A Procedure for Modeling A.C. Microgrids Controlled by The Droop Technique

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Abstract- **This paper presents a procedure for obtaining the small-signal model of an inverter-based microgrid, where every inverter is controlled by means of the droop technique. The proposed procedure is based on methodologies found in current literature, notwithstanding, unlike those methodologies, the model obtained simplifies the deduction of the equations that describe the microgrid electrical circuit. Additionally, the obtained model allows the analysis of the microgrid response to variations on the set points of the active and reactive power. Besides, unlike other procedures found in literature, this work presents all the steps needed to obtain the model, including the equation deduction and the calculation of the quiescent points. The model obtained by following the proposed procedure is verified through Simulink MatLab simulations.**

Keywords: Dynamic phasors, A.C. microgrid, dynamic model, small signal.

I.   INTRODUCTION

Given the increasing penetration of non-conventional energy sources in the electrical distribution system, distributed generation and electrical microgrids have become of great importance [1]. Microgrids have advantages over traditional generation systems due to the use of renewable sources. In addition, among the advantages of these, there is the gradual reduction of the adverse effects of climate change, the possibility of dealing with the reduction of non-renewable sources where there is an increasing demand for energy, the reduction of losses in traditional transmission lines, among others [2].

Microgrids can be defined as electrical grids that have their respective electrical loads, power source systems, and energy storage units that are controlled to reliably and efficiently supply power to local loads [3-4]. In addition, inverter-based microgrids can operate in either the grid-connected or grid-isolated mode [5].

Inverter-based microgrids that are controlled by the droop technique have the advantage of not requiring a communication system for power distribution. On the other hand, it is possible that small disturbances appear during load changes, eventually producing transient oscillations, due to this, it is important to improve the dynamic characteristics of the microgrid. For this improvement to occur, it is necessary to have models that consider the complete dynamics of the microgrid, including the droop controller, the dynamic of the connection lines, the loads, and the dynamics of the inverters. Several procedures have been presented in the literature to model the microgrid [6-9].

In this work, a procedure is proposed that adds an improvement in the construction of a small signal model with respect to the procedures found in the literature. In addition, it allows the analysis of the microgrid when subjected to variations in droop curve set points. Thus, this document describes the complete procedure to obtain the model of a small signal microgrid. For this, the parts that conform the model are initially explained, then the procedure to deduce the mathematical expressions that represent each part and how they are joined is shown. Subsequently, it is shown how to obtain the quiescent points and a verification is carried out by simulation. Finally, the conclusions of the work are presented.

II.   MICROGRID'S DYNAMIC MODEL

To obtain the dynamic model of the microgrid of inverters controlled by droop curves, the procedure used by [9] was followed, which in turn was adapted from [6]. This procedure can be understood with the diagram in figure 1, where three main blocks are shown, each indicating its inputs on the left side, its outputs on the right side and the relationship that these variables have with the other blocks is indicated by arrows.

The block called "Droop dynamic Model" comprises a representation in state space of the small-signal dynamics of the droop control law. Specifically, this block models the dynamics of small variations in the frequency of the *n*-th invert $\Delta\omega_n(t)$ and of the components of its voltage phasor (real and imaginary part) $\Delta E_{n,d}(t)$ and $\Delta E_{n,q}(t)$. This is done considering the measurement filter applied to the active and reactive power readings of each inverter and the two droop curves of each inverter. Thus, since the model contained in this block does not consider the calculation of active and reactive power, the main input of this is the small variations in the active and reactive power of each inverter $\Delta P_n(t)$ and $\Delta Q_n(t)$. Furthermore, unlike the model used in [9] and [6], here, small variations are considered as extra inputs that can be applied to the adjustment points of the droop curves of each inverter $\Delta E_{0n}(t)$ and $\Delta\omega_{0n}(t)$. These small variations allow to model displacements on the axis of the abscissa of the droop curves (No-load frequency and voltage), so they can be used for adjusting the distribution of active and reactive power; accordingly, they can be used to adjust the system frequency or voltage at a given point on the microgrid.

The block called "Dynamic model of the microgrid passive elements" comprises a representation in state space of the dynamics of the real and imaginary components of the phasor of the currents of each inductor and of the voltages of each capacitor connected to the grid of connections. (connection lines, distribution lines and loads). This model is obtained through the use of dynamic phasors and has as input the components of the voltage phasors of each of the inverters. Regarding this, note that this block has as input the small variations of the components of the voltage phasor of each inverter, so that although the model contained in this block is not a small-signal model (it can be used to model large variations), it will produce as outputs, small variations of the components of the voltage and current phasors contained in it.

Fig 1: Block diagram of the microgrid model and its parts

Given that the model in state space contained in the block "Dynamic model of the microgrid passive elements" considers the components of the currents in the inductors of the connection lines ($I_{Cn,d}(t)$, $I_{Cn,q}(t)$), these can be fed back to the block, "Active and reactive power small variation calculation", thus closing the control loop. On the other hand, the phasor components of the other currents and voltages modeled in the block "Dynamic model of the microgrid passive elements" ($X_{An,d}(t)$, $X_{An,q}(t)$) are not used in the control law, so they are not fed back.

In the following parts of this document, the procedure to obtain each of the blocks that comprises the complete model of the microgrid is described.

A. droop dynamic model (inverters)

To obtain the dynamic model of the set of inverters, in the same way as proposed in [6], it is assumed that the control law for the droop curves are given by:

$$\omega(t) = \omega\_0\,(t) - k\_P\,(P\_flt\,(t) - P\_0\,)$$
$$E(t) = E_0(t) - k_q\big(Q_{flt}(t) - Q_0\big) \tag{1}$$

Where ω is the inverter's frequency, E is the inverter's RMS voltage, ω_0, E_0, P_0 and Q_0 are constants that are adjusted according to the operating point and the capacity of the inverter, while k_p and k_q are the slopes of droop curves. $P_{flt}(t)$ and $Q_{flt}(t)$ are the measured and filtered values of

the active and reactive power of the same inverter, whose filtering is given by [6]:

$$P_{flt}(s) = \frac{\omega_c}{s + \omega_c} P_{inv}(s)$$
$$Q_{flt}(s) = \frac{\omega_c}{s + \omega_c} Q_{inv}(s) \tag{2}$$

Where ω_c is the cutoff frequency of the filter, while $P_{inv}(s)$ and $Q_{inv}(s)$ are the active and reactive power delivered by the inverter.

Unlike [6] and [9], to obtain the small signal model, in this work are also considered the small variations of the inverter's frequency and voltage adjustment points (ω_0 and E_0), that is:

$$\Delta\omega(t) = \Delta\omega_{0(t)} - k_P\Delta P_{flt}(t)$$
$$\Delta E(t) = \Delta E_0(t) - k_q\Delta Q_{flt}(t) \tag{3}$$

Where $\Delta\omega(t), \Delta\omega_0(t), \Delta P_{flt}(t), \Delta E(t), \Delta E_0(t)$ and $\Delta Q_{flt}(t)$ are the small variations of $\omega(t), \omega_0(t), P_{flt}(t), E(t), E_0(t)$ and $Q_{flt}(t)$ around its quiescent point, respectively. Thus, following a procedure similar to that developed by [6], it can be shown that:

$$\frac{d\Delta\omega(t)}{dt} = -k_P\omega_c\Delta P_{inv}(t) - \omega_c\Delta\omega(t) + \frac{d\Delta\omega_0(t)}{dt} + \omega_c\Delta\omega_0(t)$$
$$\frac{d\Delta E(t)}{dt} = -k_q\omega_c\Delta Q_{inv}(t) - \omega_c\Delta E(t) + \frac{d\Delta E_0(t)}{dt} + \omega_c\Delta E_0(t) \tag{4}$$

Furthermore, considering that the inverter voltage can be represented by phasors, that is:

$$\vec{E} = E\angle\delta = E_d + jE_q \tag{5}$$

Where:

