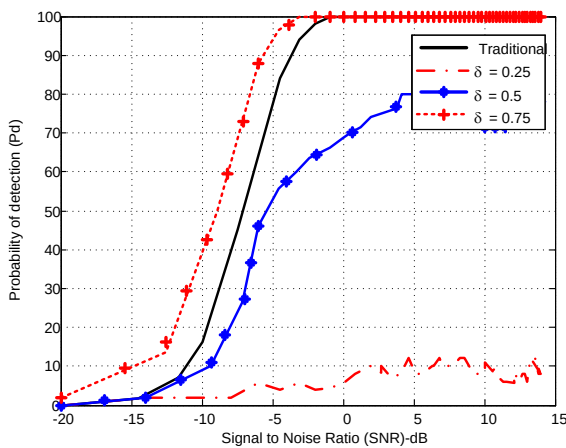


Figure 11 ROC of the proposed approach at different reduction ratios in range samples and the traditional LFM radar for case III for $P_{fa} = 10^{-5}$.

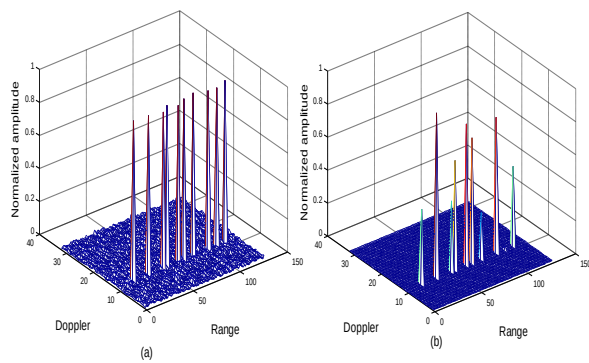


Figure 12 Range-Doppler processing representation for case III, SNR = -5 dB and $P_{fa} = 10^{-5}$.
(a) Traditional LFM radar.
(b) The proposed approach.

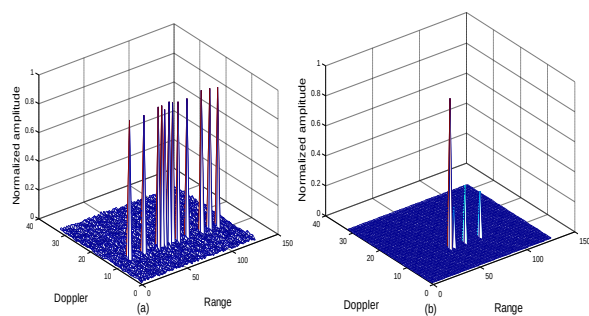


Figure 13 Range-Doppler representation of the proposed approach at 50% reduction ratio in range samples and the traditional FMCW radar for twelve targets, SNR = -5 dB and $P_{fa} = 10^{-5}$.
(a) Traditional LFM radar.
(b) The proposed approach.

Table (1) summarizes the relation between the number of maximum targets to be detected (K) and the measurement samples (M) achieving the minimum reduction ratio ($\delta = M/N$) for a total number of range samples (N) of 128.

Table 1 The relation between the number of targets (K) and the minimum number of the measurement samples (M) for the proposed approach

Number of targets (K)	Minimum number of measurements (M)	Minimum reduction ratio ($\delta = M/N$)
1	21	0.16
9	58	0.46
10	67	0.51

5.2 Resolution Performance

The resolution performance of the proposed approach in range and Doppler domains for LFM radar signal processing is compared with that of the traditional one. Range resolution is measured by simulating two targets at two range cells number 65 and 66 respectively. Fig.(14) shows the output representation in range domain of these targets at 50% reduction ratio in range samples ($\delta = 0.5$), SNR of -5 dB and P_{fa} of 10^{-5} .

Doppler resolution is also measured by simulating two targets with two Doppler cells (numbers 11 and 12 respectively). Fig.(15) shows the output representation in Doppler domain of these targets at the selected Doppler cells for SNR of -5 dB and for P_{fa} of 10^{-5} .

From these figures, it is found that, the resolution performance for the proposed approach and the traditional one in both range and Doppler domains is the same.

