International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 1 Issue 4, June - 2012

A NOTE ON CONFORMALLY RECURRENT KAHLERIAN MANIFOLDS

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ABSTRACT

Present paper delineates to the study of conformally recurrent kahlerian manifolds. In this paper, few interesting results have been obtained. In the last, conformally recurrent kahlerian manifold is flat if its scalar curvature is zero.

Key words: Ricci Tensor, Riemannian Curvature Tensor, Scalar Curvature Tensor, Recurrent Vector, Conformal Curvature Tensor.

1. INTRODUCTION :

Let g_{ji} is a positive definite metric and F^h_i be the structure tensor of a real 2n - dimentional Kahlerian space. Then we have the following relations :

$$\nabla_k F^i_j = 0,$$

$$\nabla_k g_{ji} = 0$$

$$F^r_j F^i_r = - \delta^i_j \, , \tag{1.1}$$

$$g_{rt} F^r_j F^t_i = g_{ji}$$

$$F_{ji} = g_{ri} F^r_j$$

$$F_{ij} = - F_{ji}$$

$$F^{ji} = g^{jr} F^i_r$$

$$F^{ij} = - F^{ji}$$

Let R_{ji} be the Ricci tensor and $R^h{}_{kji}$ be the Riemann curvature tensor. Then, we have the following relations:

$$R = R_{ji} g^{ji}$$

$$R_{kjih} = R^r{}_{kji} g_{rh}$$

$$H_{ij} = (1/2) R_{ijkl} F^{kl}$$

Then the following relations hold [4]:

(1.2) $$H_{ij} = - H_{ji} ,$$

(1.3) $$R_{ks} F^{s}_{j} = H_{kj} ,$$

(1.4) $$H_{ks} F^{s}_{j} = - R_{kj} ,$$

(1.5) $$H_{kj} F^{kj} = - R ,$$

(1.6) $$\nabla_l H_{kj} + \nabla_k H_{jl} + \nabla_j H_{lk} = 0 .$$

2. CONFORMALLY RECURRENT KAHLERIAN MANIFOLDS :

Definition 2.1 :

A 2n - dimensional ($n \neq 1,2$) Kaehler space which satisfies the relation

(2.1) $$\nabla_l C^{h}_{kji} = \lambda_l C^{h}_{kji}$$

Wherein λ_l is a non - zero vector is called recurrence vector and C^{h}_{kji} is the conformal curvature tensor and ∇_l denotes covariant differentiation with regard to the Riemannian metric of the space. Such a space is called a conformally recurrent Kaehler space [2].

We have the following relation [2]

(2.2) $$\nabla_l C_{kjih} = \lambda_l C_{kjih}$$

wherein

$$C^{ijkh} = C^{i\,jk}_{r} g^{rh}$$

(2.3) $$C^{ijkh} = R^{ijkh} + g^{ih} L^{jk} + g^{jk} L^{ih}$$
$$- g^{jh} L^{ik} - g^{ik} L^{jh}$$

(2.4) $$L_{ji} = -\{1/2(n-1)\} R_{ji} + \{1/4(n-1)(2n-1)\} R g_{ji}$$

Equation (2.2) in covariant form can be written as

(2.5) $$\nabla_l R_{kjih} + g_{kh}\nabla_l L_{ji} - g_{jh}\nabla_l L_{ki} + g_{ji}\nabla_l L_{kh} - g_{ki}\nabla_l L_{jh}$$
$$= \lambda_l [R_{kjih} + g_{kh} L_{ji} - g_{jh} L_{ki} + g_{ji} L_{kh} - g_{ki} L_{jh}]$$

Transvecting equation (2.5) with F^{ih} yields

(2.6) $$\nabla_l [H_{kj} + \{1/(n-1)\}H_{jk}] + \{1/2(n-1)(2n-1)\}F_{jk}\nabla_l R = \lambda_l [H_{kj} + \{1/(n-1)\} H_{jk} + \{1/2(n-1)(2n-1)\}R\, F_{jk}]$$

Next, transvecting equation (2.6) with F^{kj} and using the equation (1.5), we get

(2.7) $$(\nabla_l R - \lambda_l R)\, [F_{kj} F^{kj} + 2\,(2n-1)(n-2)] = 0$$

wherein

(2.8) $$\nabla_l R = \lambda_l R$$

Inserting equation (2.8) into equation (2.6), we obtain

(2.9) $$\nabla_l H_{kj} = \lambda_l H_{kj}$$

From equation (1.3), we get

$$F^s_j \nabla_l R_{ks} = \nabla_l H_{kj} = \lambda_l H_{kj}$$

Hence

$$F^j_m F^s_j \nabla_l R_{ks} = \lambda_l H_{kj} F^j_m,$$

From this it follows that

(2.10) $$\nabla_l R_{km} = \lambda_l R_{km}$$

By virtue of equation (2.4), we obtain

(2.11) $$\nabla_l L_{ji} = \lambda_l L_{ji}$$

From equations (2.11) and (2.5), we obtain

(2.12) $$\nabla_l R_{kjih} = \lambda_l R_{kjih}$$

In this regard, we have

(2.13) $$R_{kjih} R^{kjih} = R^2$$

Remark 2.1 :

It is noteworthy that if we take $R = 0$, then we get $R_{kjih} = 0$, i.e. the space is flat.

Theorem 2.1 :

In a Kahler space, the scalar curvature is zero and different from zero if a conformally recurrent is flat and a simple recurrent one.

Taking co-variant derivative of equation (2.4) with respect to x^m, we get

$$\nabla_m L^{jk} = \nabla_m [-\{1/2(n-1)\} R^{jk} + \{1/4(n-1)(2n-1)\} R g^{jk}]$$

i.e.

(2.14) $$\nabla_m L^{jk} = -\{1/2(n-1)\} \nabla_m R^{jk} + \{1/4(n-1)(2n-1)\} g^{jk} \nabla_m R$$

Inserting equation (2.8) into equation (2.14), we obtain

(2.15) $$\nabla_m L^{jk} = -\{1/2(n-1)\} \lambda_m [R^{jk} - \{1/2(2n-1)\} R g^{jk}]$$

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