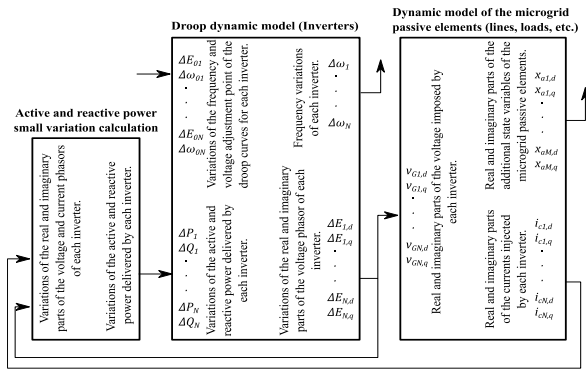$$E_d = E\cos\delta \text{ y } E_q = E\sin\delta \tag{6}$$

With:

$$E = \sqrt{E_q^2 + E_d^2} \text{ y } \delta = \tan^{-1}\left(\frac{E_q}{E_d}\right) \tag{7}$$

Using the Taylor series as done in [6], it can be shown that:

$$\begin{bmatrix} \dfrac{d\Delta E_d(t)}{dt} \\ \dfrac{d\Delta E_q(t)}{dt} \\ \dfrac{d\Delta\omega(t)}{dt} \end{bmatrix} = \mathbf{A_{E\omega}} \begin{bmatrix} \Delta E_d(t) \\ \Delta E_q(t) \\ \Delta\omega(t) \end{bmatrix} + \mathbf{B_{PQ}} \begin{bmatrix} \Delta P_{inv}(t) \\ \Delta Q_{inv}(t) \end{bmatrix}$$
$$+ \mathbf{B_{E\omega0}} \begin{bmatrix} \dfrac{d\Delta E_0(t)}{dt} \\ \dfrac{d\Delta\omega_0(t)}{dt} \end{bmatrix}$$
$$+ \omega_c\mathbf{B_{E\omega0}} \begin{bmatrix} \Delta E_0(t) \\ \Delta\omega_0(t) \end{bmatrix} \tag{8}$$

Where:

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$\mathbf{A_{E\omega}}$

$$= \begin{bmatrix} \dfrac{-\omega_c k_{DEq} k_{EEd}}{k_{EEd} k_{DEq} - k_{EEq} k_{DEd}} & \dfrac{-\omega_c k_{DEq} k_{EEq}}{k_{EEd} k_{DEq} - k_{EEq} k_{DEd}} & k_E \\ \dfrac{\omega_c k_{DEd} k_{EEd}}{k_{EEd} k_{DEq} - k_{EEq} k_{DEd}} & \dfrac{\omega_c k_{DEd} k_{EEq}}{k_{EEd} k_{DEq} - k_{EEq} k_{DEd}} & k_E \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{B_{PQ}} = \begin{bmatrix} 0 & \dfrac{-k_q \omega_c k_{DEq}}{k_{EEd} k_{DEq} - k_{EEq} k_{DEd}} \\ 0 & \dfrac{k_q \omega_c k_{DEd}}{k_{EEd} k_{DEq} - k_{EEq} k_{DEd}} \\ -k_P \omega_c & 0 \end{bmatrix} \text{ y } \mathbf{B_{E\omega 0}} =$$

$$\begin{bmatrix} \dfrac{k_{DEq}}{k_{EEd} k_{DEq} - k_{EEq} k_{DEd}} & 0 \\ \dfrac{-k_{DEd}}{k_{EEd} k_{DEq} - k_{EEq} k_{DEd}} & 0 \\ 0 & 1 \end{bmatrix} \tag{9}$$

With [6]:

$$k_{DEd} = \frac{-E_{qe}}{E_{de}^2 + E_{qe}^2}, k_{DEq} = \frac{E_{de}}{E_{de}^2 + E_{qe}^2}, k_{EEd}$$
$$= \frac{E_{de}}{\sqrt{E_{de}^2 + E_{qe}^2}}, k_{EEq}$$
$$= \frac{E_{qe}}{\sqrt{E_{de}^2 + E_{qe}^2}} \tag{10}$$

Where the constants E_{de} and E_{qe} are the quiescent points of the two components of the inverter output voltage.

Thus, since the system of equations obtained contains the derivative of the input variables, to obtain the representation of the system in state variables, 3 auxiliary variables are defined as follows:

$$\begin{bmatrix} \Delta x_d(t) \\ \Delta x_q(t) \\ \Delta x_\omega(t) \end{bmatrix} = \begin{bmatrix} \Delta E_d(t) \\ \Delta E_q(t) \\ \Delta \omega(t) \end{bmatrix} - \mathbf{B_{E\omega 0}} \begin{bmatrix} \Delta E_0(t) \\ \Delta \omega_0(t) \end{bmatrix} \tag{11}$$

In this way, by changing the variables, it is possible to obtain:

$$\begin{bmatrix} \dfrac{d\Delta x_d(t)}{dt} \\ \dfrac{d\Delta x_q(t)}{dt} \\ \dfrac{d\Delta x_\omega(t)}{dt} \end{bmatrix} = \mathbf{A_{E\omega}} \begin{bmatrix} \Delta E_d(t) \\ \Delta E_q(t) \\ \Delta \omega(t) \end{bmatrix} + \mathbf{B_{PQ}} \begin{bmatrix} \Delta P_{inv}(t) \\ \Delta Q_{inv}(t) \end{bmatrix}$$
$$+ \omega_c \mathbf{B_{E\omega 0}} \begin{bmatrix} \Delta E_0(t) \\ \Delta \omega_0(t) \end{bmatrix} \tag{12}$$

Then, from the two previous equations, the small-signal model of the output voltage and frequency of an inverter, which is controlled by the droop curves used, expresses as:

$$\begin{bmatrix} \dfrac{d\Delta x_d(t)}{dt} \\ \dfrac{d\Delta x_q(t)}{dt} \\ \dfrac{d\Delta x_\omega(t)}{dt} \end{bmatrix} = \mathbf{A_{E\omega}} \begin{bmatrix} \Delta x_d(t) \\ \Delta x_q(t) \\ \Delta x_\omega(t) \end{bmatrix}$$
$$+ (\mathbf{A_{E\omega} B_{E\omega 0}} + \omega_c \mathbf{B_{E\omega 0}}) \begin{bmatrix} \Delta E_0(t) \\ \Delta \omega_0(t) \end{bmatrix}$$
$$+ \mathbf{B_{PQ}} \begin{bmatrix} \Delta P_{inv}(t) \\ \Delta Q_{inv}(t) \end{bmatrix} \tag{13}$$

Knowing the representation in state space for one of the inverters, a model which includes *N* inverters can be built by forming an increased system such as:

$$\frac{d}{dt}(\Delta x_{invs}) = \mathbf{A_{invs}} \Delta x_{invs} + \mathbf{B_{invsEW}} \Delta u_{invsEW}$$
$$+ \mathbf{B_{invsPQ}} \Delta u_{invsPQ} \tag{14}$$
$$\Delta e_{invs} = \mathbf{C_{invs}} \Delta x_{invs} + \mathbf{D_{invs}} \Delta u_{invsEW} \tag{15}$$

With:

$$\mathbf{A_{invs}} = diag(\mathbf{A_{E\omega,1}}, \mathbf{A_{E\omega,2}}, \dots, \mathbf{A_{E\omega,N}})_{3Nx3N}$$
$$\mathbf{B_{invsPQ}} = diag(\mathbf{B_{PQ,1}}, \mathbf{B_{PQ,2}}, \dots, \mathbf{B_{PQ,N}})_{3Nx2N}$$
$$\mathbf{B_{invsEW}} = diag(\mathbf{A_{E\omega,1} B_{E\omega 0,1}}$$
$$+ \omega_c \mathbf{B_{E\omega 0,1}}, \mathbf{A_{E\omega,2} B_{E\omega 0,2}}$$
$$+ \omega_c \mathbf{B_{E\omega 0,2}}, \dots, \mathbf{A_{E\omega,N} B_{E\omega 0,N}}$$
$$+ \omega_c \mathbf{B_{E\omega 0,N}})_{3Nx2N} \tag{16}$$

$$\mathbf{C_{invs}} = diag\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \dots, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\right)_{2Nx2N}$$

$$\mathbf{D_{invs}} = diag\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{B_{E\omega 0,1}}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{B_{E\omega 0,2}}, \dots, \begin{bmatrix} 1 & 0 \\ 0 & \end{bmatrix}\right)$$

