A Novel grouping of Supremum and Infimum Function of a Fuzzy Set

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Abstract. **In this study we suggest a new group decision-making method which is based on some basic fuzzy set operations. By using this method, we can get two types of results. First, we can identify which alternative is the best. The second result, a crucial point of this work, is the screening of decision- makers. The decision-makers should be serious and responsible in giving their opinions otherwise the process will eliminate them because of their inappropriate evaluations to the alternatives compared with that of the other decision-makers. We also discuss an application to demonstrate the process of the method.**

Key Words and Phrases: Fuzzy sets; α-level set; group decision-making.

INTRODUCTION

An individual or group is frequently faced with the problem of choosing one alternative from a feasible alternative set. For an individual people, the problem is the identification of the most preferred alternative according to his/her preference structure. However, group decision making, except the above task, another important problem is how to aggregate the expert's opinion to obtain an acceptable result for the group. In general, the preference relations take the form of multiplicative preference relations whose elements estimate the dominance of one alternative over another and take the form of exact numerical values. It is more suitable to provide their preferences by means of linguistic variables rather than numerical ones developed a practical method for group decision making with linguistic preference relations. To aggregate the preference information and to rank the given alternatives presented a method which is called the ordered weighted averaging operator.

SOME PRELIMINARIES

In this section, we describe some preliminary definitions of fuzzy set operations that will be used in this paper. More details and historical background of fuzzy set theory can be found in [8, 17, 25].

In a universe U, a fuzzy set $\bar{A}$ is defined as

$$\bar{A} = \{(x, \mu_{\bar{A}}(x)) : x \in U, \mu_{\bar{A}}(x) \in [0,1]\} \qquad (1)$$

Where the function $\mu_{\bar{A}}(x)$ is called membership function. The value of the membership function $\mu_{\bar{A}}(x)$ specifies the grade or degree to any element x in U Large value of $\mu_{\bar{A}}(x)$ indicate higher degree of membership. We will identify any fuzzy set with its membership function and use these two concepts as interchanged.

Let $\bar{A}$ be a fuzzy set in the universe U as in (1). Then the support of $\bar{A}$ is defined as

$$\inf \bar{A} = \{x : x \in U, \mu_{\bar{A}}(x) < 0\} \ \& \ \sup \bar{A} = \{x : x \in U, \mu_{\bar{A}}(x) > 0\} \qquad (2)$$

The cardinality of a crisp set A. denoted as $|A|$, is the number of elements of the set A and the cardinality of a fuzzy set $\bar{A}$ is denoted as $card\,\bar{A} = \sum_{x \in U} \mu_{\bar{A}}(x)$ (3)

The mean relative cardinality of $\bar{A}$ is defined as $mrc\bar{A} = \dfrac{card\,\bar{A}}{|\sup p\bar{A}|}$ (4)

And then the α – level set of $\bar{A}$ is defined as $\bar{A}_\alpha = \{x : x \in U, \mu_{\bar{A}}(x) \geq \alpha\}$ (5)

The proposed method

When using this method first we decision makers give their evaluations according their own opinions for all the considered alternatives in the form of a fuzzy set. Each alternative is evaluated with a value in the interval [0,1] from the point of view of each decision makers.

Let $A=\{a_1,a_2.....a_n\}$ be an alternative set and let $B=\{b_1,b_2.....b_m\}$ be a decision maker set in a finite universe set U_a & U_b respect. Then this method can be described by the following steps in k- cycles.

Step 1: let the evaluation of decision maker $b_j \in B_k$

$$\overline{A_{b_j}} = \{(\alpha, \mu_{A_{b_j}}(\alpha)) : \alpha \in A, \mu_{A_{b_j}}(\alpha) = b_j(\alpha)\} \quad (6)$$

Step 2: we using the Arithmetic Mean for probability concept for the fuzzy set as

$$\overline{A_{b_k}} = \left\{(\alpha, \mu_{A_{b_k}}(\alpha)) : \alpha \in A, \mu_{A_{b_k}}(\alpha) = \frac{1}{|b_k|}\sum_{b_i \in B_k} \mu_{A_{b_k}}(\alpha)\right\} \quad (7)$$

Step 3: The distances between the sets A_{b_j} and A_{b_k} for all $b_i \in B_k$ as

$$\overline{A_k}(b_i) = \{(\alpha, \mu_{A_b(b_i)}(\alpha)) : \alpha \in A, \mu_{A_b(b_i)}(\alpha) = |\mu_{A_b(b_i)}(\alpha) - \mu_{A_b(b_k)}(\alpha)|\} \quad (8)$$

CONCLUSION

The decision-making is a process of choosing the most desirable alternative among a set of alternatives and is, therefore, important in many disciplines including social, physical, medical and engineering sciences. When choosing a preferable alternative, people must set a priority for each available alternative. This process is not easy for an individual. It is even more difficult when there is more than one decision-maker involved in the process. How to aggregate the individual choices into a group preference has always been a hot topic.

REFERENCES

[1] Asai, K., Tanaka, H. and Okuda, T., Decision making and its goal in a fuzzy environment, In *Fuzzy Sets and Their Applications to Cognitive and Decision Processes* Ed. Zadeh at al, pp.257–277, Academic Press, 1975.

[2] Bellman, R.E. and Zadeh, L.A., Decision-making in a fuzzy environment, *Manage. Sci.*17 (4) 141–154. 1970.

[3] Ben-Arieh, D. and Chen, Z., A new linguistic label aggregation method and consen- sus measure for group deciison-making, *Proceedings of the 2004 IIE Annual Conference*, Houston, TX, U.S.A., May 2004.

[4] Bezdek, J.C., Analysis of Fuzzy Information, vol. II, Artificial Intelligence and Decision Systems, Boca Raton, 1987.

[5] Blin, J.M., Fuzzy relations in group decision theory, *J. of Cybernetics* 4 17–22. 1974.

[6] Bordogna, G., Fedrizzi, M., Passi, G.A, linguistic modeling of consensus in group deci- sion making based on OWA operator,27 126Ű132. 1997.

[7] Chiclana, F., Herrera, F. and Herrera-Viedma, E., Integrating multiplicative preference relations in a multipurpose decision making model based on fuzzy preference relations, Fuzzy Sets and Systems 122 277Ű-291. 2001.

[8] Dubois, D. and Prade, H., *Fuzzy Set and Systems: Theory and Applications,* Academic Press, New York, 1980.