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A non-Linear Optimization Approach to the Identification of Unit Hydrograph Discrete Kernels

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Abstract

The process of identification of discrete kernels of unit hydrographs is an important step in its application to flood forecast. The process of discrete kernels estimation of unit hydrographs has several interesting features, like the overestimation condition that appears when there are more equations than variables to be estimated, this condition often produces negative ordinates of the unit hydrograph and sometimes produces changing values that give oscillating values on the unit hydrograph. In order to avoid such undesirable outcomes, it is proposed in the paper to use the well-known Rosenbrock´s non-linear multi-variable optimization procedure in the estimation phase of the ordinates of unit hydrographs coupled with an objective function that minimize the sum of the squared errors between the forecasted and actual direct runoff hydrographs. Two examples of application are contained in the paper and through them the applicability and the goodness of fit of the proposed methodology is depicted.

1. Introduction

The spread use of the unit hydrograph (UH), as an essential tool for real-time river flows forecasting models, and due to the generalized use of computers to perform the operations required by the phase of identification of the UH ordinates, in order to produce forecasted hydrographs, the process of identification becomes to be an important step of the whole process of real-time hydrograph forecasting. Often the identification phase of UH ordinates produces undesirable outcomes, like negative values and oscillations in the UH ordinates, which even though they mathematically correct they are unrealistic form the point of view of hydrology and are difficult to explain from the physical basis of the phenomenon of streamflow.

Some efforts have been made to avoid such problems, the use of optimization to obtain UH ordinates for flood forecasting has been proposed, [1]. Two approaches to identify UH ordinates, based in linear programming, namely MINISAD, in which the sum of absolute deviations is minimized, and MINIMAD, which minimize the maximum absolute deviation, were proposed [2]. The application of a non-linear optimization procedure to analyze the impact of several forms of the objective function in the identification phase of the UH ordinates, has been proposed, [3]. The use of quadratic programming to stabilize the UH ordinates via the difference norms and they found an advantage over the standard ridge regression, where the penalties are placed on oscillations of the UH ordinates rather than on the size of its ordinates, has been proposed, [4]. The use of discrete time-kernels to route discrete-time inflow hydrographs, has been proposed, [5].

2. Discrete kernels of unit hydrograph ordinates identification through non-Linear optimization

The discrete form of the convolution between effective rainfall and direct runoff is, [6]:

$$q(n)=\sum_{\nu=1}^{n}\delta(n-\nu+1)\,r(\nu) \qquad (1)$$

for n = 1, 2, …

where:

q(n) is the direct runoff hydrograph ordinate at time n

δ(.) is the UH discrete kernel

r(.) is the mean effective rainfall rate

In equation (1), q(.) and r(.) must have consistent units given that δ(.) are dimensionless. From eq. (1), the error river forecast, defined as e(n), is, [6]:

$$e(n)=q^{0}(n)-\sum_{\nu=1}^{n}\delta(n-\nu+1)\,r(\nu) \qquad (2)$$

for n = 1, 2, …

and the mean squared error of the river forecast is, [6]:

$$MSE(e)=\left[\sum_{n=1}\frac{e^{2}(n)}{N}\right]^{1/2} \qquad (3)$$

where:

MSE(e) is the mean squared error of the river forecast

N is the number of ordinates of the direct runoff hydrograph

Now, in order to set properly the optimization problem to be solved, the following objective function is used:

$$\min_{\underline{\delta}}(MSE(e))=\min_{\underline{\delta}}\left[\sum_{n=1}^{N}\frac{e^{2}(n)}{N}\right]^{1/2} \qquad (4)$$

subject to the following constraints:

a) Non-negativity constraints:

$$\delta\geq 0 \qquad (5)$$

for I = 1, …, M

where M is the memory time

a) Ordering constraints:

$$\delta(1)\leq\delta(2)\leq\ldots\leq\delta(p)$$
$$\delta(p)\geq\delta(p+1)\geq\ldots\geq\delta(N) \qquad (6)$$

where:

δ(p) is the ordinate which corresponds to the peak of the UH

The optimization part of the problem to identify the discrete kernels of the UH, has been carried out through the well-known non-linear optimization Rosenbrock´s method for constrained multiple variables, [7]. This procedure is named MINIMSE herein.

3. Results and discussion

The proposed methodology was applied to two different cases, in the first place the proposed methodology was applied to the data contained in [2]. The resulting discrete kernels for the unit hydrograph are shown in table 1. These results are depicted in figure 1. In here, it is easy to see that the least appropriate approach is that of MINISAD, those of MINIMAD and MINIMSE produce similar discrete kernels unit hydrographs.

The values of the objective functions for the approaches MINISAD, MINIMAD and MINIMSE are contained in table 2. With regard to the objective functions, when the sum of absolute deviations (SAD) is minimized, the MINISAD produces the best value and MINIMAD the worst, being MINIMSE almost in the middle of such values. When the maximum absolute deviation (MAD) is under consideration, the MINIMAD produces the best value but the MINIMSE approach is very close to this value and the MINISAD approach provides the worst value. In the case of the mean squared error (MSE), the MINIMSE method produced the best value followed, not very close, by those of MINISAD and MINIMAD.

