Published by :
http://www.ijert.org

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 6 Issue 04, April-2017

A Novel Technique for the Modeling of Power Line Noise for PLC Low Voltage Applications

A. M. Nyete
Electrical and Information Engineering
University of Nairobi,
Kenya.

T. J. O. Afullo
Electrical and Electronic Engineering
University of KwaZulu-Natal,
South Africa.

Abstract **- Most of the published work on impulsive noise and its models is based on parametric modelling techniques. Even though these techniques are good in that they represent the noise using some parameters like the variance and mean, they have a big shortcoming in that their mathematical expressions that determine the underlying distribution are fixed. The fixed form of many parametric distributions therefore introduces rigidity in the form that the noise probability density function and cumulative distribution function may take. Thus, fitting measured noise data using parametric distributions may result in an overestimation of the actual data structure that may lead to some of the salient features of the noise distribution being missed. To overcome the rigidity associated with parametric estimators, the use of nonparametric techniques is necessary. In this paper, a novel application of nonparametric kernel density estimators to develop reference models of the power line noise measured in low voltage indoor power networks in both time and frequency domain is presented. Nonparametric kernel density techniques estimate the underlying distribution of the noise directly from the measured data, without imposing any restrictions or making any assumptions as to the particular form of the data structure. As such, no fixed parameters are used to model the data, and therefore the data is modelled as it is. The kernel density method is the most efficient and popular nonparametric estimator. In fact almost all nonparametric algorithms are asymptotically kernel methods. It is continuous and overcomes the challenges of the other popular but primitive nonparametric estimator; the histogram. This estimator is very good going by the models developed and the validation results obtained. The objectives of this paper are:**

1. **To present the definitions and features of nonparametric kernel density estimators as a stochastic and probabilistic modeling tool.**
2. **To proof that the said estimator is actually very suitable for modeling of power line noise, more so as a reference (baseline) modeling technique.**
3. **To introduce this wonderful technique to the PLC research community and therefore stimulate further research in line with the findings in this paper as well as exploration of other nonparametric estimators.**

Key words: Powerline Communication, Kernel Density, Powerline Noise.

1. PLC IMPULSIVE NOISE MODELS

This subsection revisits PLC impulsive noise models briefly. Noise in PLC systems falls into three main categories: impulsive noise, coloured background noise and narrowband interference. Asynchronous impulsive noise that is periodic to the frequency of the mains power, as well as narrowband and coloured background noises that usually tend to remain stationary for periods that range between seconds and minutes or even hours, and therefore are generally classified as background noise. However, impulsive noise that is asynchronous with the mains frequency, and synchronous impulsive noise that is periodic with the mains frequency are time variant from microseconds to milliseconds. The spectral density of the noise power rises significantly during impulsive events and may result in bit or burst errors during transmission of data [1-6].

Since narrowband interference is mainly ingressed into the network, its effect on the system performance is not as severe as compared to the other two. On the other hand, background noise is stationary and can be modelled as a classical Gaussian process. Additionally, impulsive noise; in all its three sub-classes poses the greatest threat to the performance of the PLC channel. This type of noise has been modelled using different parametric models; but, the most commonly used ones are the two-term mixture Gaussian model and the Middleton's class-A impulsive noise model [5-13]. In two recent publications by Shongwe *et al.* [14, 15], a comprehensive study/survey of impulsive noise and its models was presented.

Even though the two-term mixture Gaussian model is simple and is used frequently in the analysis of PLC systems [16-18], it is deficient in that it does not provide a very accurate representation of the true impulse noise. Another parametric model, the Middleton's class-A impulsive noise model, which is also widely used and counters the shortcomings of this model, is discussed next. This is a rather simple model based on a Poisson-Gaussian process and incorporates both background and impulsive noise, and was first suggested in [19]. Due to its slightly higher accuracy in the modelling and characterization of the real impulsive noise, this model has been employed by several authors in their analysis of the performance of impulsive systems [20-22]. However, this model was not designed for PLC systems and also does not tell us whether or not the noise is impulsive in the time domain.

Another very flexible model that has recently featured in the study of impulsive inference/noise is the alpha stable distribution. This distribution is characterized by tails that are much fatter/longer than those of the Gaussian distribution, a characteristic synonymous with impulsive processes, like PLC noise. It is also more flexible than the other two parametric noise models described above. This distribution is able to capture the impulsive nature of the noise and can model very extreme cases; ranging from very impulsive noise cases to pure background noise since the Gaussian distribution is one of the limiting cases [4, 13, 23, 24]. This distribution is defined by its characteristic function $\emptyset(t)$, given by [4, 13, 23, 24]:

$$\emptyset(t) = exp\{j\delta t - \gamma^{\alpha}|t|^{\alpha}[1 + j\beta sign(t)\omega(t,\alpha)]\} \quad (1)$$

Where,

$$\omega(t,\alpha) = \begin{cases} \tan\frac{\pi\alpha}{2}, & \alpha \neq 1 \\ \frac{2}{\pi}\log|t|, & \alpha = 1 \end{cases} \qquad (2)$$

$$sign(t) = \begin{cases} 1 & for\ t > 0 \\ 0 & for\ t = 0 \\ -1 & for\ t < 0 \end{cases} \qquad (3)$$

And: $-\infty < \delta < \infty, \gamma > 0, 0 < \alpha \leq 2,$

$$-1 \leq \beta \leq 1 \qquad (4)$$

α is the characteristic index or exponent, δ is the location parameter, γ is the dispersion or scale parameter and β is the symmetry parameter.

The impulsive noise models discussed above, and virtually all literature on noise modelling in PLC systems is based on parametric methods. A few parameters, expressed in fixed parametric mathematical equations are used in all the cases to describe the noise distribution. These parametric methods therefore introduce rigidity in the particular structure that the noise distribution may take. Thus, if these models are fitted straight to some measured noise data, as seen in [10] for example, the resultant models are just approximations to the measurements and may actually miss out on some of the salient features of the measured data distribution in most cases. Nonparametric density estimators are able to overcome the rigidity associated with parametric methods. To this end, in this paper, we introduce an alternative reference power line noise modelling framework that is based on nonparametric kernel density estimation techniques. The density estimate is obtained straight from the data itself and therefore "hugs" the measured data almost 100%. Thus, these models are seen as reference models of the measured noise distribution, and give an actual feel of any data distribution. Also, kernel density methods are the most accurate and widely used nonparametric estimators. This modelling framework is applied to measured noise characteristics and found to produce very good baseline results.

