

A Novel Fuzzy Vehicle Detection and Tracking Framework Using Fuzzy Edge Membership and Motion Similarity Relations

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Abstract - Vehicle detection and tracking are essential components of intelligent transportation systems, traffic surveillance, and autonomous driving applications. Conventional detection and tracking approaches employ crisp decision-making mechanisms that often suffer from uncertainty caused by illumination variations, occlusions, shadows, camera vibrations, and complex traffic scenarios. To overcome these limitations, this paper proposes a novel Fuzzy Vehicle Detection and Tracking Framework (FVDTF) based on fuzzy edge membership, fuzzy motion similarity, and fuzzy association relations. The proposed framework incorporates fuzzy set theory to model uncertainty during vehicle detection, feature extraction, object association, and trajectory estimation. A Cartesian fuzzy vehicle relation is defined to measure the similarity between vehicle observations across consecutive frames. Several theoretical properties of the proposed model, including symmetry, boundedness, monotonicity, and closure under Cartesian products, are derived and proved mathematically. The proposed framework provides a strong theoretical foundation for the development of intelligent vehicle monitoring systems and future fuzzy-based traffic surveillance models. The methodology can be extended to intuitionistic fuzzy, Pythagorean fuzzy, and q-rung Ortho pair fuzzy environments for enhanced decision-making under uncertainty.

Keywords: Vehicle Detection, Vehicle Tracking, Fuzzy Logic, Fuzzy Vehicle Relations, Traffic Surveillance, Intelligent Transportation Systems, Fuzzy Motion Similarity, Pattern Recognition.

1. INTRODUCTION

Vehicle detection and tracking are fundamental tasks in intelligent transportation systems, traffic surveillance, autonomous driving, and traffic flow management. Existing vehicle monitoring systems employ Gaussian Mixture Models (GMM), Support Vector Machines (SVM), Kalman Filters, Particle Filters, AdaBoost classifiers, and feature-based tracking methods for detecting and tracking moving vehicles. These techniques have demonstrated satisfactory performance under controlled environments; however, their effectiveness decreases significantly in the presence of shadows, illumination changes, partial occlusions, camera vibrations, and complex traffic conditions.

Recent developments in computer vision have shown that uncertainty is an inherent characteristic of traffic scenes. Vehicle boundaries are often ambiguous, feature correspondences between consecutive frames may be uncertain, and object trajectories can be partially lost during occlusion. Traditional crisp decision-making approaches assign either vehicle or non-vehicle labels, which may not adequately represent these uncertainties. Consequently, more flexible mathematical frameworks are required to model gradual transitions between detection states and tracking confidence levels.

Fuzzy set theory, introduced by Zadeh, provides a powerful foundation for representing and processing uncertain information [14]. The theory has evolved through numerous extensions including fuzzy groups, anti-fuzzy groups, intuitionistic fuzzy structures, flexible Q-fuzzy groups, and q-rung Ortho pair fuzzy systems [14,17–30]. These developments have successfully addressed uncertainty in algebraic systems, pattern recognition, decision making, and intelligent control applications [7–30].

In traffic surveillance environments, the concept of fuzzy membership is particularly suitable because an observed object may belong to the vehicle class with varying degrees of confidence rather than possessing a strict binary classification. Similarly, the similarity between tracked features across successive frames can be represented through fuzzy relations, thereby reducing the effect of tracking ambiguities and temporary feature losses. Motivated by fuzzy algebraic structures, fuzzy PS-ideals, Cartesian fuzzy relations, and recent developments in flexible Q-fuzzy systems [21–30], this work introduces a fuzzy vehicle detection and tracking framework capable of handling uncertain observations and feature associations.

The major contributions of this paper are as follows:

1. Definition of fuzzy vehicle detection and fuzzy tracking operators based on membership functions.
2. Construction of fuzzy vehicle relations using cartesian product concepts.
3. Development of fuzzy motion similarity and fuzzy association measures for trajectory estimation.
4. Theoretical derivation and proof of fundamental properties of the proposed fuzzy framework.
5. Establishment of a mathematical foundation for future extensions using intuitionistic fuzzy, Pythagorean fuzzy, picture fuzzy, and q-rung Ortho pair fuzzy environments.

2. PRELIMINARIES

Let $V = \{v_1, v_2, v_3, \dots, v_n\}$ be the set of all vehicle candidates extracted from a video sequence. Throughout this paper, the symbols μ , μ_E , μ_M , and R_F denote fuzzy memberships associated with vehicle existence, edge information, motion information, and vehicle relations respectively.

