# A Numerical Study of the Dynamics of Parachute Aided Descent 

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#### Abstract

Computational Fluid Dynamics (CFD) has been successfully applied to many engineering problems. This paper presents the influence of three different two-equation turbulence models on computing the drag coefficient for both hemispherical canopy and Ram air canopy at different velocities. The aim of this paper is to offer guidance to select appropriate turbulence models for this application. Using available different turbulence models from ANSYS FLUENT the drag coefficient of hemispherical canopy (HC) and Ram air canopy ( RC ) of different sizes were calculated in the range of velocity ( $2 \mathrm{~m} / \mathrm{s}-50 \mathrm{~m} / \mathrm{s}$ ). These results were used to evaluate the terminal velocity and time taken to reach the ground by the paratrooper by solving the differential equations numerically using RK4 technique. Comparisons have been made for both the canopies based on the effects of shape and also the area of the canopies. It was observed that all three models predicted the drag coefficient to be a constant in the above mentioned velocity range and results also indicate that the canopy's shape does not influence the terminal velocity.


Keywords: Turbulence Models, Parachute's Canopy, Drag Coefficient, Terminal Velocity.

## I. INTRODUCTION

Skydiving is an interesting sport and an amazing airborne technique used in military operations. Though named differently the physics involved is Newton's second law of motion which is mathematically expressed as

$$
\begin{equation*}
F=m a \tag{1}
\end{equation*}
$$

Where " F " denotes the net force in the system, " $m$ " denotes the mass of the object and "a" denotes the acceleration of fall. Parachutes are used to land safely from high altitude free fall. Basically there are two types of parachute in which one is the dome canopy and the other ram-air canopy model. In this project both these canopies are studied under no cross wind assumption. In the dome canopy the air is trapped in the canopy's envelope which create a high pressure that retards movement in the direction opposite to the entering air flow whereas in the ram-air canopy model, the parafoil acts as a wing, allowing the jumper to fly towards a target.

## II. PHYSICS INVOLVED

Newton's Second Law: "The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object". As mentioned before it is mathematically expressed as (1).

The force term ( F ) consists of both the paratrooper's weight (W) and the air resistance $\left(F_{D}\right)$ which opposes the fall. Skydiving consists of two phase namely,

- Free Fall,
- Parachute Aided Fall.

Free fall is the period which starts from the point of jump and end when the parachute is deployed and the remaining period where the descent is using parachute is called Parachute aided fall. At the point of jump the paratrooper's velocity would be zero and he falls the velocity will increase rapidly and at some point the velocity becomes independent of time. That velocity is known as "Terminal velocity". This happens when the weight (W) and drag force $\left(F_{D}\right)$ becomes equal. It is known that the weight always remains constant and the drag force changes with velocity and when the velocity is such that the magnitude of drag force equals the weight then the net force (F) acting on the system would be zero.

$$
\begin{equation*}
F=0 \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
W=F_{D} \tag{3}
\end{equation*}
$$

Drag force $\left(F_{D}\right)$ takes into account the mass, shape of the object and it is mathematically expressed as
$F_{D}=0.5 \rho A C_{d} v_{t}{ }^{2}$
Where
$\rho=$ Density of the air $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$,
$A=$ Projected area $\left(m^{2}\right)$,
$C_{d}=$ Drag coefficient,
$v_{t}=$ Terminal velocity $(\mathrm{m} / \mathrm{s})$.
The paratrooper attains terminal velocity during both the phases if the fall is sufficiently large. Initially it is important to know the drag coefficients of both the canopies and the paratrooper's body if he were to fall tummy facing the ground.

## III. TURBULENCE MODELS

Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquids, when they flow past solid surfaces or even when neighbouring streams of the same fluid flow past or over one another.

## Characteristics of Turbulent flow

- Non-uniform,
- Inertial dominant flow,
- High Reynolds number,
- Unsteady,
- Presence of eddies,
- Large length scale between eddies,
- Large time scale between eddies,
- Highly diffusive,
- Highly dissipative,
- Flow will be 3 dimensional.

Turbulent flows are highly complex such that the physics of the problem changes for every moment. Therefore time averaging the Navier-Strokes (NS) equation is important in order to get time averaged results.