2. NONPARAMETRIC DENSITY ESTIMATION

Existing PLC noise models, including the ones discussed in Subsection 1 are based on parametric statistical distributions. This is to say that the estimation of the probability density function (pdf) that defines a set of noise data is done via a fixed set of parameters, which introduces rigidity in the particular shape that a data structure can take. As such, these fixed parameters mean that a data set structure can only take an approximate parametric distribution fit. The function $f(x)$ is assumed to belong to some distributions that are parametric like Gamma, Rayleigh, Lognormal, Erlang, Gaussian and Exponential, among others. With the parametric data modelling technique, the estimation of the parameters that relate the data to a particular distribution is usually the main task. Some error analysis to validate the assumption is then done. For instance, if the Gaussian distribution is used to model some data set, then the estimator would be given by:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{(x-\mu)^{\wedge}2 / 2\sigma^2}, \quad x\epsilon R \qquad (5)$$

where, $\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$. Parametric data modelling is fine as long as the distribution that is assumed to fit the data is correct or not seriously wrong at the very least. This method is quite easy to apply in many cases, and gives rise to relatively stable estimates. But, the fixed forms of the parametric distributions render them rigid, and this comes across as the biggest disadvantage of this data modelling technique. The data can only be trained to take on a particular structure whose final outcome is fixed. This would imply that in most of the cases, key aspects of the data structure like skewness, actual tail probabilities, peakiness and bimodality may be missed out and therefore a misinterpretation of the underlying data distribution occurs. For instance, the Gaussian estimator above gives rise to models that are symmetrical and dome-shaped, and therefore this renders it inappropriate for modelling data that is bimodal, heavily tailed, and/or skewed [8, 25].

To address the rigidity concerns associated with parametric distributions, nonparametric methods are used. These methods estimate the probability density straight from the raw data without any prior assumptions as to the characteristic structure for the underlying distribution. As such, no fixed parameters are used to model the data, and therefore the data is modelled as it is. The histogram was the only known nonparametric density estimation technique until the 1950s, when meaningful progress was made in both spectral density estimation and density estimation. However, the histogram suffers from serious shortcomings that include the sharp transitions (discontinuities) between the bins, which results in a step-like data structure that is usually difficult to interpret, as well as an exponential growth in bin numbers with the number of dimensions, and that the data structure also depends on the bins' start and end points. For many practical cases, these shortcomings render histograms useless, and they are therefore only useful in quick visualizations of data in both one and two dimensions.

On the other hand, smooth nonparametric density estimation methods like kernel density estimators overcome the drawbacks associated with the histogram. One of the earliest papers on kernel density estimators is by Rosenblatt in 1956 [26]. The kernel density estimator is motivated as an averaged shifted histogram limiting case. Some of the techniques that can be applied in demonstrating its superior qualities, as well as providing a deeper understanding of it include numerical analysis and finite differences, smoothing by convolution and orthogonal series approximations. In fact almost all nonparametric algorithms are asymptotically kernel methods, a fact that was clearly demonstrated by Walter and Blum [27] and later proven in a rigorous way by Terrell and Scott [28]. The kernel density estimator for a random variable n is given as [28-30]:

$$f(n) = \frac{1}{kh}\sum_{i=1}^{k} K\left(\frac{n - X_i}{h}\right) \qquad (6)$$

where k is the number of data points, h is the smoothing parameter, also referred to as the bandwidth or window width, X_i is the i^{th} data point and $K(\cdot)$ is the kernel function. The kernel function is symmetric in most cases which means that $K(u) = K(-u)$. A second order kernel function is defined by the following properties [28-30]:

$$\int_{-\infty}^{+\infty} K(u)du = 1 \qquad (7)$$

$$\int_{-\infty}^{+\infty} uK(u)du = 0 \qquad (8)$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 6 Issue 04, April-2017

$$\int_{-\infty}^{+\infty} u^2 K(u)du > 0 \qquad (9)$$

From Equations (8) and (9), we conclude that the kernel function has a zero first order moment and a finite second one respectively. Equation (7) on the hand shows that the kernel function is a true pdf. The kernel density estimate, as seen in Equation (6) is controlled by two main factors, the kernel function and the smoothing parameter. Optimal selection of the bandwidth is the most important aspect of modelling using kernel density estimators. A very small value of the smoothing parameter results in a very peaky (spiky) and spurious under-smoothed density estimate that is hard to interpret while a very large value of the same parameter results in over-smoothed densities that would mask the data structure. A simple illustration on how the density estimation is carried out in the kernel technique is shown in Fig. 1 below. From this figure, as well as from Equation (6), we conclude that the density estimate is a summation of "bumps" centred on every data point in the neighbourhood of the point of estimation. The bump's shape is determined by the kernel function while the value of the bandwidth determines how spread (wide) they are.

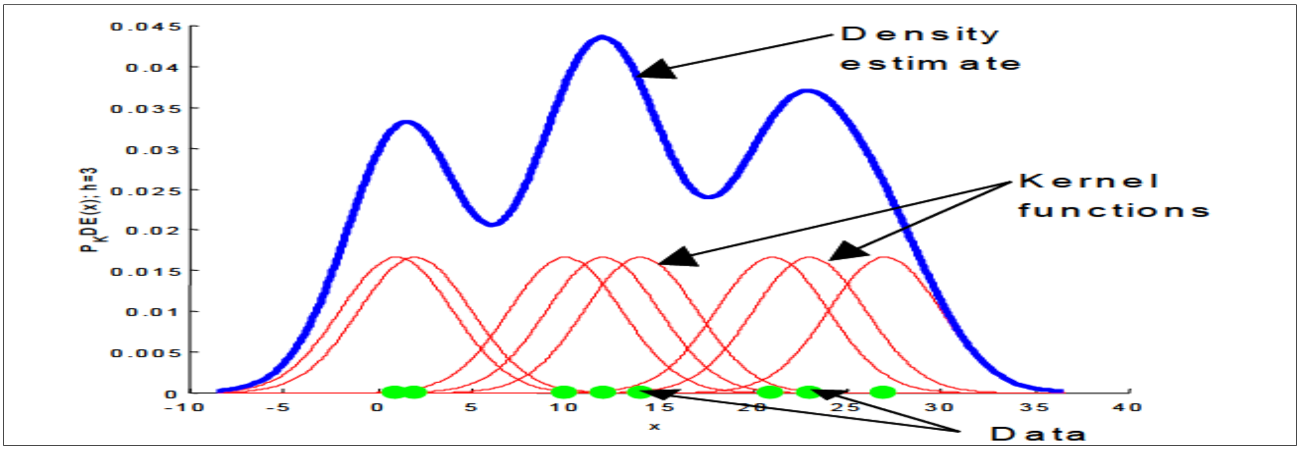

Figure 1: Kernel density estimation illustration

There are several techniques that are used in the determination of the optimal value of the smoothing parameter. These include automatic techniques like plug-in and classical methods, and the reference to a distribution methodology. Plug-in methods refers to those ones that find a pilot estimate of the density using a pilot value of the smoothing parameter then use the estimated density to determine the error. Classical methods are basically extensions of methods used in parametric estimations and they include: least squares cross-validation, biased cross-validation, likelihood cross-validation and indirect cross-validation. Even though these optimal smoothing parameter selection methods have been explored by many authors, none is considered the best in every situation [29]. There are many second order and higher order kernels available in literature but some of the common second order kernels are shown in Table 1 below.