Definition 2.1 (Fuzzy Vehicle Set)

A fuzzy vehicle set is a mapping $\mu: V \rightarrow [0,1]$ such that $\mu(v)$ represents the degree of membership of object v to the class of vehicles. For every vehicle candidate v_i , $0 \leq \mu(v_i) \leq 1$. Values near 1 indicate a high probability that the object represents a vehicle, whereas values near 0 indicate weak vehicle evidence.

Definition 2.2 (Fuzzy Edge Membership)

Let e be an edge point detected in a frame. The fuzzy edge membership of e is defined as $\mu_E(e) = \frac{|G(e)|}{G_{\max}}$, where $G(e)$ denotes the gradient magnitude at edge point e and $G_{\max}$ denotes the maximum gradient magnitude in the image. Hence, $0 \leq \mu_E(e) \leq 1$. A higher value of $\mu_E(e)$ indicates stronger evidence of a vehicle boundary.

Proposition 2.1

The fuzzy edge membership function is bounded.

Proof

Since $0 \leq |G(e)| \leq G_{\max}$, dividing both sides by $G_{\max} > 0$, $0 \leq \frac{|G(e)|}{G_{\max}} \leq 1$.

Thus, $0 \leq \mu_E(e) \leq 1$. Hence the fuzzy edge membership is bounded.

Definition 2.3 (Fuzzy Motion Membership)

Let s denote the observed speed of a vehicle candidate and s_r be a reference speed.

The fuzzy motion membership is defined as $\mu_M(s) = \exp\left(-\frac{|s-s_r|}{\sigma}\right)$ where $\sigma > 0$ is a scaling parameter. The function measures how closely the observed motion matches the expected vehicle motion.

Proposition 2.2

The fuzzy motion membership satisfies $0 < \mu_M(s) \leq 1$.

Proof

Since $|s - s_r| \geq 0$ and $\sigma > 0$, we have $-\frac{|s-s_r|}{\sigma} \leq 0$.

Therefore, $0 < \exp\left(-\frac{|s-s_r|}{\sigma}\right) \leq 1$. Hence $0 < \mu_M(s) \leq 1$.

The maximum value occurs when $s = s_r$. Thus the proposition follows.

Definition 2.4 (Fuzzy Vehicle Relation)

Define $R_F: V \times V \rightarrow [0,1]$ by $R_F(v_i, v_j) = \min\{\mu(v_i), \mu(v_j)\}$. The relation measures similarity between vehicle candidates. If two vehicle observations possess high membership values, then their relation value becomes correspondingly high.

Example 2.1

Suppose $\mu(v_1) = 0.8$, $\mu(v_2) = 0.6$. Then $R_F(v_1, v_2) = \min\{0.8, 0.6\} = 0.6$.

Thus the similarity relation between v_1 and v_2 equals 0.6.

Definition 2.5 (Cartesian Fuzzy Vehicle Relation)

Let $\mu_1, \mu_2: V \rightarrow [0,1]$ be two fuzzy vehicle sets. The Cartesian product relation is defined by

$$(\mu_1 \times \mu_2)(v_i, v_j) = \min\{\mu_1(v_i), \mu_2(v_j)\}.$$

Theorem 2.1

The Cartesian product of two fuzzy vehicle sets is a fuzzy relation.

Proof

For every $v_i, v_j \in V$, we have $0 \leq \mu_1(v_i) \leq 1$ and $0 \leq \mu_2(v_j) \leq 1$.

Therefore, $0 \leq \min\{\mu_1(v_i), \mu_2(v_j)\} \leq 1$.

Hence $(\mu_1 \times \mu_2)(v_i, v_j) \in [0,1]$.

Consequently, $\mu_1 \times \mu_2$ forms a fuzzy relation on $V \times V$.

Thus the theorem is proved.

Theorem 2.2

The fuzzy vehicle relation R_F is symmetric.

Proof

For any $v_i, v_j \in V$, $R_F(v_i, v_j) = \min \{\mu(v_i), \mu(v_j)\}$.

By commutativity of the minimum operator,

$$\min \{\mu(v_i), \mu(v_j)\} = \min \{\mu(v_j), \mu(v_i)\}.$$

Hence $R_F(v_i, v_j) = R_F(v_j, v_i)$.

Therefore R_F is symmetric.

Theorem 2.3

The fuzzy vehicle relation R_F is reflexive.

Proof

Consider any $v_i \in V$.

Then $R_F(v_i, v_i) = \min \{\mu(v_i), \mu(v_i)\}$.

Since the minimum of two identical numbers equals that number, $R_F(v_i, v_i) = \mu(v_i)$.

Thus every vehicle candidate is related to itself with membership value $\mu(v_i)$.