The idea of Reynolds Time Averaging is to express the variable which is a function of space and time as the sum of mean and a fluctuating component.
$\varphi(x, t)=\overline{\varphi(x, t)}+\varphi^{\prime}(x, t)$
Where,
$\overline{\varphi(x, t)}=$ Averaged value of $\varphi(x, t)$,
$\varphi^{\prime}(x, t)=$ Fluctuatuing value of $\varphi(x, t)$.
Definition of time averaging of a property $\varphi$,
$\bar{\varphi}=\lim _{T \rightarrow \infty} \frac{1}{T} \int_{0}^{T} \varphi d t$
Therefore by definition time average of a function independent of time,

$$
\overline{\bar{\varphi}}=\lim _{T \rightarrow \infty} \frac{1}{T} \int_{0}^{T} \bar{\varphi} d t=\bar{\varphi}\left(\lim _{T \rightarrow \infty} \frac{1}{T} T\right)=\bar{\varphi}
$$

And time average of a fluctuating component is,
$\overline{\varphi^{\prime}}=\lim _{T \rightarrow \infty} \frac{1}{T} \int_{0}^{T} \varphi^{\prime} d t=0$
Time Averaging of NS equations give
Continuity equation:
$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}=0$
$x$ - momentum equation:
$\rho\left(\frac{\partial \bar{u}}{\partial t}+\bar{u} \frac{\partial \bar{u}}{\partial x}+\bar{v} \frac{\partial \bar{u}}{\partial y}\right)=-\frac{\partial \bar{p}}{\partial x}$

$$
\begin{align*}
& +\mu\left(\frac{\partial^{2} \bar{u}}{\partial x^{2}}+\frac{\partial^{2} \bar{u}}{\partial y^{2}}\right) \\
& -\rho\left(\frac{\partial \overline{u^{\prime} u^{\prime}}}{\partial x}+\frac{\partial \bar{u}^{\prime} v^{\prime}}{\partial y}\right) \tag{8}
\end{align*}
$$

$y$ - momentum equation:
$\rho\left(\frac{\partial \bar{v}}{\partial t}+\bar{u} \frac{\partial \bar{v}}{\partial x}+\bar{v} \frac{\partial \bar{v}}{\partial y}\right)=-\frac{\partial \bar{p}}{\partial y}$

$$
\begin{align*}
& +\mu\left(\frac{\partial^{2} \bar{v}}{\partial x^{2}}+\frac{\partial^{2} \bar{v}}{\partial y^{2}}\right) \\
& -\rho\left(\frac{\partial \overline{u^{\prime} v}}{\partial x}+\frac{\partial \bar{v}^{\prime} v^{\prime}}{\partial y}\right) \tag{9}
\end{align*}
$$

After time averaging the momentum equations, few fluctuating terms survive. These fluctuating terms are known as Reynolds stresses and are related to mean velocity gradients using Boussinesq hypothesis which introduces turbulence or eddy viscosity $\left(\mu_{t}\right)$.

Standard $k-\varepsilon$, RNG $k-\varepsilon$ and SST $k-\omega$ computes eddy viscosity as a function of turbulent kinetic energy $(k)$ and either turbulence dissipation rate $(\varepsilon)$ or specific turbulence dissipation rate $(\omega)$ respectively.

## Standard $k-\varepsilon$ Model

Standard $k-\varepsilon$ is a two-equation model in which the solution of two separate transport equation allows the turbulent velocity and length scales to be determined. This model is valid only for fully turbulent flows [2, 3]. The turbulence kinetic energy, $k$, and its rate of dissipation, $\varepsilon$, are obtained from the following transport equations:

$$
\begin{align*}
\frac{\partial}{\partial t}(\rho k)+ & \frac{\partial}{\partial x_{i}}\left(\rho k u_{i}\right)=\frac{\partial}{\partial x_{j}}\left[\left(\mu+\frac{\mu_{t}}{\sigma_{k}}\right) \frac{\partial k}{\partial x_{j}}\right] \\
& +G_{k}+G_{b}-\rho \varepsilon \\
& -\mathrm{Y}_{M}+S_{k}  \tag{10}\\
\frac{\partial}{\partial t}(\rho \varepsilon)+ & \frac{\partial}{\partial x_{i}}\left(\rho \varepsilon u_{i}\right)=\frac{\partial}{\partial x_{j}}\left[\left(\mu+\frac{\mu_{t}}{\sigma_{\varepsilon}}\right) \frac{\partial \varepsilon}{\partial x_{j}}\right] \\
& +C_{1 \varepsilon} \frac{\varepsilon}{k}\left(G_{k}+C_{3 \varepsilon} G_{b}\right) \\
& -C_{2 \varepsilon} \rho \frac{\varepsilon^{2}}{k}+S_{\varepsilon} \tag{11}
\end{align*}
$$

$k-\varepsilon$ Model computes $\mu_{t}$ as a function of $k \& \varepsilon$,
$\mu_{t}=\rho C_{\mu} \frac{k^{2}}{\varepsilon}$
Model constants: $C_{\mu}=0.09, C_{1 \varepsilon}=1.44, C_{2 \varepsilon}=1.92, \sigma_{k}=$ $1, \sigma_{\varepsilon}=1.3$

## RNG $k-\varepsilon$ Model

RNG $k-\varepsilon$ is the refined version standard $k-\varepsilon$. The standard version is a high-Reynolds-number model but the RNG version provides an option to also account for low-Reynolds-number effects [2]. This feature makes this model more accurate and reliable for a wider class of flows than the standard version. This model is often recommended for strongly strained turbulent flows [5] and this model also reduces the over production of turbulence around the stagnation points [7]. The transport equations for this model are as follows:

$$
\begin{align*}
& \frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_{i}}\left(\rho k u_{i}\right)=\frac{\partial}{\partial x_{j}} {\left[\alpha_{k} \mu_{e f f} \frac{\partial k}{\partial x_{j}}\right]+G_{k} } \\
&+G_{b}-\rho \varepsilon-\mathrm{Y}_{M}+S_{k}
\end{aligned} \begin{aligned}
\frac{\partial}{\partial t}(\rho \varepsilon)+\frac{\partial}{\partial x_{i}}\left(\rho \varepsilon u_{i}\right)=\frac{\partial}{\partial x_{j}} & {\left[\alpha_{\varepsilon} \mu_{e f f} \frac{\partial \varepsilon}{\partial x_{j}}\right] }  \tag{13}\\
& +C_{1 \varepsilon} \frac{\varepsilon}{k}\left(G_{k}+C_{3 \varepsilon} G_{b}\right) \\
& -C_{2 \varepsilon} \rho \frac{\varepsilon^{2}}{k}-\mathrm{R}_{\varepsilon}+S_{\varepsilon}
\end{align*}
$$

The formula for computing $\mu_{t}$ in both standard $k-\varepsilon$ and $k-\varepsilon$ RNG are same for high-Reynolds-number.
$\mu_{t}=\rho C_{\mu} \frac{k^{2}}{\varepsilon}$
Model constant: $C_{\mu}=0.0845$
SST $k-\omega$ Model

Shear Stress Transport $k-\omega$ combines standard $k-\omega$ and high Reynolds number version of the $k-\varepsilon$ model to capture the physics in the near wall region independent of far field and the physics in the farther portion of the boundary layer independent of near wall [2]. The standard $k-\varepsilon$ model is formulated into $k-$ $\omega$ in order to achieve the above mentioned purpose. A blending function is used in SST $k-\omega$ which acts a sort of joint between the standard $k-\omega$ and the transformed $k-$ $\varepsilon$ and this blending function plays a very interesting part. It is designed that such that it activates the $k-\omega$ in the near wall region and the transformed $k-\varepsilon$ far from the wall [2]. This feature ensures that the physics at both the regions are captured properly. The transport equations [6] for this model are as follows:

$$
\begin{gather*}
\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_{i}}\left(\rho k u_{i}\right)=\frac{\partial}{\partial x_{j}}\left[\left(\mu+\frac{\mu_{t}}{\sigma_{k}}\right) \frac{\partial k}{\partial x_{j}}\right] \\
+\tilde{G}_{k}-\mathrm{Y}_{k}+S_{k}  \tag{16}\\
\frac{\partial}{\partial t}(\rho \omega)+\frac{\partial}{\partial x_{i}}\left(\rho \omega u_{i}\right)=\frac{\partial}{\partial x_{j}}\left[\left(\mu+\frac{\mu_{t}}{\sigma_{\omega}}\right) \frac{\partial \omega}{\partial x_{j}}\right] \\
+G_{\omega}-Y_{\omega}+D_{\omega}+S_{\omega} \tag{17}
\end{gather*}
$$