2.1 BANDWIDTH SELECTION FOR KERNEL DENSITY ESTIMATORS

Kernel density estimation effectiveness as a modelling tool is dependent on the choice of the smoothing parameter more than even the choice of the kernel function itself. The value of the smoothing parameter determines the biasness and the variance of the estimated model.

Table 1: Common kernel functions

Kernel function	Mathematical expression, $K(u)$
Epanechnikov	$K(u) = \begin{cases} \frac{3}{4\sqrt{5}}\left(1-\frac{1}{5}u^2\right), -\sqrt{5} \le u \le \sqrt{5} \\ 0, elsewhere \end{cases}$
Triangular	$K(u) = \begin{cases} (1-\lvert u \rvert), -1 \le u \le 1 \\ 0, elsewhere \end{cases}$
Gaussian	$K(u) = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{u^2}{2}\right)}, -\infty < u < \infty$
Rectangular	$K(u) = \begin{cases} \frac{1}{2}, -1 \le u \le 1 \\ 0, elsewhere \end{cases}$

If the value of the chosen smoothing parameter is very small compared to the optimal one, the resulting model is usually under-smoothed and very spiky, and therefore difficult to interpret. On the other hand, high values of the smoothing parameter results in density estimates that are over-smoothed, and therefore obscure the data structure. In practical applications, an optimal choice of the smoothing parameter is done based on the kernel function, number of data samples, as well as their variance. The common method used to determine a "rough estimate" of the optimal smoothing parameter is Silverman's rule of thumb [31, 32] which assumes that the function *f (n)* is one of the standard distributions. For the Normal distribution with mean μ and variance σ^2, the roughness of the kernel function *K(u)*, $R(f'')$, is given by [33]:

$$R(f'') = \frac{3}{8\sigma^5\sqrt{\pi}} \quad (10)$$

If the kernel density estimator and the form of *f (n)* are known, the optimal bandwidth can be chosen. For the Gaussian kernel case, the respective values of *R(K)* and $\mu_2^2(K)$ are calculated to be $(2\sqrt{\pi})^{-1}$ and 1 respectively, and the optimal bandwidth is given by [25, 31]:

$$h_{opt} = \sigma(4/3k)^{\frac{1}{5}} \quad (11)$$

where σ is the standard deviation. The value of h_{opt} is quickly calculated by estimating σ from the observed data.

If the size of the sample data is small and the data density is close to the normal distribution, another bandwidth expression is used [34]:

$$h_{opt} = 0.79(\hat{q}_3 - \hat{q}_1)k^{\frac{-1}{5}} \quad (12)$$

where $\hat{q}_3$ and $\hat{q}_1$ are respectively the third quartile and the first quartile of the sample data. When the data density is not close to the normal distribution, the following expression is used:

$$h_{opt} = 0.9 \min\left(\hat{\sigma}, \frac{\hat{q}_3 - \hat{q}_1}{1.349}\right) k^{\frac{-1}{5}} \quad (13)$$

where $\hat{\sigma}$ is the standard deviation of the sample data. All in all, extensive modelling with kernel density estimators shows that there is no single

plug-in formula that is applicable in all situations. Usually, simple plug-in formulas are available for the "first rough estimate" of the smoothing parameter from the data set, from which other validation and goodness of fit tests can be applied to obtain the optimal model. To this end, the error between the kernel model and the measured data can be minimized with respect to the kernel under consideration. Silverman showed that the optimal kernel is Epanechnikov kernel. Thus, the efficiency of any kernel is estimated by comparing it to that of the Epanechnikov kernel. But, in our modeling with kernel density estimators, we found out that the results obtained are almost the same as long as the optimization of all the models is carried out appropriately. The optimal bandwidths and efficiencies of some common kernel functions are given in Table 2. The value of sample variance σ is estimated from the sample data with *k* observations.

Table 2: Common kernel efficiencies and bandwiths plug-in formulae

Kernel	Efficiency (%)	Optimal bandwidth
Epanechnikov	100	$\frac{2.34\sigma}{k^{\frac{1}{5}}}$
Triangular	98.6	$\frac{2.58\sigma}{k^{\frac{1}{5}}}$
Gaussian	95.1	$\frac{1.06\sigma}{k^{\frac{1}{5}}}$
Rectangular	93	$\frac{1.84\sigma}{k^{\frac{1}{5}}}$

3. KERNEL DENSITY LOW VOLTAGE POWER LINE NOISE MODELING

Noise in power lines is complex and cannot be modelled and characterized using pure mathematical derivations. This is the reason why almost all existing noise models are derived from measurements. Two sample measurements (among thousands that we used in our modelling) both in frequency and time domains are shown below in Figures 2 and 3 respectively. The noise measurement setup is shown in Figure 4.