Where the size of the state vector Δx_{invs} is $3N$ positions, while the input vectors Δu_{invsPQ}, which contains the variations of the active and reactive power of each inverter and Δu_{invsEW}, which contains the variations to the set points, are $2N$ positions each. For its part, the output vector Δe_{invs} contains the variations of the two components of the voltage of each inverter. In this way, these vectors can be written as:

$$\Delta x_{invs} = [[\Delta x_{1,d} \quad \Delta x_{1,q} \quad \Delta x_{1,\omega}] \quad [\Delta x_{2,d} \quad \Delta x_{2,q} \quad \Delta x_{2,\omega}] \quad \cdots$$
$$\Delta u_{invsPQ} = [[\Delta P_1 \quad \Delta Q_1] \quad [\Delta P_{inv,2} \quad \Delta Q_2] \quad \cdots \quad [\Delta P_{inv,N} \quad \Delta Q_N$$
$$\Delta u_{invsEW} = [[\Delta E_{01} \quad \Delta \omega_{01}] \quad [\Delta E_{02} \quad \Delta \omega_{02}] \quad \cdots \quad [\Delta E_{0N} \quad \Delta \omega_0$$
$$\Delta e_{invs} = [[\Delta E_{1,d} \quad \Delta E_{1,q}] \quad [\Delta E_{2,d} \quad \Delta E_{2,q}] \quad \cdots \quad [\Delta E_{N,d} \quad \Delta E$$
$$\tag{17}$$

To obtain the small variations in the active and reactive power delivered by each inverter, it is decided to make the calculation from the voltage and current components, as in [7] and [9], that is:

$$\begin{bmatrix} \Delta P(t) \\ \Delta Q(t) \end{bmatrix} = \mathbf{I_e} \begin{bmatrix} \Delta E_d(t) \\ \Delta E_q(t) \end{bmatrix} + \mathbf{E_e} \begin{bmatrix} \Delta I_d(t) \\ \Delta I_q(t) \end{bmatrix} \tag{18}$$

With:

$$\mathbf{I_e} = \begin{bmatrix} I_{de} & I_{qe} \\ -I_{qe} & I_{de} \end{bmatrix}, \mathbf{E_e} = \begin{bmatrix} E_{de} & E_{qe} \\ E_{qe} & -E_{de} \end{bmatrix} \qquad (19)$$

Where E_{de} and E_{qe} are the quiescent points of the two components of the inverter voltage, while I_{de} and I_{qe} are those of the components of the inverter's current. Thus, as done with the inverter model, the calculation of small variations in the active and reactive power of the N inverters can be written as:

$$\begin{bmatrix} \begin{bmatrix} \Delta P_1(t) \\ \Delta Q_1(t) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \Delta P_N(t) \\ \Delta Q_N(t) \end{bmatrix} \end{bmatrix} = \mathbf{I_{invse}}\Delta e_{invs} + \mathbf{E_{invse}}\Delta i_{invs} \qquad (20)$$

Where Δi_{invs} contains the variations of the two current components of each inverter and has $2N$ positions. It can be written as:

$$\Delta i_{invs} = [[\Delta I_{C1,d}(t) \quad \Delta I_{C1,q}(t)] \quad [\Delta I_{C2,d}(t) \quad \Delta I_{C2,q}(t)] \quad \cdots$$
$$(21)$$

In this case, the subscript C was added to each component of each current because the current of the inverter is the same current of the connection line between the inverter and other connections of the microgrid. On the other hand, $\mathbf{I_{invse}}$ and $\mathbf{E_{invse}}$ contain the matrices of the quiescent points of current and voltage for each inverter, respectively:

$$\mathbf{I_{invse}} = diag(\mathbf{I_{e,1}}, \mathbf{I_{e,2}}, \dots, \mathbf{I_{e,N}})_{2Nx2N}, \mathbf{E_{invse}}$$
$$= diag(\mathbf{E_{e,1}}, \mathbf{E_{e,2}}, \dots, \mathbf{E_{e,N}})_{2Nx2N} \qquad (22)$$

B. Dynamic model of the microgrid passive elements

Since the model of the internal connections of the microgrid receives, as input, the two voltage components of each inverter $\mathbf{u_{ur}}$ and has as outputs, the components of the current delivered by each inverter i_{invs}, the representation, in state space, of the model contained in this block must have the form:

$$\frac{d(x_{ur})}{dt} = \mathbf{A_{ur}}x_{ur} + \mathbf{B_{Uur}}u_{ur} \qquad (23)$$

$$i_{invs} = \mathbf{C_{ur}}x_{ur} \qquad (24)$$

With:

$$x_{ur} = [I_{C1,d} \quad I_{C1,q} \quad \cdots \quad I_{CN,d} \quad I_{CN,q} \quad X_{A1,d} \quad X_{A1,q} \quad X_{A2}$$

$$i_{invs} = [I_{C1,d} \quad I_{C1,q} \quad I_{C2,d} \quad I_{C2,q} \quad \cdots \quad I_{CN,d} \quad I_{CN,q}]^T$$

$$u_{ur} = [E_{1,d} \quad E_{1,q} \quad E_{2,d} \quad E_{2,q} \quad \cdots \quad E_{N,d} \quad E_{N,q}]^T$$

Where the state vector x_{ur} of this representation contains the two components of the current of each inductor and of the voltage of each capacitor connected to the microgrid.

For this case study, it is considered the grid shown in figure 2, which is based on the microgrid studied in [10]. This figure shows the electrical circuit that represents one of the phases of the microgrid. The generators are represented with the sources $v_{Gn}(t)$ (n=1,..,4), the line used for the connection of each generator to the circular grid is represented with R_{Cn} and L_{Cn} (n=1,..,4), the lines that make up the circular grid that help in the distribution are represented with R_{Dn} and L_{Dn} (n=1,..,5), inductive loads are

represented by R_{Ln} and L_{Ln} (n=1,..,5). On the other hand, the resistors and capacitors R_{Can} and C_{an} (n=1,..,5) are components added to the original grid to facilitate obtaining the dynamic model.

Using the same subscripts, the current in the inverters connection line is denoted by $i_{Cn}(t)$ (n=1,...,4), the current in the distribution lines is indicated as $i_{Dn}(t)$ (n=1,...,5), the current in the loads are $i_{Ln}(t)$ (n=1,...,5) and the voltage over each load (added capacitors) is denoted by $v_{Can}(t)$ (n=1,...,5). Note also that the directions of the currents and the polarities of the voltages were all defined in the same way for all branches of the same type.

Unlike the grid studied in [10], in this work at each node where there is a load (inductive) there is placed, in parallel, a capacitor and a resistor. This is done based on the strategy proposed in [7], where a grounded resistor is added to each node of the original grid so that the node voltage is well defined. The addition of this resistor allows to write the sum of currents in each node in a convenient way to simplify obtaining the model in state space, however, it is necessary to select that resistor so that it does not significantly affect the behavior of the original circuit (resistor with relatively high value).
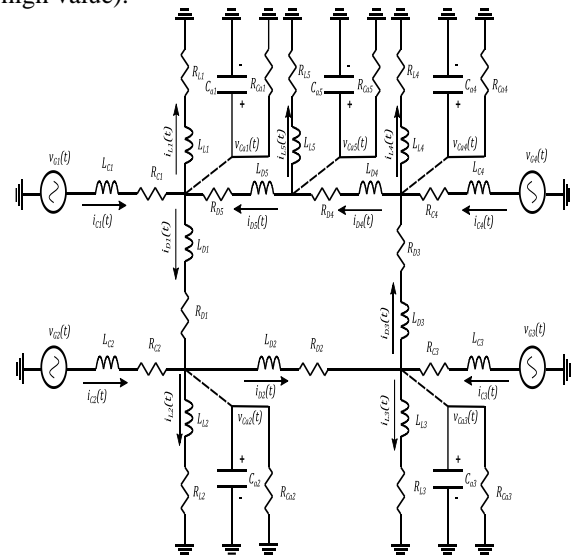

Fig 2: Circuit representation of one of the phases of the microgrid connections

Although adding that resistor makes it easier to deduce the model, some matrix manipulations still need to be done to get the final dynamic model. On the other hand, by adding a capacitor in parallel to each of those resistors, makes the sum of currents to be easily written and reduce the additional manipulations that are necessary, using only the mentioned resistor. These capacitors should also be chosen so that their insertion has little effect on the behavior of the original grid (capacitor with relatively high reactance). In counterpart, the representation in state space of the circuit with said capacitors has two more states for each node.