Therefore R_F is reflexive in the fuzzy sense.

Definition 2.6 (Strongest Fuzzy Vehicle Relation)

Let μ be a fuzzy vehicle set on V . The strongest fuzzy vehicle relation generated by μ is

$$\mu_R(v_i, v_j) = \min \{\mu(v_i), \mu(v_j)\}.$$

Theorem 2.4

If μ is a fuzzy vehicle set, then the strongest fuzzy vehicle relation μ_R is symmetric and bounded.

Proof

From Definition 2.6, $\mu_R(v_i, v_j) = \min \{\mu(v_i), \mu(v_j)\}$.

Since $0 \leq \mu(v_i), \mu(v_j) \leq 1$, it follows that $0 \leq \mu_R(v_i, v_j) \leq 1$.

Hence boundedness holds.

Also, $\mu_R(v_i, v_j) = \min \{\mu(v_i), \mu(v_j)\} = \min \{\mu(v_j), \mu(v_i)\} = \mu_R(v_j, v_i)$.

Therefore symmetry holds.

Hence the strongest fuzzy vehicle relation is symmetric and bounded.

Remark

The above concepts establish the mathematical basis for the proposed fuzzy vehicle detection and tracking framework. The fuzzy memberships represent uncertain detection evidence, while the fuzzy vehicle relations model similarity and association among vehicle candidates observed across consecutive video frames. These preliminary results will be used in the subsequent section to develop fuzzy vehicle detection, fuzzy feature association, and fuzzy trajectory tracking algorithms.

3. Proposed Fuzzy Vehicle Tracking Method

The proposed Fuzzy Vehicle Detection and Tracking Framework (FVDTF) consists of three major stages:

1. Fuzzy Vehicle Detection
2. Fuzzy Feature Association
3. Fuzzy Tracking Confidence Evaluation

The objective of this framework is to reduce uncertainty arising from illumination changes, shadows, occlusions, and camera motion while maintaining accurate vehicle trajectories.

3.1 Fuzzy Vehicle Detection

Vehicle detection begins by evaluating edge and shape information extracted from the image frame.

Let $v \in V$ be a vehicle candidate. Define the fuzzy detection membership function as

$$\mu_D(v) = \min \{\mu_E(v), \mu_S(v)\}$$

Where $\mu_E(v)$ denotes the edge membership and $\mu_S(v)$ denotes the shape membership.

A vehicle candidate is accepted whenever $\mu_D(v) \geq \alpha$ where $\alpha \in [0,1]$ is a predefined detection threshold.

Example 3.1

Suppose $\mu_E(v) = 0.85$ and $\mu_S(v) = 0.72$.

Then $\mu_D(v) = \min \{0.85, 0.72\} = 0.72$. If $\alpha = 0.70$, then $\mu_D(v) = 0.72 > \alpha$.

Hence the candidate is classified as a vehicle.

Theorem 3.1

The fuzzy detection membership function is bounded.

Proof

Since $0 \leq \mu_E(v) \leq 1$ and $0 \leq \mu_S(v) \leq 1$, the minimum operator satisfies

$$0 \leq \min \{\mu_E(v), \mu_S(v)\} \leq 1.$$

Thus $0 \leq \mu_D(v) \leq 1$. Hence the fuzzy detection membership is bounded.

Theorem 3.2

The fuzzy detection operator is monotonic.

Proof

Suppose $\mu'_E(v) \geq \mu_E(v)$ and $\mu'_S(v) \geq \mu_S(v)$.

Then $\min \{\mu'_E(v), \mu'_S(v)\} \geq \min \{\mu_E(v), \mu_S(v)\}$.

Therefore $\mu'_D(v) \geq \mu_D(v)$. Hence the detection operator is monotonic.

3.2 Fuzzy Feature Association

Once vehicles are detected, feature association between consecutive frames is performed.

Let v_i^t be a vehicle candidate in frame t and v_j^{t+1} be a vehicle candidate in frame $t + 1$.

Define $A(v_i^t, v_j^{t+1}) = \min \{\mu_M, \mu_P, \mu_A\}$ where μ_M = motion similarity,

μ_P = position similarity, and μ_A = appearance similarity.

The pair having the largest association value is selected as the corresponding tracked object.

Motion Similarity

Motion similarity is given by $\mu_M = \exp\left(-\frac{|s_i - s_j|}{\sigma}\right)$ where s_i and s_j are vehicle speeds.

Position Similarity

Position similarity is defined as $\mu_P = \exp\left(-\frac{d_{ij}}{\lambda}\right)$ where d_{ij} is the Euclidean distance between two vehicle positions.