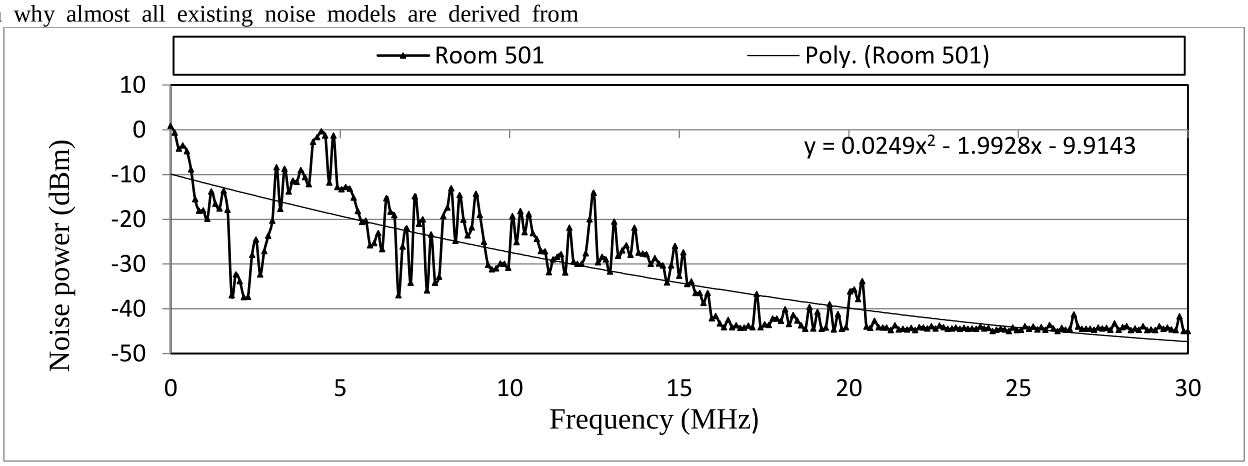

Figure 2: Electromagnetic laboratory (Room 501) frequency domain noise

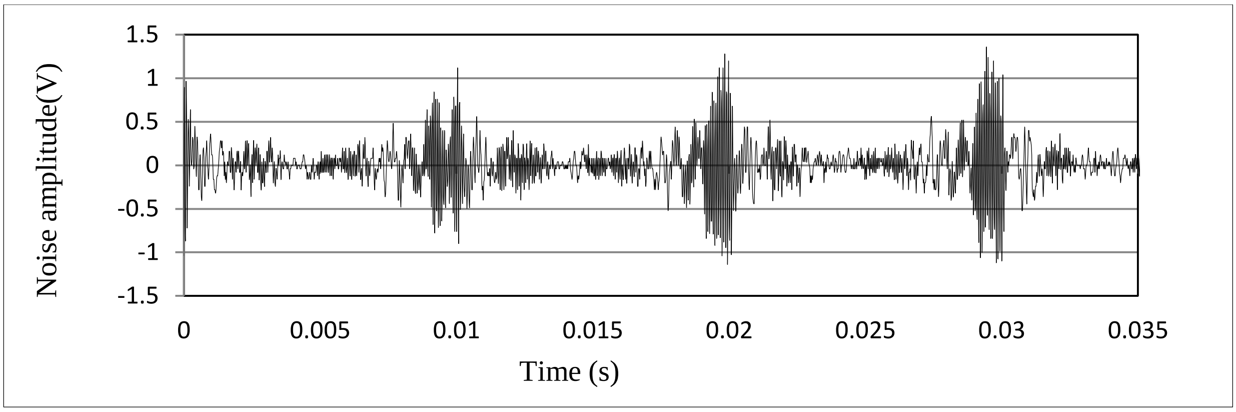

Figure 3: Radio Frequency laboratory (Room R01) time domain noise

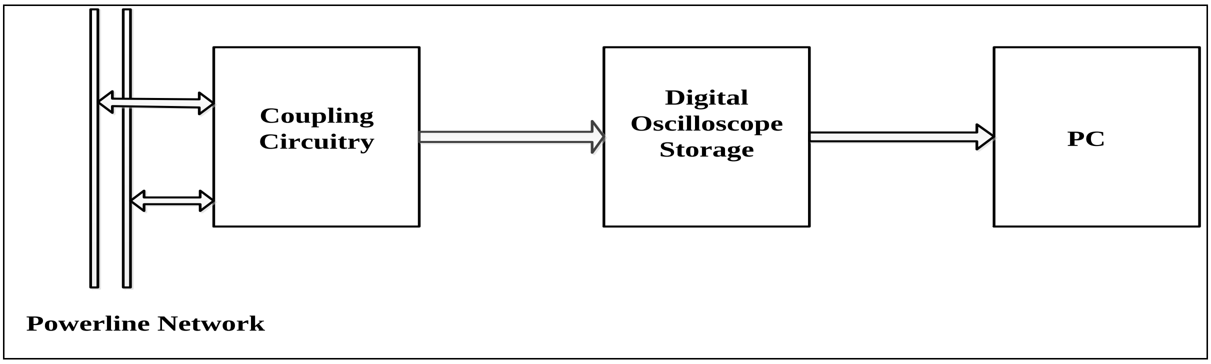

Figure 4: Noise measurement set up schematic

In order to capture the random concentration of the noise distribution across different frequency bands in the frequency domain or the voltage level variation concentration (which is an indicator of the impulse power), statistical tools need to be employed to model and characterize the noise into certain probability density functions (pdfs) and cumulative frequency distributions (cdfs). This is crucial because, with the corresponding parameters derived from the noise measurements for the pdf or cdf plot, we can then give a full statistical description of the overall noise characteristics. As pointed out earlier, the models presented in Subsection 2 are rigid and hence are unsuitable for the reference modelling (initial fitting) of power line noise. In this research work therefore, we introduce a flexible modelling tool for the power line noise measured in this study, based on the novel application of nonparametric kernel density estimators for the modelling of PLC noise.

The four kernels shown in Table 1 above are used in the derivation of simple nonparametric models that expresses the noise characteristics in form of some tractable mathematical forms. These models are optimized through an error-based optimization procedure. The determination of the optimum kernel models is based on the optimal choice of the bandwidth. The model optimization procedure developed in this study follows an iterative methodology that is described below:

1. From the bandwidth plug-in formula for each kernel as shown in Table 1, determine the first rough estimate of the bandwidth.
2. Derive the kernel models for each kernel and compute the error between the measured and modelled pdf.
3. Choose a slightly larger or smaller value of the bandwidth and repeat step 2.
4. Compare the errors calculated in step 2 and 3, and choose a smaller or larger value of the bandwidth.
5. Repeat step 2 to 4 until the error computed is minimum, determined when the otherwise diminishing error starts increasing.
6. The bandwidth that gives the minimum error is the optimal value.

Different global measures of accuracy can be used in the optimization of the kernel density estimates, but in this study we chose the mean integral square error (MISE), which is by far the most popular error criteria used in ensuring accurate results with kernel modelling.
From the basic definition of the MISE, we have:

$$MISE = E[\int_{-\infty}^{\infty} [f(n) - f^*(n)]^2 \, dk] \qquad (14)$$

Where $f(n)$ is the measured pdf values and $f^*(n)$ is the kernel model values. The asymptotic MISE (AMISE) is given by:

$$MISE(f) = \frac{R(K)}{kh} + \frac{1}{4} h^4 \mu_2^2 R(f'') \qquad (15)$$

where k is the size of the sample data, $R(K) = \int K^2(u)\,du$ is the roughness of $K(u)$, $\mu_2^2(K) = \int \left(u^2 K(u)\right)^2 du$ is the second moment squared of the pdf defined by the kernel $K(u)$, and $R(f'') = \int (f''(n))^2 \, dn$ is a roughness measure. A bias-variance trade-off ensures that the MISE obtained is minimum. Assuming the second derivative of the density is square integrable and absolutely continuous, then by a Taylor series expansion of $f(n - yh)$ about n we obtain:

$$f(n-yh) = f(n) - hyf'(n) + \frac{1}{2}h^2y^2f''(n) + o(h^2) \qquad (16)$$

From which, the bias of the density estimate is:

$$Bias\left(f(n)\right) = \frac{h^2}{2}f''(n)\mu_2(K) + O(h^2) \qquad (17)$$

Also, the estimated function variance is given by:

$$Var\left(f(n)\right) = \frac{1}{kh}\int K^2(y)\,f(n-yh)dy - \frac{1}{k}\left(E\left(f(n)\right)\right)^2 \quad (18a)$$

$$= \frac{1}{kh}\int K^2(y)\,\{f(n) + O(1)\}dy = \frac{1}{k}\{f(n)+O(1)\}^2 \qquad (18b)$$

$$= \frac{1}{kh}K^2(y)dyf(n) + O\left(\frac{1}{kh}\right) \qquad (18c)$$

$$= \frac{1}{kh}R(K)f(n) + O\left(\frac{1}{kh}\right) \qquad (18d)$$

Decreasing the bias leads to a very noisy estimate (large variance) while decreasing the variance leads to over-smoothed estimates (large bias). The variance-bias trade-off ensures consistency in the density estimation.