Even though a particular grid is addressed here, the circuit of Fig 2 allows illustrating the procedure to obtain the model without loss of generality. In this way, to obtain the model of the grid of connections, its analysis is divided into 4 parts.

C. Conductors between the generator and its connection point (connection impedance)

The voltage on one of the conductors for any generator can be written as:

$$v_{G1}(t) - v_{Ca1}(t) = L_{C1}\frac{di_{C1}(t)}{dt} + R_{C1}i_{C1}(t)$$
$$\vdots \qquad (26)$$
$$v_{G4}(t) - v_{Ca4}(t) = L_{C4}\frac{di_{C4}(t)}{dt} + R_{C4}i_{C4}(t)$$

Using the first term in the series of the representation with dynamic phasors, then:

$$\vec{V}_{G1}(t) - \vec{V}_{Ca1}(t) = R_{C1}\vec{I}_{C1}(t) + L_{C1}\frac{d\left(\vec{I}_{C1}(t)\right)}{dt} + j\omega L_{C1}\vec{I}_1$$
$$\vdots \qquad (27)$$
$$\vec{V}_{G4}(t) - \vec{V}_{Ca4}(t) = R_{C4}\vec{I}_{C4}(t) + L_{C4}\frac{d\left(\vec{I}_{C4}(t)\right)}{dt} + j\omega L_{C4}\vec{I}_4$$

Splitting equations in their real and imaginary parts, for any of the 4 generators (n=1, 2, 3, 4), it is valid to write that:

$$\begin{bmatrix} V_{Gn,d}(t) \\ V_{Gn,q}(t) \end{bmatrix} - \begin{bmatrix} V_{Can,d}(t) \\ V_{Can,q}(t) \end{bmatrix}$$
$$= \mathbf{Z_{Cn}}\begin{bmatrix} I_{Cn,d}(t) \\ I_{Cn,q}(t) \end{bmatrix} \qquad (28)$$
$$+ \mathbf{L_{Cn}}\frac{d}{dt}\begin{bmatrix} I_{Cn,d}(t) \\ I_{Cn,q}(t) \end{bmatrix}$$

Where:

$$\mathbf{Z_{Cn}} = \begin{bmatrix} R_{Cn} & -\omega L_{Cn} \\ \omega L_{Cn} & R_{Cn} \end{bmatrix}, \mathbf{L_{Cn}} = \begin{bmatrix} L_{Cn} & 0 \\ 0 & L_{Cn} \end{bmatrix} \qquad (29)$$

Thus, since the voltage of the generators is actually the voltage of the inverters, that is, $E_{1,d}(t) = V_{G1,d}(t)$, $E_{1,q}(t) = V_{G1,q}(t)$, $E_{2,d}(t) = V_{G2,d}(t)$, $E_{2,q}(t) = V_{G2,q}(t)$, etc; Together, the equations of the four generators ca be written in a matrix form as:

$$\mathbf{I_{8x8}}E_{dq} - \mathbf{I_C}V_{Ca} = \mathbf{Z_C}I_C + \mathbf{L_C}\frac{d}{dt}I_C \qquad (30)$$

With:

$$\mathbf{L_C} = diag(\mathbf{L_{C1}}, \mathbf{L_{C2}}, \mathbf{L_{C3}}, \mathbf{L_{C4}})_{8x8}, \mathbf{Z_C} = diag(\mathbf{Z_{C1}}, \mathbf{Z_{C2}}, \mathbf{Z_{C3}}, \mathbf{Z_{C4}})_{8x8}$$
$$\mathbf{I_C} = [\mathbf{I_{8x8}} \quad \mathbf{0_{8x2}}]_{8x10}$$

$$E_{dq} = [E_{1,d}(t) \quad E_{1,q}(t) \quad E_{2,d}(t) \quad E_{2,q}(t) \quad E_{3,d}(t) \quad E_{3,q}$$
$$V_{Ca} = [V_{Ca1,d}(t) \quad V_{Ca1,q}(t) \quad V_{Ca2,d}(t) \quad V_{Ca2,q}(t) \quad \cdots \quad V$$
$$I_C = [I_{C1,d}(t) \quad I_{C1,q}(t) \quad I_{C2,d}(t) \quad I_{C2,q}(t) \quad I_{C3,d}(t) \quad I_C$$
$$(31)$$

D. Line conductors (Distribution impedance)

The voltage on one of the conductors of lines 1 to 5 can be written as:

$$v_{Ca1}(t) - v_{Ca2}(t) = L_{D1}\frac{di_{D1}(t)}{dt} + R_{D1}i_{D1}(t)$$
$$\qquad (32)$$
$$v_{Ca2}(t) - v_{Ca3}(t) = L_{D2}\frac{di_{D2}(t)}{dt} + R_{D2}i_{D2}(t)$$

$$v_{Ca3}(t) - v_{Ca4}(t) = L_{D3}\frac{di_{D3}(t)}{dt} + R_{D3}i_{D3}(t)$$
$$v_{Ca4}(t) - v_{Ca5}(t) = L_{D4}\frac{di_{D4}(t)}{dt} + R_{D4}i_{D4}(t)$$
$$v_{Ca5}(t) - v_{Ca1}(t) = L_{D5}\frac{di_{D5}(t)}{dt} + R_{D5}i_{D5}(t)$$

Following the same procedure of the previous section, for lines 1 to 4 (n=1, 2, 3, 4), there is:

$$\begin{bmatrix} V_{Can,d}(t) \\ V_{Can,q}(t) \end{bmatrix} - \begin{bmatrix} V_{Ca(n+1),d}(t) \\ V_{Ca(n+1),q}(t) \end{bmatrix}$$
$$= \mathbf{Z_{Dn}}\begin{bmatrix} I_{Dn,d}(t) \\ I_{Dn,q}(t) \end{bmatrix} \qquad (33)$$
$$+ \mathbf{L_{Dn}}\frac{d}{dt}\begin{bmatrix} I_{Dn,d}(t) \\ I_{Dn,q}(t) \end{bmatrix}$$

With (n=1, 2, 3, 4, 5):

$$\mathbf{Z_{Dn}} = \begin{bmatrix} R_{Dn} & -\omega L_{Dn} \\ \omega L_{Dn} & R_{Dn} \end{bmatrix}, \mathbf{L_{Dn}} = \begin{bmatrix} L_{Dn} & 0 \\ 0 & L_{Dn} \end{bmatrix} \qquad (34)$$

And for line 5:

$$\begin{bmatrix} V_{Ca5,d}(t) \\ V_{Ca5,q}(t) \end{bmatrix} - \begin{bmatrix} V_{Ca1,d}(t) \\ V_{Ca1,q}(t) \end{bmatrix}$$
$$= \mathbf{Z_{D5}}\begin{bmatrix} I_{D5,d}(t) \\ I_{D5,q}(t) \end{bmatrix} \qquad (35)$$
$$+ \mathbf{L_{D5}}\frac{d}{dt}\begin{bmatrix} I_{D5,d}(t) \\ I_{D5,q}(t) \end{bmatrix}$$

Thus, the equations of the five lines can be written in matrix form as:

$$\mathbf{I_{DC}}V_{Ca} = \mathbf{Z_D}I_D + \mathbf{L_D}\frac{d}{dt}I_D \qquad (36)$$

With:

$$I_D = [I_{D1,d}(t) \quad I_{D1,q}(t) \quad I_{D2,d}(t) \quad I_{D2,q}(t) \quad \cdots \quad I_{D5,d}(t$$
$$\mathbf{L_D} = diag(\mathbf{L_{D1}}, \mathbf{L_{D2}}, \mathbf{L_{D3}}, \mathbf{L_{D4}}, \mathbf{L_{D5}})_{10x10}, \mathbf{Z_D} = diag(\mathbf{Z_{D1}}, \mathbf{Z_{D2}}, \mathbf{Z_{D3}}, \mathbf{Z_{D4}}, \mathbf{Z_{D5}})_{10x10}$$

$$\mathbf{I_{DC}} = \begin{bmatrix} \mathbf{I_{2x2}} & -\mathbf{I_{2x2}} & & & \\ & \mathbf{I_{2x2}} & -\mathbf{I_{2x2}} & & \\ & & \mathbf{I_{2x2}} & -\mathbf{I_{2x2}} & \\ & & & \mathbf{I_{2x2}} & -\mathbf{I_{2x2}} \\ -\mathbf{I_{2x2}} & & & & \mathbf{I_{2x2}} \end{bmatrix}_{10x10}$$
$$(37)$$

Where all empty positions correspond to null matrices of size 2x2.