Appearance Similarity

Appearance similarity is computed as $\mu_A = 1 - \frac{|F_i - F_j|}{F_{\max}}$

where F_i and F_j denote appearance feature vectors.

Example 3.2

Suppose $\mu_M = 0.91, \mu_P = 0.84, \mu_A = 0.77$.
 Then $A(v_i^t, v_j^{t+1}) = \min \{0.91, 0.84, 0.77\} = 0.77$.
 Therefore the association confidence equals 0.77.

Theorem 3.3

The fuzzy association function is bounded.

Proof

Since $0 \leq \mu_M, \mu_P, \mu_A \leq 1$, there exists $m = \min \{\mu_M, \mu_P, \mu_A\}$.
 Hence $0 \leq m \leq 1$. Therefore $0 \leq A(v_i^t, v_j^{t+1}) \leq 1$.
 Thus the fuzzy association function is bounded.

Theorem 3.4

The fuzzy association relation is symmetric.

Proof

Assume

$$\mu_M(v_i, v_j) = \mu_M(v_j, v_i)$$

$$\mu_P(v_i, v_j) = \mu_P(v_j, v_i)$$

and

$$\mu_A(v_i, v_j) = \mu_A(v_j, v_i).$$

Then

$$\begin{aligned} A(v_i, v_j) &= \min \{\mu_M, \mu_P, \mu_A\} \\ &= \min \{\mu_M(v_j, v_i), \mu_P(v_j, v_i), \mu_A(v_j, v_i)\} \\ &= A(v_j, v_i). \end{aligned}$$

Hence the association relation is symmetric.

3.3 Fuzzy Tracking Confidence Evaluation

After feature association, the reliability of vehicle trajectories is estimated through fuzzy tracking confidence. Let $\mu_1(v), \mu_2(v), \dots, \mu_k(v)$ be membership values obtained from k consecutive observations. Define $TC(v) = \frac{1}{k} \sum_{i=1}^k \mu_i(v)$.

A trajectory is accepted whenever $TC(v) \geq \beta$, where $\beta \in [0, 1]$ is the tracking threshold.

4. Theoretical Results

In this section, several theoretical properties of the proposed fuzzy vehicle detection and tracking framework are established. These results demonstrate the mathematical consistency and applicability of the proposed fuzzy relations and tracking operators.

Theorem 4.1 (Symmetry of Fuzzy Vehicle Relation)

The fuzzy vehicle relation $R_F: V \times V \rightarrow [0, 1]$ defined by $R_F(v_i, v_j) = \min \{\mu(v_i), \mu(v_j)\}$ is symmetric.

Proof

For any $v_i, v_j \in V$,
 we have

$$R_F(v_i, v_j) = \min \{\mu(v_i), \mu(v_j)\}.$$

Since the minimum operator is commutative,

$$\min \{\mu(v_i), \mu(v_j)\} = \min \{\mu(v_j), \mu(v_i)\}.$$

Therefore,

$$R_F(v_i, v_j) = R_F(v_j, v_i).$$

Hence the fuzzy vehicle relation is symmetric.

Theorem 4.2 (Reflexivity of Fuzzy Vehicle Relation)

The fuzzy vehicle relation R_F is reflexive.

Proof

Let $v_i \in V$.

Then

$$R_F(v_i, v_i) = \min \{\mu(v_i), \mu(v_i)\}.$$

Since

$$\min \{a, a\} = a,$$

we obtain

$$R_F(v_i, v_i) = \mu(v_i).$$

Since

$$\mu(v_i) \in [0, 1],$$

each vehicle candidate is related to itself with membership value $\mu(v_i)$.

Therefore R_F is reflexive in the fuzzy sense.

Theorem 4.3 (Boundedness of Fuzzy Vehicle Relation)

The fuzzy vehicle relation is bounded.

Proof

Since $0 \leq \mu(v_i) \leq 1$ and $0 \leq \mu(v_j) \leq 1$,

it follows that

$$0 \leq \min \{\mu(v_i), \mu(v_j)\} \leq 1.$$

Therefore,

$$0 \leq R_F(v_i, v_j) \leq 1.$$

Hence the relation is bounded.

Theorem 4.4 (Cartesian Product Property)

The Cartesian product of two fuzzy vehicle sets forms a fuzzy vehicle relation.

Proof

Let $\mu_1, \mu_2: V \rightarrow [0, 1]$.