The turbulent viscosity is computed as
$\mu_{t}=\frac{\rho k}{\omega} \frac{1}{\max \left[\frac{1}{\alpha^{*}} \frac{S F_{2}}{a_{1} \omega}\right]}$
Where $S$ is the strain rate magnitude and
$\sigma_{k}=\frac{1}{\frac{F_{1}}{\sigma_{k, 1}}+\frac{\left(1-F_{1}\right)}{\sigma_{k .2}}}$
$\sigma_{\omega}=\frac{1}{\frac{F_{1}}{\sigma_{\omega, 1}+\frac{\left(1-F_{1}\right)}{\sigma_{\omega .2}}}}$
Where $F_{1}, F_{2}$ are the blending functions, are given by
$F_{1}=\tanh \left(\Phi_{1}{ }^{4}\right)$
$F_{2}=\tanh \left(\Phi_{2}{ }^{2}\right)$
Model constants: $\sigma_{k, 1}=1.176, \sigma_{\omega, 1}=2, \sigma_{k .2}=1, \sigma_{\omega .2}=$ 1.168, $a_{1}=0.31$

## IV. FREE FALL

Initially after the jump the paratrooper falls at a varying rate starting from $0 \mathrm{~m} / \mathrm{s}$ because the drag force and weight of the paratrooper won't be equal. And then the drag force increases and finally becomes equal to the weight of the paratrooper thereby the acceleration goes to zero. At this point the paratrooper attains terminal velocity (Net force $F=0$ ).

$$
\therefore W=F_{D}
$$

$v_{t}=\sqrt{\frac{W}{0.5 \rho C_{d} A}}$
Equation (20) gives the terminal velocity attained by the paratrooper under free fall. The terminal velocity can be the maximum velocity attained by the paratrooper. Both drag force and drag coefficient are unknown. So it is imperative to find the drag force either experimentally or using CFD tools. For obvious reasons CFD tools are chosen over experimentation to find the drag force on the paratrooper under free fall.
During free fall the paratrooper's body is modelled as a short cylinder. So the paratrooper's height is taken as cylinder height (L) and the paratrooper's chest measurement as cylinder diameter (D).

According to Indian Army the minimum physical standards are [4]

$$
\begin{aligned}
\text { Height } & =166 \mathrm{~cm}, \\
\text { Chest } & =77 \mathrm{~cm}, \\
\text { Weight } & =48 \mathrm{~kg}
\end{aligned}
$$

The typical measurements are given below
TABLE I. Typical Measurements of a Male

| S.No. | Height, L <br> $(\mathrm{m})$ | Chest, D <br> $(\mathrm{m})$ | L/D | Weight <br> $(\mathrm{kg})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1.66 | 0.77 | 2.155 | 62 |
| 2 | 1.725 | 0.9271 | 1.86 | 69 |
| 3 | 1.775 | 1.0026 | 1.77 | 73 |
| 4 | 1.825 | 1.08 | 1.69 | 78 |
| 5 | 1.88 | 1.155 | 1.62 | 82 |
| 6 | 1.935 | 1.23 | 1.57 | 88 |
| 7 | 1.98 | 1.31 | 1.51 | 91 |

Simulations were done for velocities $1 \mathrm{~m} / \mathrm{s} \& 100 \mathrm{~m} / \mathrm{s}$ using the typical physical measurements and the difference between $C_{d}(1 \mathrm{~m} / \mathrm{s})$ and $C_{d}(100 \mathrm{~m} / \mathrm{s})$ was about 0.01 .