E. RL Charges (Load impedance)

The phase voltage on each of the connected loads can be written as:

$$v_{Ca1}(t) = L_{L1}\frac{di_{L1}(t)}{dt} + R_{L1}i_{L1}(t)$$
$$\vdots \qquad (38)$$
$$v_{Ca5}(t) = L_{L5}\frac{di_{L5}(t)}{dt} + R_{L5}i_{L5}(t)$$

Following the same procedure of the first part, for the 5 charges (n=1, 2, 3, 4, 5), it can be shown that:

$$\begin{bmatrix} V_{Can,d}(t) \\ V_{Can,q}(t) \end{bmatrix} = \mathbf{Z_{Ln}}\begin{bmatrix} I_{Ln,d}(t) \\ I_{Ln,q}(t) \end{bmatrix} + \mathbf{L_{Ln}}\frac{d}{dt}\begin{bmatrix} I_{Ln,d}(t) \\ I_{Ln,q}(t) \end{bmatrix} \qquad (39)$$

With:

$$\mathbf{Z_{Ln}} = \begin{bmatrix} R_{Ln} & -\omega L_{Ln} \\ \omega L_{Ln} & R_{Ln} \end{bmatrix}, \mathbf{L_{Ln}} = \begin{bmatrix} L_{Ln} & 0 \\ 0 & L_{Ln} \end{bmatrix} \quad (40)$$

Thus, the equations of the five loads can be expressed as:

$$\mathbf{I_{10x10}}V_{Ca} = \mathbf{Z_L}I_L + \mathbf{L_L}\frac{d}{dt}I_L \quad (41)$$

With:

$$I_L = [I_{L1,d}(t) \quad I_{L1,q}(t) \quad I_{L2,d}(t) \quad I_{L2,q}(t) \quad \cdots \quad I_{L5,d}(t)$$
$$\mathbf{L_L} = diag(\mathbf{L_{L1}, L_{L2}, L_{L3}, L_{L4}, L_{L5}})_{10x10}, \ \mathbf{Z_L} = diag(\mathbf{Z_{L1}, Z_{L2}, Z_{L3}, Z_{L4}, Z_{L5}})_{10x10}$$

$$(42)$$

F. Resistance and capacitance added to each RL load (additional impedance)

The current sums in one phase of each load can be written as:

$$i_{C1}(t) + i_{D5}(t) = i_{L1}(t) + i_{D1}(t) + \frac{v_{Ca1}(t)}{R_{a1}}$$
$$+ C_{a1}\frac{d(v_{Ca1}(t))}{dt}$$
$$i_{C2}(t) + i_{D1}(t) = i_{L2}(t) + i_{D2}(t) + \frac{v_{Ca2}(t)}{R_{a2}}$$
$$+ C_{a2}\frac{d(v_{Ca2}(t))}{dt}$$
$$i_{C3}(t) + i_{D2}(t) = i_{L3}(t) + i_{D3}(t) + \frac{v_{Ca3}(t)}{R_{a3}} \quad (43)$$
$$+ C_{a3}\frac{d(v_{Ca3}(t))}{dt}$$
$$i_{C4}(t) + i_{D3}(t) = i_{L4}(t) + i_{D4}(t) + \frac{v_{Ca4}(t)}{R_{a4}}$$
$$+ C_{a4}\frac{d(v_{Ca4}(t))}{dt}$$
$$i_{D4}(t) = i_{L5}(t) + i_{D5}(t) + \frac{v_{Ca5}(t)}{R_{a5}}$$
$$+ C_{a5}\frac{d(v_{Ca5}(t))}{dt}$$

Following the same procedure of the first part, for load 1 it can be written that:

$$\begin{bmatrix} I_{C1,d}(t) \\ I_{C1,q}(t) \end{bmatrix} + \begin{bmatrix} I_{D5,d}(t) \\ I_{D5,q}(t) \end{bmatrix}$$
$$= \begin{bmatrix} I_{L1,d}(t) \\ I_{L1,q}(t) \end{bmatrix} + \begin{bmatrix} I_{D1,d}(t) \\ I_{D1,q}(t) \end{bmatrix}$$
$$+ \mathbf{Y_{A1}}\begin{bmatrix} V_{Ca1,d}(t) \\ V_{Ca1,q}(t) \end{bmatrix} \quad (44)$$
$$+ \mathbf{C_{A1}}\frac{d}{dt}\begin{bmatrix} V_{Ca1,d}(t) \\ V_{Ca1,q}(t) \end{bmatrix}$$

For loads 2 to 4 (*n*=2, 3, 4):

$$\begin{bmatrix} I_{Cn,d}(t) \\ I_{Cn,q}(t) \end{bmatrix} + \begin{bmatrix} I_{D(n-1),d}(t) \\ I_{D(n-1),q}(t) \end{bmatrix}$$
$$= \begin{bmatrix} I_{Ln,d}(t) \\ I_{Ln,q}(t) \end{bmatrix} + \begin{bmatrix} I_{Dn,d}(t) \\ I_{Dn,q}(t) \end{bmatrix}$$
$$+ \mathbf{Y_{An}}\begin{bmatrix} V_{Can,d}(t) \\ V_{Can,q}(t) \end{bmatrix} \quad (45)$$
$$+ \mathbf{C_{An}}\frac{d}{dt}\begin{bmatrix} V_{Can,d}(t) \\ V_{Can,q}(t) \end{bmatrix}$$

And for load 5:

$$\begin{bmatrix} I_{D4,d}(t) \\ I_{D4,q}(t) \end{bmatrix} = \begin{bmatrix} I_{L5,d}(t) \\ I_{L5,q}(t) \end{bmatrix} + \begin{bmatrix} I_{D5,d}(t) \\ I_{D5,q}(t) \end{bmatrix}$$
$$+ \mathbf{Y_{A5}}\begin{bmatrix} V_{Ca5,d}(t) \\ V_{Ca5,q}(t) \end{bmatrix} \quad (46)$$
$$+ \mathbf{C_{A5}}\frac{d}{dt}\begin{bmatrix} V_{Ca5,d}(t) \\ V_{Ca5,q}(t) \end{bmatrix}$$

With (*n*=1, 2, 3, 4, 5):

$$\mathbf{Y_{An}} = \begin{bmatrix} \frac{1}{R_{an}} & -\omega C_{an} \\ \omega C_{an} & \frac{1}{R_{an}} \end{bmatrix}, \mathbf{C_{An}} = \begin{bmatrix} C_{an} & 0 \\ 0 & C_{an} \end{bmatrix} \quad (47)$$

Thus, the equations of the five loads can be written as follows:

$$\mathbf{I_{AC}}I_C + \mathbf{I_{AD}}I_D = \mathbf{I_{10x10}}I_L + \mathbf{Y_A}V_{Ca} + \mathbf{C_A}\frac{d}{dt}V_{Ca} \quad (48)$$