The time domain and frequency domain results obtained for the four kernels from the above procedure are shown in Table 3. In this table, we demonstrate the three special classes in power line noise modelling using kernel density estimation, namely: under-smoothing, optimal smoothing and over-smoothing. The first value of the bandwidth for each kernel is an under-smoothed case while the second is the optimal value, with last one representing an over-smoothed one. The errors for each value of the smoothing parameter are shown; where we observe that the optimal bandwidth value gives the minimum error.

Table 3: Power line noise kernel modelling errors

Kernel	Time domain		Frequency domain	
	h	MISE	h	MISE
Triangular	0.03	0.0083	0.02	0.0014
	0.21	0.0058	0.481	0.0002
	0.9	0.0077	1.4	0.0008
Gaussian	0.05	0.0079	0.04	0.0013
	0.163	0.0056	0.423	0.0001
	1.0	0.0078	2.0	0.0011
Epanechnikov	0.03	0.0081	0.06	0.0012
	0.1841	0.0052	0.445	0.0002
	1.3	0.0078	1.7	0.0010
Rectangular	0.05	0.0084	0.10	0.0008
	0.189	0.0057	0.512	0.0001
	1.0	0.0076	1.2	0.0015

The minimum error for the optimal time domain models is obtained for the Epanechnikov kernel while the same is obtained for both the Gaussian and Rectangular kernels respectively for the frequency domain. This means that the kernels are equally efficient as long as the optimization is done right. Also we see that the error values are quite close across the four kernel used in the modelling, from which we conclude that the choice of the optimal value of the bandwidth has more weight on the resulting density estimate than the kernel function itself.

The time domain noise kernel model plots are shown in Figs. 5 to 8, while the frequency domain noise kernel models are shown in Figs. 9 to 12. From these noise models, we see that the time domain models are quite symmetrical (bell-shaped) while the frequency domain ones are clearly not. The frequency domain models actually exhibit long tails, with the tail probabilities representing the probability of occurrence of the impulsive noise. We see that much of the noise spectrum density is concentrated between -49.7 dBm and -45.5 dBm. This noise density band essentially represents the background noise density in this study. It also points to the fact that the background noise has a much higher probability of occurrence than impulsive noise, by comparing this noise band with the tails of the models. We note here that the measured noise is modelled as it is, which means that no effort has been made to separate the noise components.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 6 Issue 04, April-2017