| TABLE II. Drag Coefficients for Different L/D values |  |  |  |
| :---: | :---: | :---: | :---: |
| L/D | Velocity $(\boldsymbol{m} / \boldsymbol{s})$ | $\boldsymbol{C}_{\boldsymbol{d}}$ | Difference <br> in $\%$ |
|  | 1 | 0.639 | 1.9 |
|  | 100 | 0.627 |  |
| 1.57 | 1 | 0.649 | 1.5 |
|  | 100 | 0.639 |  |
| 1.62 | 1 | 0.652 | 1.8 |
|  | 100 | 0.64 |  |
| 1.77 | 1 | 0.653 | 1.8 |
|  | 100 | 0.641 |  |
| 1.86 | 1 | 0.673 | 1.6 |
|  | 100 | 0.662 |  |
|  | 1 | 0.682 | 1 |
|  | 100 | 0.675 |  |

In all the cases the difference between $C_{d}(1 \mathrm{~m} / \mathrm{s})$ and $C_{d}(100 \mathrm{~m} / \mathrm{s})$ is about 0.01 and thus $C_{d}$ can be concluded to be a constant based on L/D.


Fig. 1 Variation of $C_{d}$ with L/D

## V. CFD SIMULATION

The canopies have been designed and meshed in Gambit 2.4.6 and analysed in ANSYS FLUENT 6.3.26. The hemispherical canopy is designed with a vent hole of diameter 0.305 m . Several hemispherical canopies are analysed but the vent hole dimension remains constant for all the canopies.


Fig. 2 Gambit model for HC


Fig. 3 Gambit model for RC

## VI. GRID INDEPENDENCY TEST

The hemispherical canopy and ram air canopy were designed and meshed using Gambit 2.4.6 (3D modelling software) and analysis were done using

ANSYS FLUENT (6.3.26). Standard $k-\varepsilon$ was used in analysing the canopies for grid independency.
TABLE III. Properties of Air at $20^{\circ} \mathrm{C}[1]$

| Properties of air |  | Unit |
| :---: | :---: | :---: |
| Density | 1.205 | $\mathrm{~kg} / \mathrm{m}^{3}$ |
| Dynamic viscosity | $1.814^{*} 10^{-5}$ | $\mathrm{Ns} / \mathrm{m}^{2}$ |
| Kinematic viscosity | $1.506 * 10^{-5}$ | $\mathrm{~m}^{2} / \mathrm{s}$ |

Hemispherical Canopy (HC)
Parachute diameter $=8.1048 \mathrm{~m}$
Projected area $=51.59 \mathrm{~m}^{2}$
The analysis was done for two different mesh count and they provide results which deviate by $1 \%$ which is negligible.

TABLE IV. Comparison of Drag Coefficient Values for Two Different Mesh Count (HC)

| Velocity m/s | $C_{d}$ <br> 4 lakh <br> elements | $C_{d}$ <br> 8.9 lakh <br> elements | Difference in <br> $\%$ |
| :---: | :---: | :---: | :---: |
| 2 | 1.144 | 1.158 | 1.2 |
| 10 | 1.141 | 1.155 | 1.2 |
| 20 | 1.142 | 1.157 | 1.3 |
| 30 | 1.146 | 1.163 | 1.5 |
| 40 | 1.15 | 1.167 | 1.4 |
| 50 | 1.152 | 1.171 | 1.6 |



Fig. 4 Grid independent result for HC
The mesh count used varies by a large number but the results are almost constant and it can be concluded that the drag coefficient for hemispherical canopy is independent of velocity in the range ( $2 \mathrm{~m} / \mathrm{s}-50 \mathrm{~m} / \mathrm{s}$ ).