With:

$$\mathbf{C_A} = diag(\mathbf{C_{A1}, C_{A2}, C_{A3}, C_{A4}, C_{A5}})_{10x10}, \ \mathbf{Y_A} = diag(\mathbf{Y_{A1}, Y_{A2}, Y_{A3}, Y_{A4}, Y_{A5}})_{10x10}$$

$$\mathbf{I_{AD}} = \begin{bmatrix} -\mathbf{I_{2x2}} & & & & \mathbf{I_{2x2}} \\ \mathbf{I_{2x2}} & -\mathbf{I_{2x2}} & & & \\ & \mathbf{I_{2x2}} & -\mathbf{I_{2x2}} & & \\ & & \mathbf{I_{2x2}} & -\mathbf{I_{2x2}} & \\ & & & \mathbf{I_{2x2}} & -\mathbf{I_{2x2}} \end{bmatrix}_{10x10}$$

$$\mathbf{I_{AC}} = \begin{bmatrix} \mathbf{I_{2x2}} & & & \\ & \mathbf{I_{2x2}} & & \\ & & \mathbf{I_{2x2}} & \\ & & & \mathbf{I_{2x2}} \\ & & & \end{bmatrix}_{10x8}$$

$$(49)$$

Where all empty positions correspond to null matrices of size 2x2.

G. Complete system

Once the equations of the 4 parts are considered, the state vector is taken as:

$$x_{ur} = [I_C{}^T \quad I_D{}^T \quad I_L{}^T \quad V_{Ca}{}^T]^T \quad (50)$$

Rewriting (30) in terms of the state vector, then:

$$\mathbf{I_{8x8}}E_{dq} - [\mathbf{Z_C} \quad \mathbf{0_{8x10}} \quad \mathbf{0_{8x10}} \quad \mathbf{I_C}]x_{ur}$$
$$= \mathbf{L_C}\frac{d}{dt}I_C \quad (51)$$

Similarly for (36):

$$[\mathbf{0_{10x8}} \quad -\mathbf{Z_D} \quad \mathbf{0_{10x10}} \quad \mathbf{I_{DC}}]x_{ur} = \mathbf{L_D}\frac{d}{dt}I_D \quad (52)$$

And also rewriting (41), it can be obtained that:

$$[\mathbf{0}_{10x8} \quad \mathbf{0}_{10x10} \quad -\mathbf{Z_L} \quad \mathbf{I}_{10x10}]x_{ur} = \mathbf{L_L}\frac{d}{dt}I_L \qquad (53)$$

Finally, for (48):

$$[\mathbf{I_{AC}} \quad \mathbf{I_{AD}} \quad -\mathbf{I}_{10x10} \quad -\mathbf{Y_A}]x_{ur} = \mathbf{C_A}\frac{d}{dt}V_{Ca} \qquad (54)$$

From the four previous expressions the complete system is constructed as:

$$\begin{bmatrix} \mathbf{I_{8x8}} \\ \mathbf{0_{10x8}} \\ \mathbf{0_{10x8}} \\ \mathbf{0_{10x8}} \end{bmatrix}E_{dq}$$

$$+ \begin{bmatrix} -\mathbf{Z_C} & \mathbf{0_{8x10}} & \mathbf{0_{8x10}} & -\mathbf{I_C} \\ \mathbf{0_{10x8}} & -\mathbf{Z_D} & \mathbf{0_{10x10}} & \mathbf{I_{DC}} \\ \mathbf{0_{10x8}} & \mathbf{0_{10x10}} & -\mathbf{Z_L} & \mathbf{I_{10x10}} \\ \mathbf{I_{AC}} & \mathbf{I_{AD}} & -\mathbf{I_{10x10}} & -\mathbf{Y_A} \end{bmatrix}x_{ur} \qquad (55)$$

$$= \begin{bmatrix} \mathbf{L_C} & \mathbf{0_{8x10}} & \mathbf{0_{8x10}} & \mathbf{0_{8x10}} \\ \mathbf{0_{10x8}} & \mathbf{L_D} & \mathbf{0_{10x10}} & \mathbf{0_{10x10}} \\ \mathbf{0_{10x8}} & \mathbf{0_{10x10}} & \mathbf{L_L} & \mathbf{0_{10x10}} \\ \mathbf{0_{10x8}} & \mathbf{0_{10x10}} & \mathbf{0_{10x10}} & \mathbf{C_A} \end{bmatrix}\frac{d}{dt}x_{ur}$$

Thus, from the previous expression, it can be shown that the matrices of (23) and (24) end up being:

$$\mathbf{A_{ur}}$$
$$= \begin{bmatrix} \mathbf{L_C} & \mathbf{0_{8x10}} & \mathbf{0_{8x10}} & \mathbf{0_{8x10}} \\ \mathbf{0_{10x8}} & \mathbf{L_D} & \mathbf{0_{10x10}} & \mathbf{0_{10x10}} \\ \mathbf{0_{10x8}} & \mathbf{0_{10x10}} & \mathbf{L_L} & \mathbf{0_{10x10}} \\ \mathbf{0_{10x8}} & \mathbf{0_{10x10}} & \mathbf{0_{10x10}} & \mathbf{C_A} \end{bmatrix}^{-1} \begin{bmatrix} -\mathbf{Z_C} & \mathbf{0_{8x10}} \\ \mathbf{0_{10x8}} & -\mathbf{Z_D} \\ \mathbf{0_{10x8}} & \mathbf{0_{10x1(}} \\ \mathbf{I_{AC}} & \mathbf{I_{AD}} \end{bmatrix}$$

$$\mathbf{B_{ur}}$$
$$= \begin{bmatrix} \mathbf{L_C} & \mathbf{0_{8x10}} & \mathbf{0_{8x10}} & \mathbf{0_{8x10}} \\ \mathbf{0_{10x8}} & \mathbf{L_D} & \mathbf{0_{10x10}} & \mathbf{0_{10x10}} \\ \mathbf{0_{10x8}} & \mathbf{0_{10x10}} & \mathbf{L_L} & \mathbf{0_{10x10}} \\ \mathbf{0_{10x8}} & \mathbf{0_{10x10}} & \mathbf{0_{10x10}} & \mathbf{C_A} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I_{8x8}} \\ \mathbf{0_{10x8}} \\ \mathbf{0_{10x8}} \\ \mathbf{0_{10x8}} \end{bmatrix}, \mathbf{C_{ur}}$$

$$= [\mathbf{I_{8x8}} \quad \mathbf{0_{8x30}}]$$
$$(56)$$

H.. Closed-loop dynamic model

From equations (14), (15), (20), (23) and (24) the closed-loop model of the complete system for N inverters can be written as:

$$\frac{d(x_{ur})}{dt} = \mathbf{A_{ur}}x_{ur} + \mathbf{B_{Uur}}u_{ur} \qquad (57)$$

$$\begin{bmatrix} \Delta P_1(t) \\ \Delta Q_1(t) \\ \vdots \\ \Delta P_N(t) \\ \Delta Q_N(t) \\ \Delta\omega_1(t) \\ \vdots \\ \Delta\omega_N(t) \end{bmatrix} = \mathbf{C_{sis}}\begin{bmatrix} \Delta x_{invs} \\ \Delta x_{ur} \end{bmatrix} + \mathbf{D_{sis}}\Delta u_{invsEW} \qquad (58)$$

Where:

$$\mathbf{A_{sis}}$$
$$= \begin{bmatrix} (\mathbf{A_{invs}} + \mathbf{B_{invsPQ}I_{invse}C_{invs}}) & \mathbf{B_{invsPQ}E_{invse}C_{ur}} \\ \mathbf{B_{Uur}C_{invs}} & \mathbf{A_{ur}} \end{bmatrix} \quad \begin{matrix} (\\ 2 \\ ) \end{matrix}$$

$$\mathbf{B_{sis}} = \begin{bmatrix} (\mathbf{B_{invsEW}} + \mathbf{B_{invsPQ}I_{invse}D_{invs}}) \\ \mathbf{B_{Uur}D_{invs}} \end{bmatrix}$$

$$\mathbf{C_{sis}}$$
$$= \begin{bmatrix} \begin{bmatrix} [0 & 0 & 1] & & \\ & \ddots & \\ & & [0 & 0 & 1] \end{bmatrix}_{Nx3N} \mathbf{I_{invse}C_{invs}} & \mathbf{E_{invse}C_{ur}} \\ & \mathbf{0} \end{bmatrix}$$

$$\mathbf{D_{sis}}$$
$$= \begin{bmatrix} \begin{bmatrix} [0 & 0 & 1]\mathbf{B_{E\omega0,1}} & & \\ & \ddots & \\ & & [0 & 0 & 1]\mathbf{B_{E\omega0,N}} \end{bmatrix} \end{bmatrix} \mathbf{I_{invse}D_{invs}}$$

III. CALCULATION OF THE QUIESCENT POINTS OF THE MODEL

Since the construction of several of the matrices that make up the model requires knowing the quiescent values of several of the variables, it was necessary to find a way to obtain them without having to appeal to simulation. Thus, here is a way to obtain the values of the operating points.