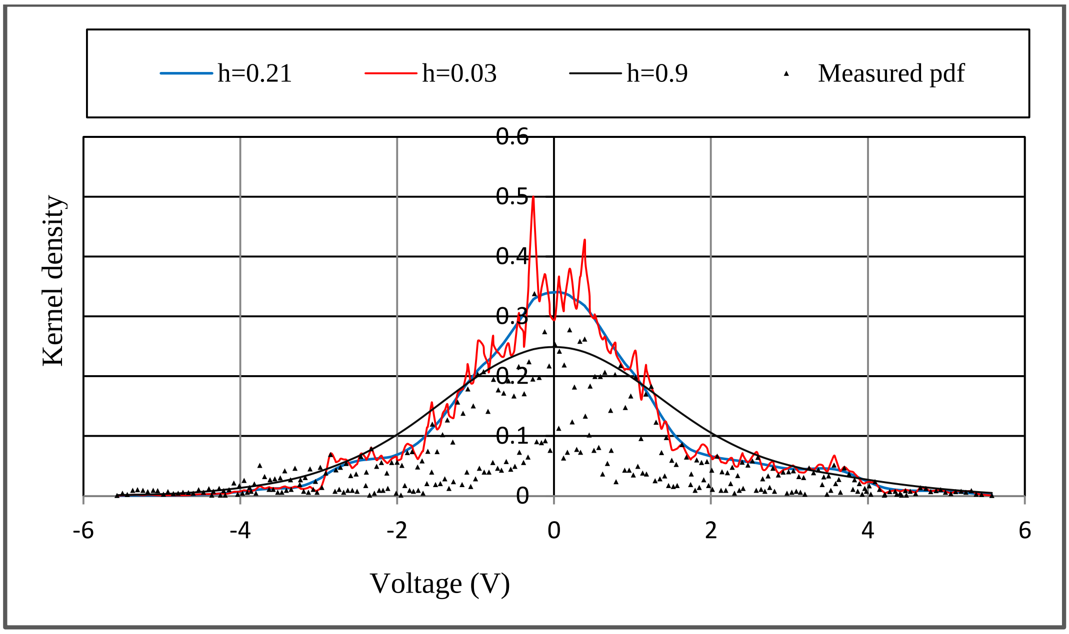

Figure 5: Triangular kernel time domain models

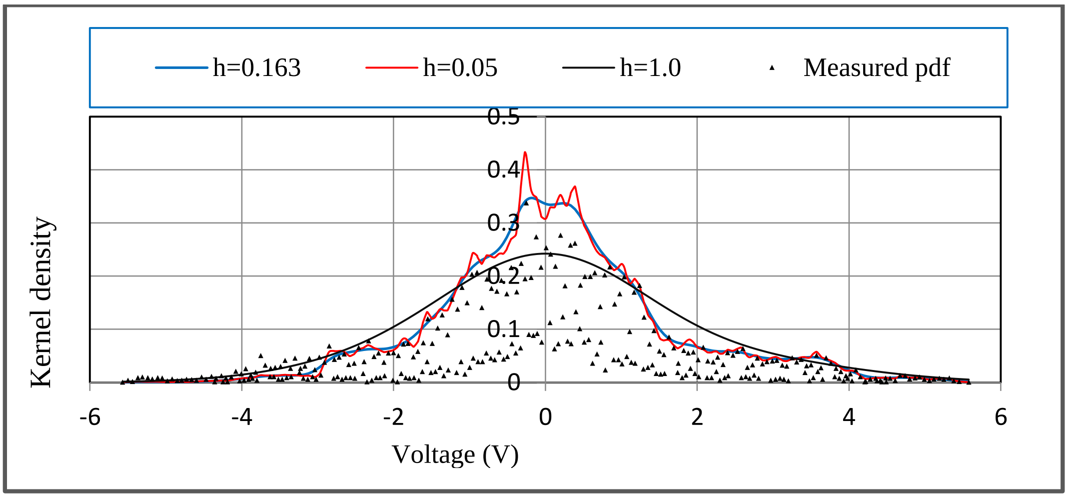

Figure 6: Gaussian kernel time domain models

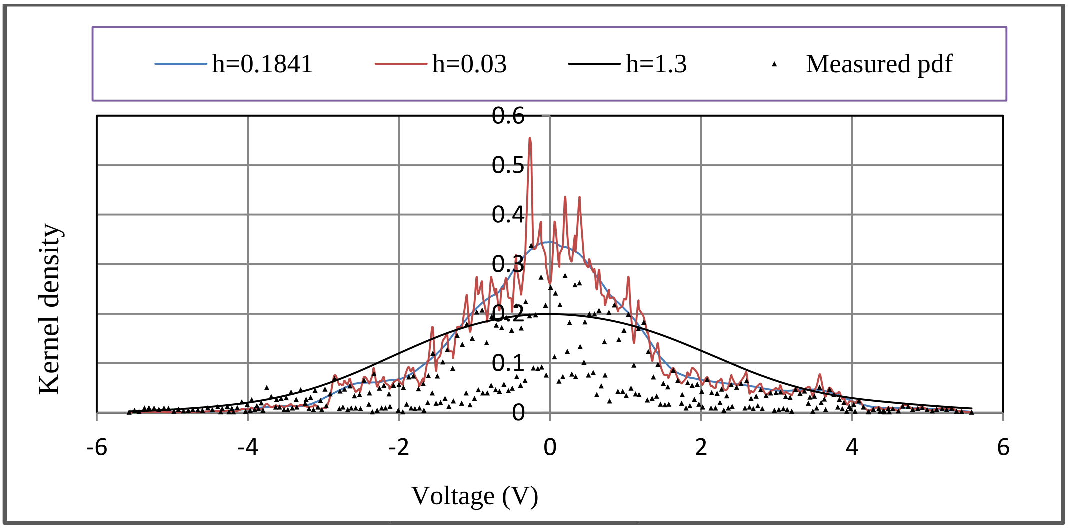

Figure 7: Epanechnikov kernel time domain noise models

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 6 Issue 04, April-2017

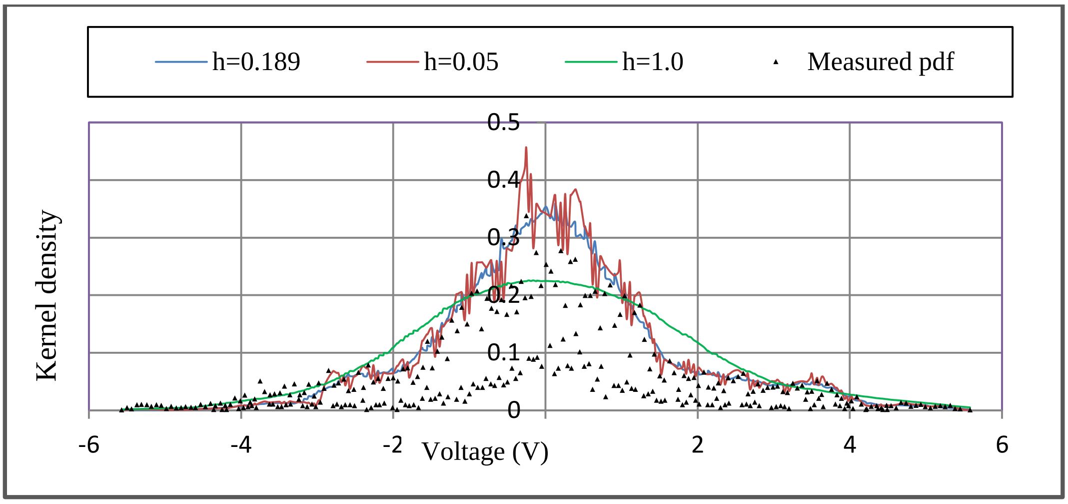

Figure 8: Rectangular kernel time domain noise models

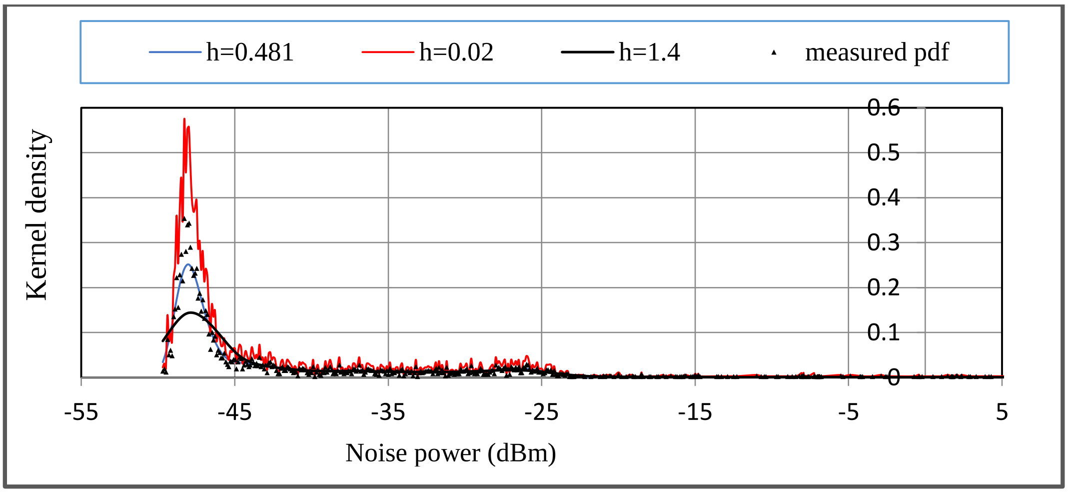

Figure 9: Triangular kernel frequency domain noise models

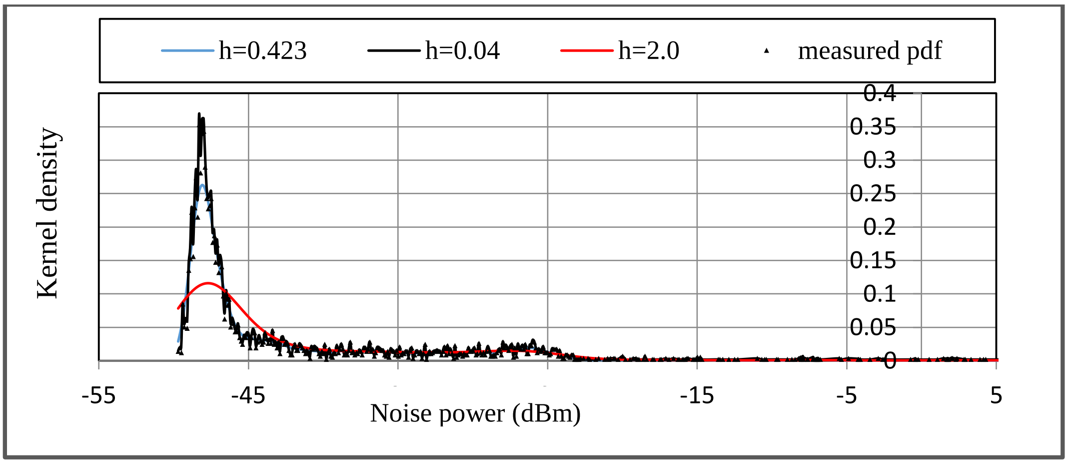

Figure 10: Gaussian kernel frequency domain noise models

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 6 Issue 04, April-2017

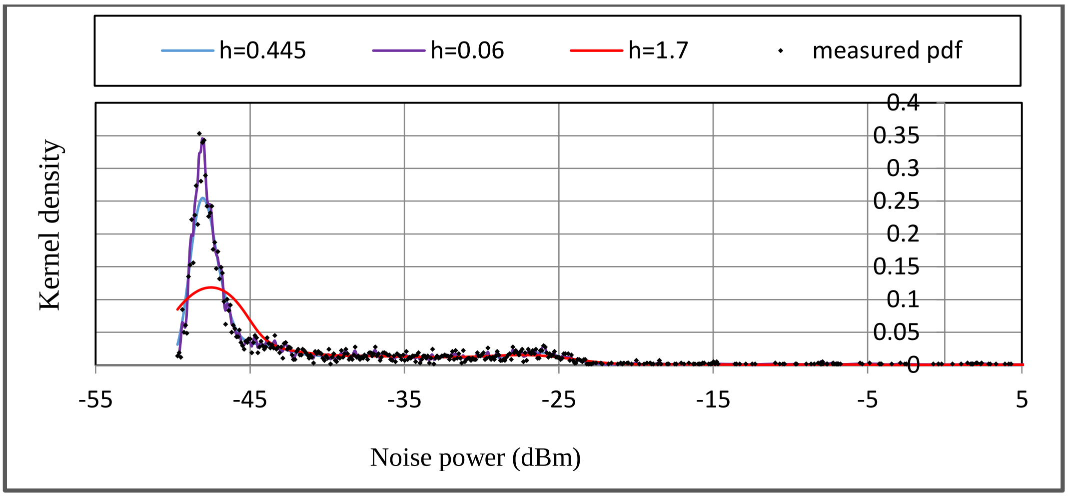

Figure 11: Epanechnikov kernel frequency domain noise models

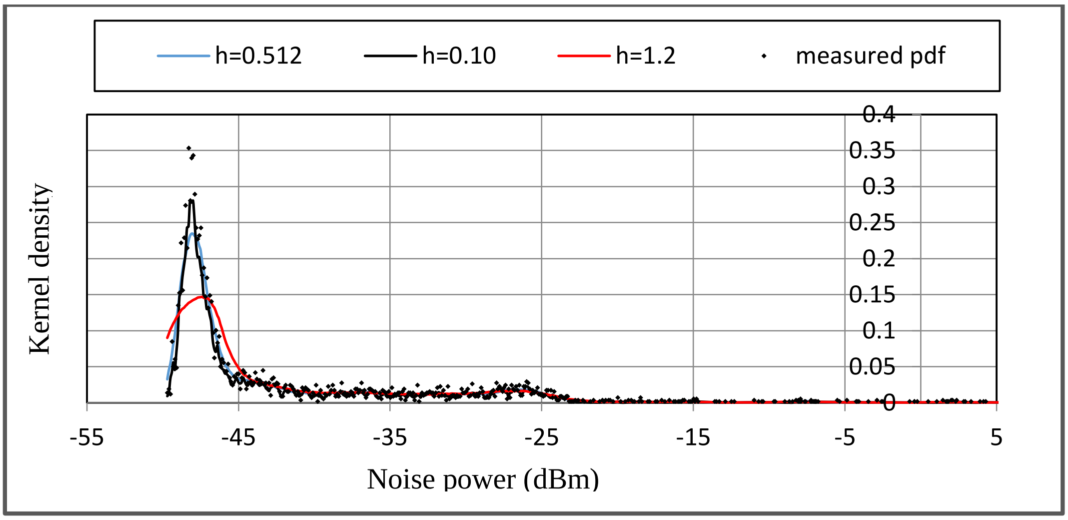

Figure 12: Rectangular kernel frequency domain noise models

Even though previously the performance of kernel models has traditionally only been evaluated via the MISE, in this research work, we go an extra mile to ascertain the stability and consistency of the optimal kernel noise models obtained in both time and frequency domains, by applying the Chi-square (χ^2) statistics test. The χ^2 statistics are computed as:

$$\chi^2 = \sum_{i=1}^{N} \frac{[f(x) - g(x)]^2}{g(x)} \qquad (19)$$

where $f(x)$ is the measured data values, $g(x)$ is the optimal kernel data values, and N is the sample data length. A tabulation of the different Chi-square test parameters is given in Table 4 below.

Table 4: Chi-square test parameters

Kernel	Time domain			
	χ^2	DF	CV	SL
Triangular	11.74	250	287.88	0.05
Gaussian	11.75	250	287.88	0.05
Epanechnikov	11.77	250	287.88	0.05
Rectangular	11.62	250	287.88	0.05
Kernel	Frequency domain			
	χ^2	DF	CV	SL
Triangular	0.871	300	341.44	0.05
Gaussian	0.812	300	341.44	0.05
Epanechnikov	0.862	300	341.44	0.05
Rectangular	1.01	300	341.44	0.05

From the results in Table 4, we see that the computed χ^2 values for both the time and frequency domains are very close to each other for all the four kernels, for the same degrees of freedom (DF) and significance level (SL). The χ^2 values are also very small compared to the critical values (CV). Given that none of the χ^2 values exceeds the CVs, we therefore accept the null hypothesis H_0 and reject the alternative hypothesis H_1 for all the cases. Thus, all the models are consistent with the measured data and, with a 95% confidence, we can say that there is no significance difference between the kernel models obtained with the optimum smoothing parameter and the models obtained from the measured data. This confirms that kernel models "hug" the measured data as close as possible, which means that they can even be used as an excellent approximation (reference models) of the measured data density, given the small errors and χ^2 values obtained in Tables 3 and 4 above. This therefore implies that the optimal kernel models can be used as a reference for parametric modeling of power line noise.