Ram-air Canopy
Span $=4.9 \mathrm{~m}$
Chord $=2.35 \mathrm{~m}$
Area $=11.59 \mathrm{~m}^{2}$
Projected span $=4.45 \mathrm{~m}$
Projected Chord $=1.94 \mathrm{~m}$
Projected area $=8.63 \mathrm{~m}^{2}$

TABLE V. Comparison of Drag Coefficient Values for Two Different

| Mesh Count (RC) |  |  |  |
| :---: | :---: | :---: | :---: |
| Velocity m/s | $C_{d}$ <br> 4 Lakh <br> elements | $C_{d}$ <br> 8 Lakh <br> elements | Difference in \% |
| 2 | 1.045 | 1.052 | 0.66 |
| 10 | 1.042 | 1.049 | 0.67 |
| 20 | 1.041 | 1.048 | 0.67 |
| 30 | 1.041 | 1.048 | 0.67 |
| 40 | 1.041 | 1.048 | 0.67 |
| 50 | 1.041 | 1.048 | 0.67 |



Fig. 5 Grid independent result for RC
Similar to hemispherical canopy analysis, using two mesh counts for ram-air canopy the drag coefficients were calculated and concluded to be a constant in the velocity range ( $2 \mathrm{~m} / \mathrm{s}-50 \mathrm{~m} / \mathrm{s}$ ).

## VII.PREDICITION OF DIFFERENT TURBULENCE MODELS

RNG $k-\varepsilon$, SST $k-\omega$ turbulence models were used to analyse both the canopies using the same dimensions and properties of air and it was observed that both the models predicted drag forces which were almost same as the drag forces of standard $k-\varepsilon$ model.
Hemispherical canopy
TABLE VI. Predictions of 3 different Two Equation Models (HC)

| Velocity <br> $\mathrm{m} / \mathrm{s}$ | $C_{d}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{k}-\varepsilon$ | RNG k- $\varepsilon$ | SST k- $\omega$ |
| 2 | 1.144 | 1.166 | 1.148 |
| 10 | 1.141 | 1.17 | 1.145 |
| 20 | 1.142 | 1.17 | 1.146 |
| 30 | 1.146 | 1.169 | 1.148 |
| 40 | 1.15 | 1.169 | 1.151 |
| 50 | 1.152 | 1.168 | 1.153 |



Fig. 6 Predictions of 3 different 2 equation models for HC

## Ram-air canopy

TABLE VII. Prediction of 3 different Two Equation Models (RC)

| Velocity m/s | $C_{d}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{k}-\varepsilon$ | RNG k- $\varepsilon$ | SST k- $\omega$ |
| 2 | 1.045 | 1.042 | 1.042 |
| 10 | 1.042 | 1.04 | 1.04 |
| 20 | 1.042 | 1.039 | 1.039 |
| 30 | 1.041 | 1.039 | 1.038 |
| 40 | 1.041 | 1.038 | 1.038 |
| 50 | 1.041 | 1.038 | 1.038 |



Fig. 7 Predictions of 3 different 2 equation models for RC
The drag coefficient for most of the object remains constant depending upon the mass and projected area when $R e>$ $10^{4}$ [8]. As mentioned before the drag coefficients predicted by all three models are same. It is concluded that working with standard $k-\varepsilon$ model will be sufficient for this application.

## VIII. NUMERICAL TECHNIQUE

Runge Kutta $4^{\text {th }}$ order method is used to solve the differential equation involved in this problem to find the terminal velocity and time taken by the paratrooper to reach the ground are given below
$m \frac{\partial^{2} H}{\partial t^{2}}=m g-F_{D}$
$\frac{\partial^{2} H}{\partial t^{2}}=g-\frac{\rho C_{d} A}{2 m}\left(\frac{\partial H}{\partial t}\right)^{2}$
Let $B=\frac{\rho C_{d} A}{2 m}$
$\therefore \frac{\partial^{2} H}{\partial t^{2}}=g-B\left(\frac{\partial H}{\partial t}\right)^{2}$
Equation (23) is a second order PDE and therefore it is first converted into two first order ordinary differential equations.
$u^{\prime}=H^{\prime}(t)=v(t)$
$v^{\prime}=g$
With initial conditions $\left\{\begin{array}{l}H(0)=0 \\ v(0)=0\end{array}\right.$
Initially let $i=0$
$k 1^{(1)}=h * v_{i}$
$k 1^{(2)}=h *\left(g-B v_{i}{ }^{2}\right)$
$k 2^{(1)}=h *\left(v_{i}+0.5 * k 1^{(2)}\right)$
$k 2^{(2)}=h *\left(g-B\left(v_{i}+0.5 k 1^{(2)}\right)^{2}\right)$
$k 3^{(1)}=h *\left(v_{i}+0.5 * k 2^{(2)}\right)$
$k 3^{(2)}=h *\left(g-B\left(v_{i}+0.5 k 2^{(2)}\right)^{2}\right)$
$k 4^{(1)}=h *\left(v_{i}+k 3^{(2)}\right)$
$k 4^{(2)}=h *\left(g-B\left(v_{i}+k 3^{(2)}\right)^{2}\right)$
$M=\left(k 1^{(1)}+2 k 2^{(1)}+2 k 3^{(1)}+k 4^{(1)}\right) / 6$
$H_{i+1}=H_{i}+M$
$N=\left(k 1^{(2)}+2 k 2^{(2)}+2 k 3^{(2)}+k 4^{(2)}\right) / 6$
$v_{i+1}=v_{i}+N$
" $h$ " is the time step and (29) and(30) can be used to find the distance travelled from the point of jump and values of velocity respectively according to the time step defined after every successful completion of the entire procedure. The above expressions were solved using C++ programming.