The number of variables is determined considering that the electrical connections of the microgrid has M_1 lines connecting the grid nodes, M_2 nodes and M_3 loads, with $2(M_1 + M_2 + M_3)$ variables.

Additionally, if there are N connected inverters, these would add two variables for each voltage and another two for their current. In addition, since in island mode, the frequency in steady state is not known, this is another variable that exists, so the total number of variables is $2(M_1 + M_2 + M_3 + 2N) + 1$.

Since the model is presented in terms of dynamic phasors, then from (23) in steady state, it can be said that:

$$0 = \mathbf{A_{ur}}x_{ur} + \mathbf{B_{Uur}}u_{ur} \qquad (60)$$

This corresponds to a system of $2(M_1 + M_2 + M_3 + N)$ linear equations that depend on the two components of the voltages of each inverter (u_{ur}) and on the two components of each current and voltage of the grid of connections (x_{ur}). Additionally, other $2N$ equations can be obtained from **Error! Reference source not found.**, where for each of the inverters it can be written that ($n=1, 2, 3, \ldots, N$):

$$\omega_e = \omega_{0n} - k_{n,p}\left(E_{n,de}I_{Cn,de} + E_{n,qe}I_{Cn,qe}\right)$$
$$\sqrt{E_{n,de}^2 + E_{n,qe}^2} = E_{0n}$$
$$\qquad - k_{n,q}\left(E_{n,qe}I_{Cn,de}\right.$$
$$\left. - E_{n,de}I_{Cn,qe}\right) \qquad (61)$$

Where ω_e is the steady-state operating frequency of the system, $E_{n,de}$, $E_{n,qe}$, $I_{Cn,de}$ and $I_{Cn,qe}$ are the quiescent points of the two components of the voltage and current of the n-th inverter, while $k_{n,p}$, $k_{n,q}$, ω_{0n} and E_{0n} are the parameters of the droop curves for the n-th inverter.

The missing equation can be obtained from the power balance principle, that is, the total power delivered by the N inverters must be equal to the sum of the power absorbed and dissipated in the loads and lines:

$$\sum_{n=1}^{N} P_n = P_{PD} + P_{PC} + P_{PRC} + P_{loads} \qquad (62)$$

Where P_{PD} and P_{PC} are the losses in the distribution and connection lines, P_{PRC} are the losses associated with the resistance connected in parallel with each capacitor, P_{loads} the power absorbed by the RL loads and P_n is the active power delivered by each inverter.

In this way, the power of each inverter can be calculated from:

$$P_n = E_{n,de}I_{Cn,de} + E_{n,qe}I_{Cn,qe} \qquad (63)$$

For a three-phase system, the losses, in each line of distribution and connection, can be calculated from the quiescent points of the components of said currents, such as:

$$P_{PD,n} = 3R_{Dn}\left(I_{Dn,de}^2 + I_{Dn,qe}^2\right)$$
$$P_{PC,n} = 3R_{Cn}\left(I_{Cn,de}^2 + I_{Cn,qe}^2\right) \qquad (64)$$

Similarly, the power absorbed by each load can be calculated based on the components of the load current as:

$$P_{carga,n} = 3R_{Ln}\left(I_{Ln,de}^2 + I_{Ln,qe}^2\right) \qquad (65)$$

Moreover, the losses associated with the resistance in parallel with each capacitor can be found as a function of the voltage components of the capacitors as:

$$P_{PRC,n} = 3\frac{V_{Can,de}^2 + V_{Can,qe}^2}{R_{Can}} \qquad (66)$$

IV. RESULTS AND DISCUSSION

The fidelity of the model obtained under the presented procedure was evaluated using Simulink/MatLab, to do so, the respective model obtained an the respective three-phase system was simulated considering the circuit of Fig 2. The parameters used for the simulation were taken from the original grid studied in [10], but the operating frequency was changed to 60Hz. On the other hand, the capacitors and resistors that were not in the original grid, were selected so that the power of each of them was much less than the power of the charge connected to the respective node. The complete parameters are summarized in table 1 **Error! Reference source not found.**and table 2**Error! Reference source not found.**.

For the verification of the mathematical model, a model was created in Simulink where the microgrid circuit was built, using the respective electrical components. Furthermore, the law of control through the droop technique was implemented using the Clarke and Park transform to perform the calculations of power and reference voltage in a synchronous framework.

Additionally, in order to make it possible to compare the assembled electrical circuit with the small-signal model obtained, in the same Simulink model, the state space representation block was used to simulate the mathematical model obtained. Since this is small-signal model, a vector with the respective quiescent points was added to the output vector of the representation in state variables to facilitate graphic comparison.

Having both representations (electrical circuit and proposed small-signal model) in the same Simulink model, both were simultaneously simulated by using the ode23t *Solver (mod.*

Stiff/Trapezoidal). Also, in order to evaluate the dynamical response of the model, small steps were applied to each of the adjustment points ($\Delta\omega_{0n}$ and ΔE_{0n}) at different instants of time; this to be able to compare the response to circuit's step and the mathematical model obtained under the same conditions. In this way, among other results, the curves shown from Fig 3 to Fig 8 were obtained. Those figures show the behavior over time for the active, reactive power and frequency of each inverter as it is subject to sudden variations in the reference voltage (Fig 3 to Fig 5**Error! Reference source not found.**) and at the reference frequency (Fig 6 to Fig 8).

Table 1 Parameters of the grid elements (adapted from [10])

	n	Resistance [Ω]	Inductance[mH]
Connection line (R_{Cn} and L_{Cn})	1	0.015	2.069
	2	0.015	2.069
	3	0.030	2.069
	4	0.030	2.069
Distribution line (R_{Dn} y L_{Dn})	1	0.015	0.318
	2	0.175	1.846
	3	0.120	0.318
	4	0.150	0.318
	5	0.175	1.846
Load (R_{Ln} y L_{Ln})	1	300	477
	2	40	64
	3	50	64
	4	50	95
	5	40	64
		Resistance [kΩ]	Capacitance [nF]
Added capacitor and resistor (R_{Can} y C_{Can})	1	407.789	0.389
	2	54.553	2.932
	3	50.000	0.003
	4	75.653	2.516
	5	54.553	2.933

Table 2 Inverter's droop curve parameters (adapted from [10])

Inverter	k_p $\left[\frac{\text{rad/s}}{\text{kW}}\right]$	k_q $\left[\frac{V_{RMS}}{kVAr}\right]$	ω_0 [rad/s]	E_0 [V_{RMS}]
1	0.934	1.3	377.619	223
2	0.934	1.3	377.619	222
3	0.125	1.5	377.619	224
4	0.125	1.5	377.619	223