CONCLUSION

In this paper, we have presented an alternative technique (new approach) for the modeling and characterization of power line noise in low voltage indoor power networks. The nonparametric kernel density estimator applied models the data as it is, and therefore the resultant models can be used as a reference/benchmark in the application of parametric techniques to model similar noise. An error-based optimization procedure has been developed and applied to derive optimal kernel models, as can be seen in the errors in Table 3. We also observe that the optimal value of the smoothing parameter has more weight on the resulting estimate than the choice of a particular kernel function, going by the errors obtained for the various optimal kernel models. Also, the models have been tested for consistency using the Chi square fitness test. This fitness test results further confirm the suitability of kernel techniques in the modeling of power line noise. As such there is no significant difference between the χ^2 values for the four kernels for both time and frequency domain cases. The computed χ^2 values are very small compared to their corresponding CVs for the same SL and DF. This is a further demonstration of the suitability of this technique for nonparametric modeling of low voltage indoor power line noise, as long as the correct values of the optimal smoothing parameters for the various kernels are used. The time domain noise models are dome-shaped and rather symmetrical, while the frequency domain ones are clearly skewed with long tails (heavy tails), which points to the impulsive nature of the measured power line noise. Overall, we have studied noise characteristics in an indoor environment and proposed simple and tractable models that can be benchmarked upon for parametric modeling of indoor power line noise. As a future work, we will develop parametric models of the measured noise characteristics based on very flexible stochastic parametric modeling tools, to be benchmarked on the kernel models developed here, and validate them appropriately.

REFERENCES

[1] M. Zimmermann and K. Dostert, "Analysis and modeling of impulsive noise in broadband powerline communications," IEEE Transactions on Electromagnetic Compatibility, vol. 44, no. 1, pp. 249-258, November 2002.