## IX. RESULTS AND DISCUSSION

When the paratrooper jumps out of the plane his initial velocity would be zero and it increases rapidly until he reaches his terminal velocity in the free fall phase and then once $3 / 4^{\text {th }}$ of the total distance is covered he deploys his parachute and due to the sudden enlargement in the projected area the velocity reduces quickly. Again after a certain distance the paratrooper attains the terminal velocity.

TABLE VIII. Dimensions of Canopies

| Canopy | Diameter m | Span m | Chord m | ${\text { Area } \mathrm{m}^{2}}^{\|c\| c\|c\|}$ |
| :---: | :---: | :---: | :---: | :---: |
| Dome | 4.7 | - | - | 17.4 |
| Ram air | - | 6.28 | 2.78 | 17.4 |

Paratrooper's dimensions (Cylinder dimensions)
Length (L) $=1.935 \mathrm{~m}$
Diameter (D) $\quad=1.23 \mathrm{~m}$
Mass (m) = 117 kg
$\mathrm{C}_{\mathrm{d}}$ (Free Fall) $=0.64$.
TABLE IX. Drag coefficient of both canopies under parachute aided fall

| Velocity m/s | Drag Coefficient, $\mathrm{C}_{\mathrm{d}}$ |  |
| :---: | :---: | :---: |
|  | Hemispherical Canopy | Ram Air Canopy |
|  | 1.037 | 1.048 |
| 10 | 1.035 | 1.046 |
| 20 | 1.034 | 1.045 |
| 30 | 1.034 | 1.045 |
| 40 | 1.033 | 1.044 |
| 50 | 1.033 | 1.044 |

Table IX provides drag coefficients for both the canopies which has same projected area.


Fig. 8 Comparison between HC and RC of the entire fall


Fig. 9 Comparison of parachute aided fall for HC and RC

The terminal velocity under parachute aided fall is 10.35 $\mathrm{m} / \mathrm{s}$ for HC and $10.25 \mathrm{~m} / \mathrm{s}$ for RC and also the time taken to reach the ground varies by just one second which is not significant so it can be concluded that the shape doesn't play any significant role in deciding the terminal velocity. The HC is subjected to 1603 Pa and RC is subjected to 1587 Pa for this particular set of values. The negligible difference between the pressures exerted on the canopies is a reason which explains why both the canopies have drag coefficients.

From (23) it can be seen that the velocity varies with respect to projected area in the form
$v_{t}=x A^{y}$
The effect of area was studied for both HC and RC and it was observed that it varies as per (30). For a particular paratrooper the effect of the canopy area was studied and the results are as follows.
Paratrooper's dimensions (Cylinder dimensions)
Length (L) $\quad=1.98 \mathrm{~m}$
Diameter (D) $\quad=1.31 \mathrm{~m}$
Mass (m) $\quad=121 \mathrm{~kg}$
$C_{d}$ (Free Fall) $=0.64$.