From the curves shown in Fig 3 and Fig 8 it can be seen that the behavior of the circuit (in blue color) is very close to that obtained with the mathematical model found (in orange color). Only in a few time intervals instantaneous differences are distinguished, however, on average the model obtained always follows the same trend of the simulated circuit.
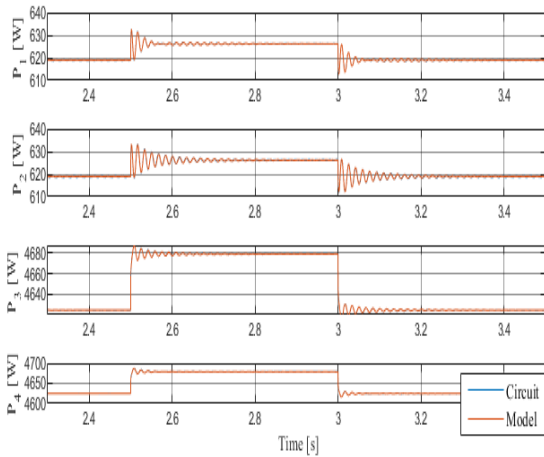
Fig 3: Behavior of active power delivered by the four inverters, against sudden variations of the voltage set points
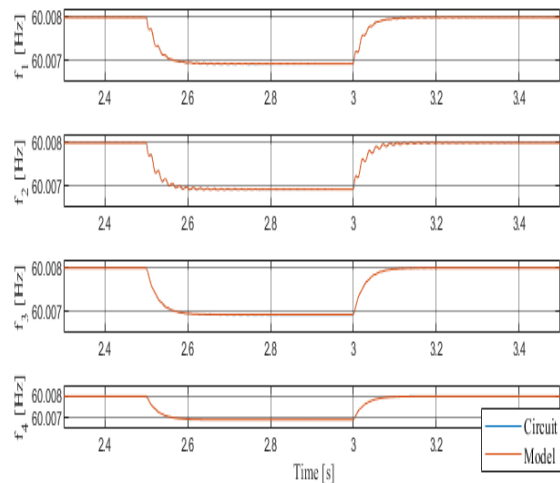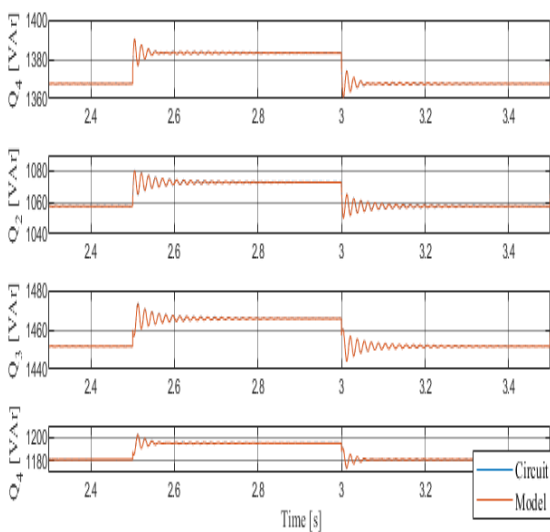

Fig 4: Behavior of reactive power delivered by the four inverters, against sudden variations of the voltage set points
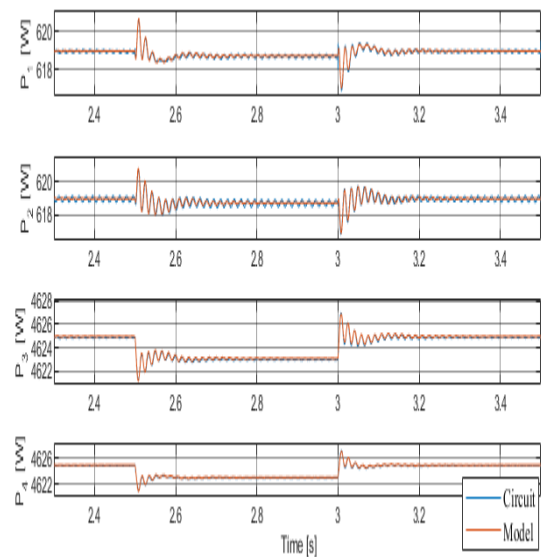
From the results obtained with the comparison between the circuit and the model obtained, it can be seen that the procedure explained in this document allows obtaining a model that reasonably predicts the behavior of the analyzed microgrid. Thus, this model could be used to evaluate the stability of the system based on the variation of various parameters, such as the characteristics of the lines and the coefficients of the droop curves, among others. Additionally, since in this model it was considered that the set points could be varied, this model would also facilitate the design of the controllers that are in the secondary level of control, in charge of frequency restoration and voltage regulation.

Fig 5: Behavior of the operating frequency of the four inverters against sudden variations of the voltage set points

Fig 6: Behavior of the active power delivered by the four inverters against sudden variations of the frequency set points
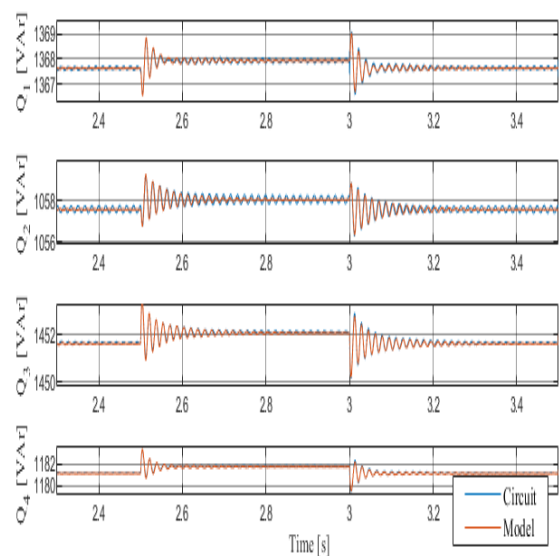

Fig 7: Behavior of the reactive power delivered by the four inverters against sudden variations of the frequency set points
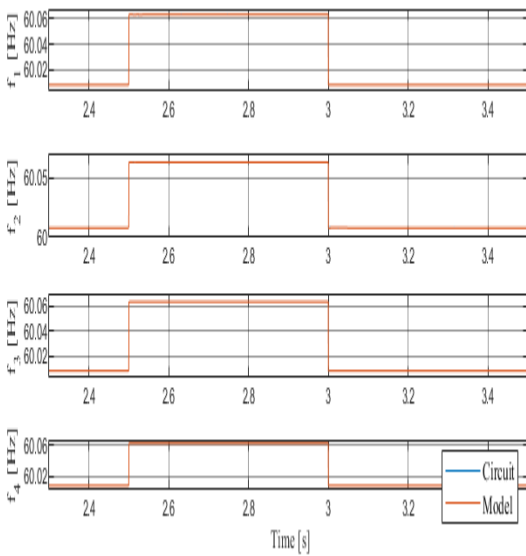
Fig 8: Behavior of the operating frequency of the four inverters against sudden variations of the frequency set points

V. CONCLUSIONS

In this work, a complete procedure was presented to obtain a model of small variations for an AC microgrid controlled by the droop technique. This procedure is based on the use of dynamic phasors, which is a tool that, according to the literature, allows to predict with greater certainty the dynamic behavior of the microgrid variables and to determine the stability of the power system from its eigenvalues.

Despite the fact that the procedure shown here is based on strategies found in the literature, in this work, two modifications are included and the step-by-step procedure is shown including the calculation of the equilibrium points, which is not always clear in the literature or sometimes they are obtained by simulation.

One of the modifications made in this procedure is the consideration of capacitors in the microgrid nodes. This eases the obtaining and manipulation of matrices associated with the connections of the grid; In addition, it facilitates the analysis of the voltages on the nodes from the representation in state variables of the system. In this work, the values of those capacitors and resistors added to the microgrid were adjusted, so that they do not significantly alter its dynamics, however, the values of these components could also be used to model the capacitance and conductance of the line or to model loads with leading power factor.

The second modification made in the shown procedure is the consideration of the variations of the adjustment points. These variations behave as inputs to the model, making it easy to assess the fidelity of the model to the simulated circuit. Furthermore, these variations can be used for the design and analysis of controllers that, based on variations of the set points, seek to regulate variables such as frequency, voltage and power.

As future works, strategies should be explored that allow the possibility of modeling load variations from exogenous variables to be added to the model obtained. Additionally, from the model obtained by this procedure, the possibility of designing control strategies for microgrids subject to load variation and changes in the configuration of the microgrid structure should be explored.

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