[2] M. Zimmermann and K. Dostert, "Analysis of the broadband noise scenario in powerline networks," in proceedings of the ISPLC Conference, 2000, pp.131-138.

[3] J.A. Cortes, L. Diez, F.J. Canete and J.J. Sanchez-Martinez, "Analysis of the indoor broadband power-line noise scenario," IEEE Transactions on Electromagnetic Compatibility, vol. 52, no. 4, pp. 849-858, November 2010.

[4] A.M. Nyete, T. J. O. Afullo, and I.E. Davidson, "Statistical analysis and characterization of power line noise for telecommunication applications," IEEE AFRICON 2015 Conference Proceedings, Addis Ababa, Ethiopia, 14–17 September 2015, pp. 213-217.

[5] A. M. Nyete, T. J. O. Afullo, and I.E. Davidson, "Power line noise measurements and statistical modelling in the time domain," Proceedings of PIERS 2015 in Prague, July 6-9 2015, Prague, Czech Republic, pp. 1569-1574.

[6] R.M. Vines, H.J. Trussell, L. Gale and J.B. O'Neal, "Noise on residential power distribution circuits," IEEE Transactions on Electromagnetic Compatibility, vol. 26, pp. 161-168, November 1984.

[7] T.Q Bui, Coded modulation techniques with bit interleaving and iterative processing for impulsive channels, MSc. Thesis, University of Saskatchewan, 2006.

[8] A.M. Nyete, T. J. O. Afullo, and I.E. Davidson, "On the application of non-parametric estimation methods for low voltage PLC network noise modelling," Southern Africa Telecommunication Networks and Applications Conference (SATNAC) 2015, pp. 59-63.

[9] L.D. Bert, P. Caldera, D. Schwingshackl, and A.M. Tonello, "On noise modeling for power line communications," in Proc. Int. Symp. Power Line Communication and its Applications, 2011, pp. 283-288.

[10] C. Assimakopoulos and N. Pavlidou, "A enhanced powerline channel noise model," WSEAS Transactions on Power Systems, Issue 1, vol. 1, pp. 239-245, January 2006.

[11] T. Esmailian, F.R. Kschischang, P.G. Gulak "In-building power lines as high-speed communication channels: channel characterization and a test channel ensemble," International Journal of Communication Systems 2003; vol. 16, pp. 381–400.

[12] H.C. Ferreira et al., "Power line communications: An overview," IEEE AFRICON 1996, 24-27 September 1996, Stellenbosch, South Africa, pp. 558-563.

[13] A. M. Nyete, T.J.O. Afullo, I.E. Davidson, "Statistical models of noise distribution in broadband plc networks," In Proceedings of PIERS 2015 Conference, July 6-9 2015, Prague, Czech Republic, 423-429.

[14] T. Shongwe, A. J. Han Vinck, H. C. Ferreira, "On impulse noise and its models," In proceedings of the18th ISPLC Conference, Glasgow, Scotland, March 30th 2014-April 2nd 2014, pp. 12-17.

[15] T. Shongwe, A. J. Han Vinck, H. C. Ferreira, "A study on impulse noise and its models," SAIEE Africa Research Journal, vol. 106, no.3, pp. 119-131, September 2015.

[16] Y. Ma, P. So, and E. Gunawan, "Performance analysis of OFDM system for broadband power line communications under impulsive noise and multipath effects," IEEE Transactions on Power Delivery, Vol. 20, pp. 674-682, April 2005.

[17] T. Faber, T. Scholand, and P. Jung, "Turbo decoding in impulsive noise environments," Electronics Letters, vol. 39, pp. 1069-1071, July 2003.

[18] H. V. Poor and M. Tanda, “Multiuser detection in flat fading non-Gaussian channels," IEEE Transactions on Communications, vol. 50, no. 11, pp. 1769-1777, November 2002.

[19] D. Middleton, “Statistical-physical models of electromagnetic interference," Electromagnetic Compatibility; Proceedings of the Second Symposium and Technical Exhibition, vol. EMC-19, pp. 331-340, June 1977.

[20] S. Miyamoto, M. Katayama, and N. Morinaga, “Performance analysis of QAM systems under Class-A impulsive noise environment," IEEE Transactions on Electromagnetic Compatibility, vol. 37, no. 2, pp. 260-267, May 1995.

[21] D. Umehara, H. Yamaguchi, and Y. Morihiro, “Turbo decoding in impulsive noise environment," in Proceedings of the IEEE Global Telecommunications Conference, 29 Nov.-3 Dec. 2004, pp. 194-198.

[22] M. Ardakani, F. R. Kschischang, and W. Yu, “Low-density parity-check coding for impulse noise correction on power line channels," in Proceedings of the 2005 ISPLC Conference, March 2005, pp. 90-94.

[23] G. A. Tsihrintzis, and C. L. Nikias, “Fast estimation of the parameters of alpha-stable impulsive interference,” IEEE Transactions on Signal Processing, vol. 44, no. 6, 1996.

[24] R. Weron, Performance of the estimators of stable law parameters, Research report HSC/95/1, Hugo Steinhaus Center for Stochastic Methods, Institute of Mathematics, Wroclaw university; Available online: http://prac.im.pwr.wroc.pl/~hugo/publ/RWeron_HSC_95_1.pdf -last accessed on 3/4/2015.

[25] W. Zucchini, Applied smoothing techniques, part 1: Kernel Density Estimation, Available: http://staff.ustc.edu.cn/~zwp/teach/Math-Stat/kernel.pdf.

[26] M. Rosenblatt, “Remarks on some nonparametric estimates of a density function,” The Annals of Mathematical Statistics, vol. 27, no. 3, pp. 832-837, 1956.

[27] G. Walter and J. Blum, “Probability density estimation using delta sequences,” The Annals of Statistics, vol. 7, no. 2, pp. 328-340, 1979.

[28] G.R. Terrell and D.W. Scott, “Variable kernel density estimation,” The Annals of Statistics, vol. 20, no. 3, pp. 1236-1265, 1992.

[29] A. Z. Zambom and R. Dias, A Review of Kernel Density Estimation with Applications to Econometrics, Cornell University library, December 2012, Available : http://arxiv.org/abs/1212.2812.

[30] S. Hovda, “Using pseudometrics in kernel density estimation,” Journal of Nonparametric Statistics, vol. 26, no. 4, pp. 669-696, 2014.

[31] D. F. Froelich and M. Rahman, ‘‘A note on the choice of the smoothing parameter in the kernel density estimate’’, BRAC University Journal, vol. VI, no.1, pp. 59-68, 2009.

[32] B. W. Silverman, Density estimation for statistics and data analysis, Chapman and Hall, London, 1986.

[33] V. C. Raykar and R. Duraiswami, “Very fast optimal bandwidth selection for univariate kernel density estimation”, Perceptual Interfaces and Reality Laboratory of Maryland University, June 28, 2006.

[34] S. J. Sheather, “Density estimation,” Institute of Mathematical statistics, Australia, vol. 19, No.4, pp. 588-597, 2004.