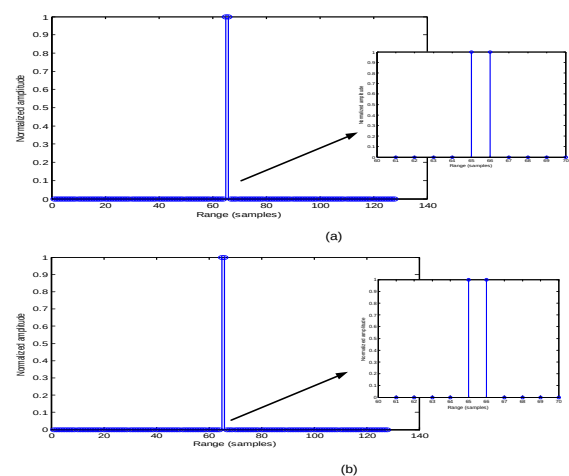


Figure 14 Resolution in range domain of both the proposed approach, and traditional LFM radar for two targets (range cell numbers 65, 66) at SNR = -5 dB and $P_{fa} = 10^{-5}$.
(a) Traditional LFM radar.
(b) The proposed approach ($\delta = 50\%$).

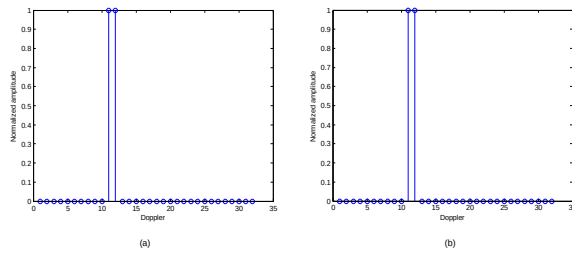


Figure 15 Resolution in Doppler domain of both the proposed approach and traditional LFM CW radar for two targets (Doppler cells numbers 10 and 11) at SNR=-5dB and $P_{fa} = 10^{-5}$.

(a) Traditional LFM CW radar.
(b) The proposed approach ($\delta=50\%$).

5.3 Hardware Complexity

The expected complexity is evaluated regarding the required memory, number of complex operation, and consequently the processing time. Initial calculations for the proposed approach of Fig.(3) compared to that of the traditional one of Fig.(1) are summarized in Table (2). This comparison is performed considering a reduction ratio of 50% in range samples ($\delta=0.5$) for the proposed approach.

From this table, it is found that, the complexity and the processing time of the proposed approach are approximately half that of the traditional one.

Table 2 Initial complexity comparison between the proposed approach at 50% reduction ratio in range samples and the traditional one, (128 range cells and 32 Doppler cells).

Comparison aspects Approach	Sampling frequency	Memory size (cells)	Time of processing (clock)	Complexity	
				operations	Calculations (complex addition and multiplication)
Traditional (N-pionts)	f_s	32x128 x2 (8192)	Two CPI (8192)	128 point FFT + 32 point FFT + CFAR threshold	1072
The proposed approach (Range reduction) (M=N/2)	$f_s/2$	32x64+ 64x128 (10240)	One CPI (4096)	32 point FFT + y * A + Thresholding	479

6. CONCLUSION

In this paper, LFM CW radar signal processing based on CS theory is performed to extract target information such as range and speed. The proposed CS-based LFM CW radar signal processor is based on range samples reduction. The reconstructed signal has been obtained using CAMP algorithm. Performance evaluation of the proposed approach is compared with that of the traditional one through ROC curves. The detection performance of the proposed approach is approximately similar to that of the traditional one in case of 50% reduction ratio in range samples. As the reduction ratio increases, the detection performance increases. Time of processing and the expected hardware complexity of the proposed approach is half that of the traditional one in case of 50% reduction ratio in range samples. The resolution performance of the proposed approach is similar to that of the traditional one in both range and Doppler domains.

The proposed approach may be considered as a good alternative to the traditional LFM CW radar signal processor with less sampling rate, less hardware complexity, and less processing time. Real time implementation of the proposed approach using FPGA may be considered as a future work.

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