## Hemispherical Canopy

TABLE X. Diameters and corresponding Terminal velocities for HC

| Diameter m | Area m ${ }^{2}$ | $\mathrm{v}_{\mathrm{t}} \mathrm{m} / \mathrm{s}$ |
| :---: | :---: | :---: |
| 5.3048 | 22.1 | 8.611 |
| 6.3048 | 31.22 | 7.197 |
| 7.3048 | 41.91 | 6.131 |
| 8.1048 | 51.59 | 5.464 |
| 9.3048 | 68 | 4.701 |



Fig. 10 Variation of terminal velocity with Area of HC
Using best fit method the variation of terminal velocity with respect to area is
$v_{t}=45.952 A^{-0.54}$
Comparing (31) and (32) it is observed that $x=45.952$ and $y=-0.54$.

Also the time taken to reach the ground ( $\mathrm{t}_{\mathrm{g}}$ ) by the paratrooper from different height of jumps (HOJ) has been studied and it was observed that as the HOJ increases for a particular parachute it varies linearly.


Fig. 11 Comparison of $\mathrm{t}_{\mathrm{g}}$ with respect to area for different of HOJ using HC

## Ram-Air Canopy

TABLE XI. Area and corresponding Terminal velocities for RC

| Span m | Chord m | Area $\mathrm{m}^{2}$ | $\mathrm{~V}_{\mathrm{t}} \mathrm{m} / \mathrm{s}$ |
| :---: | :---: | :---: | :---: |
| 3.92 | 1.71 | 6.73 | 16.501 |
| 4.45 | 1.94 | 8.63 | 14.569 |
| 4.99 | 2.19 | 10.95 | 12.931 |
| 5.49 | 2.77 | 15.18 | 11.174 |
| 6.28 | 2.78 | 17.44 | 10.423 |

Here the terminal velocity varies with area in the form
$v_{t}=42.796 A^{-0.5}$
These results also agrees with (31) and by comparison between (31) and (33) shows
$x=42.796$ and $y=-0.5$.


Fig. 12 Variation of terminal velocity with Area of HC


Fig. 11 Comparison of $\mathrm{t}_{\mathrm{g}}$ with respect to area for different of HOJ using RC

## X.CONCLUSION

CFD has been applied to this particular problem and the drag forces for short cylinders of different L/D, hemispherical and ram-air canopies have been found. The drag coefficients remained almost constant and it is known that it remains constant for most geometry which is subjected to flows having $R e>10^{4}$. And the terminal velocity and time taken to reach the ground also agrees with the common understanding. As the area increases the drag increases and thus results in lower terminal velocity and as the terminal velocity reduces the time to reach the ground will increase. And it can be concluded that any of the two equation turbulence model can be used for this application.

## XI. REFERENCE

[1] C. P. Kothandaraman, S. Subramanyan, Heat and Mass Transfer Data Book, $7^{\text {th }}$ edition, New Age International Publishers, 2010.
[2] Fluent 6.3 User's Guide, Fluent Inc, 2006
[3] H. K. Versteeg, W. Malalasekera, An introduction to Computational Fluid Dynamics, $1^{\text {st }}$ edition, Longman Scientific \& Technical, 1995.
[4] R. Vaidya, R. Bhalwar, S. Bobdey, Anthropometric Parameters of Armed Forces Personnel, MJAFI 2009; 65: 313-318
[5] O. Rouaud, M. Havet, Computation of the airflow in a pilot scale clean room using $k-\varepsilon$ turbulence models, International Journal of Refrigeration 25(2002) 351-361
[6] P. A. Costa Rocha, H. H. Barobosa Rocha, F. O. Moura Carneiro, M. E. Vieira da Silva, A.Valente Bueno, $k-\omega$ SST (shear stress transport) turbulence model calibration: A case study on a small scale horizontal axis wind turbine, Journal on Energy 65 (2014) 412418
[7] Un Yuon Jeong, Hyun-Moo Koh, Hae Sung Lee, Finite element formulation for the analysis of turbulent wind flow passing bluff structures using the RNG $k-\varepsilon$ model, Journal of Wind Engineering and Industrial Aerodynamics 90 (2002) 151-169
[8] Yunus A. Cengel, John M. Cimbala, Fluid Mechanics: Fundamentals and Applications, $1^{\text {st }}$ edition, Tata McGraw Hill, 2006.