Define

$$(\mu_1 \times \mu_2)(v_i, v_j) = \min \{\mu_1(v_i), \mu_2(v_j)\}.$$

Since

$$0 \leq \mu_1(v_i) \leq 1$$

and

$$0 \leq \mu_2(v_j) \leq 1,$$

we have

$$0 \leq \min \{\mu_1(v_i), \mu_2(v_j)\} \leq 1.$$

Thus,

$$(\mu_1 \times \mu_2)(v_i, v_j) \in [0, 1].$$

Therefore, the Cartesian product constitutes a valid fuzzy relation on $V \times V$.

Theorem 4.5 (Boundedness of Fuzzy Association Operator)

The fuzzy association operator $A(v_i^t, v_j^{t+1})$ is bounded.

Proof

The association operator is defined by

$$A(v_i^t, v_j^{t+1}) = \min \{\mu_M, \mu_P, \mu_A\}.$$

Since

$$0 \leq \mu_M, \mu_P, \mu_A \leq 1,$$

therefore

$$0 \leq \min \{\mu_M, \mu_P, \mu_A\} \leq 1.$$

Hence

$$0 \leq A(v_i^t, v_j^{t+1}) \leq 1.$$

Thus, the association operator is bounded.

Theorem 4.6 (Monotonicity of Association Operator)

The association operator is monotonic.

Proof

Suppose

$$\mu'_M \geq \mu_M,$$

$$\mu'_P \geq \mu_P,$$

and

$$\mu'_A \geq \mu_A.$$

Then

$$\min \{\mu'_M, \mu'_P, \mu'_A\} \geq \min \{\mu_M, \mu_P, \mu_A\}.$$

Therefore

$$A'(v_i^t, v_j^{t+1}) \geq A(v_i^t, v_j^{t+1}).$$

Hence the association operator is monotonic.

Theorem 4.7 (Boundedness of Tracking Confidence)

The tracking confidence function is bounded.

Proof

The tracking confidence is given by

$$TC(v) = \frac{1}{k} \sum_{i=1}^k \mu_i(v).$$

Since

$$0 \leq \mu_i(v) \leq 1,$$

we obtain

$$0 \leq \sum_{i=1}^k \mu_i(v) \leq k.$$

Dividing by k ,

$$0 \leq TC(v) \leq 1.$$

Hence tracking confidence is bounded.

Theorem 4.8 (Monotonicity of Tracking Confidence)

Tracking confidence is monotonic.

Proof

Assume

$$\mu'_i(v) \geq \mu_i(v)$$

for every observation i .

Then

$$\sum_{i=1}^k \mu'_i(v) \geq \sum_{i=1}^k \mu_i(v).$$

Dividing by k ,

$$\frac{1}{k} \sum_{i=1}^k \mu'_i(v) \geq \frac{1}{k} \sum_{i=1}^k \mu_i(v).$$

Therefore,

$$TC'(v) \geq TC(v).$$

Hence the tracking confidence is monotonic.

Theorem 4.9 (Occlusion Stability Property)

The proposed tracking confidence remains stable under partial occlusion.

Proof

Suppose a vehicle is observed in $k - r$ frames normally and in r frames with partial occlusion.

Let μ_H represent normal visibility membership and μ_L represent occluded visibility membership such that

$$0 < \mu_L < \mu_H \leq 1.$$

Then

$$TC(v) = \frac{(k - r)\mu_H + r\mu_L}{k}.$$

Since both terms are positive,

$$TC(v) > 0.$$

Thus, confidence decreases gradually rather than collapsing to zero, ensuring stable trajectory maintenance during temporary occlusions.

Hence the framework possesses occlusion stability.

Corollary 4.1

The proposed fuzzy vehicle tracking framework is mathematically valid since all detection memberships, association values, and trajectory confidence measures remain within the unit interval $[0, 1]$.

Proof

Directly follows from Theorems 4.3, 4.5, and 4.7.

5. Conclusion and Future Research Directions

This paper presented a Novel Fuzzy Vehicle Detection and Tracking Framework based on fuzzy vehicle memberships, fuzzy edge information, fuzzy motion similarity measures, and fuzzy association relations. A fuzzy vehicle relation was introduced to represent similarities among vehicle candidates, while a fuzzy tracking confidence measure was developed to evaluate the reliability of generated trajectories.

The framework provides an effective mathematical mechanism for handling uncertainty arising from illumination changes, shadows, camera vibrations, vehicle deformation, feature ambiguity, and partial occlusions. Several important theoretical properties including symmetry, reflexivity, boundedness, monotonicity, Cartesian closure, and occlusion stability were formally established and proved. These results demonstrate the theoretical soundness of the proposed fuzzy tracking methodology.

The proposed framework can serve as a mathematical foundation for advanced traffic surveillance and intelligent transportation applications.

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