The corresponding direct runoff hydrograph produced by the approaches MINISAD, MINIMAD and MINIMSE are shown in table 2 and depicted in figure 2. In this case, the direct runoff produced by MINIMAD and MINIMSE follow closely the actual direct runoff hydrograph, being the MINISAD the worst solution.

So, the MINIMSE produces better overall solutions than the ones provided by the approaches of MINISAD and MINIMAD, for this first case considered in this paper.

Table 1. Discrete kernels for the approaches MINISAD, MINIMAD and MINIMSE

Time (min)	MINISAD	MINIMAD	MINIMSE
0	0	0	0
1	0.1	0.05	0.05
2	0.17	0.118	0.114
3	0.19	0.208	0.206
4	0.16	0.208	0.202
5	0.05	0.014	0.071
6	0.01	0.014	0.011
7	0.01	0.014	0.009
8	0.01	0.014	0.006
9	0.01	0.014	0.003
10	0.01	0.014	0.003
11	0.01	0.014	0.003

Table 2. Objective functions of MINISAD, MINIMAD and MINIMSE

Method	Criterion		
	MINISAD	MINIMAD	MINIMSE
SAD	12.9	14.83	14.00
MAD	2.62	1.44	1.46
MSE	0.90	0.87	0.75

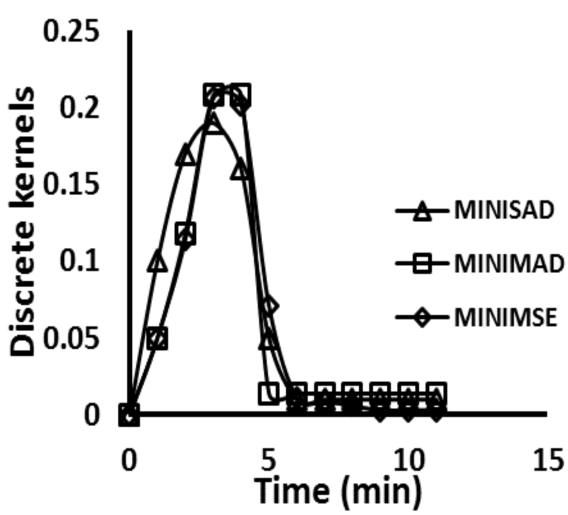

Figure 1. Unit hydrographs produces by the approaches MINISAD, MINIMAD and MINIMSE

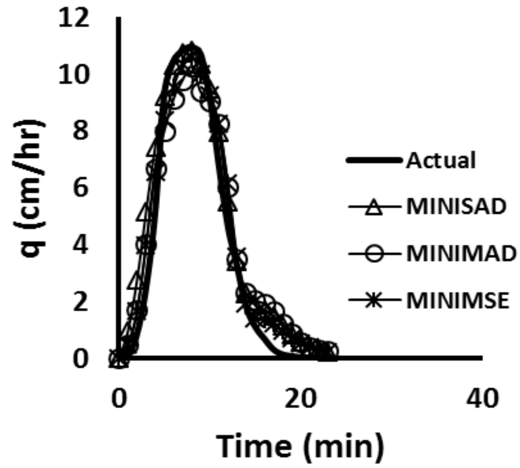

Figure 2. Actual direct runoff and the hydrographs produced by the approaches MINISAD, MINIMAD and MINIMSE

Table 3. Direct runoff hydrographs for the actual hydrograph and for the approaches MINISAD, MINIMAD and MINIMSE

(1)	(2)	(3)	(4)	(5)	(6)
0	0	0	0	0	0
1	9.65	0.13	0.97	0.48	0.48
2	11.18	0.88	2.76	1.7	1.66
3	14.48	2.52	5.18	4.05	3.99
4	13.21	5.16	7.45	6.7	6.56
5	19.56	9.32	9.22	8.01	8.41
6	15.24	10.45	10.33	9.12	9.54
7	13.72	10.83	10.73	9.79	10.36
8	14.48	10.96	10.82	10.26	10.63
9	13.97	10.71	10.37	9.38	10.01
10	7.37	9.19	9.5	9.05	9.28
11	2.03	6.8	7.95	8.25	8.25
12	1.02	4.66	5.51	6.04	6.17
13	1.52	3.27	3.4	3.51	3.61
14	2.14	1.84	2.28	2.31	1.93
15	1.51	1.13	1.89	2.06	1.42
16	1.13	0.71	1.73	1.93	1.35
17	0.88	0.33	1.5	1.68	1.22
18	0.63	0.15	1.17	1.26	0.97
19	0.5	0.1	0.85	0.91	0.73
20	0.38	0.05	0.58	0.6	0.54
21	0.25	0.03	0.41	0.41	0.4
22	0.13	0	0.32	0.34	0.3
23	0.08	0	0.24	0.27	0.22

(1) Time (min)

(2) Mean effective rainfall (cm/hr)
(3) Actual direct runoff hydrograph (cm/hr)
(4) MINISAD
(5) MINIMAD
(6) MINIMSE

As a second example of application, the MINIMSE methodology was applied to an actual set of data recorded at gauging station San Bernardo in Northwestern Mexico. The data and results are contained in table 4 and graphically depicted in figure 3.

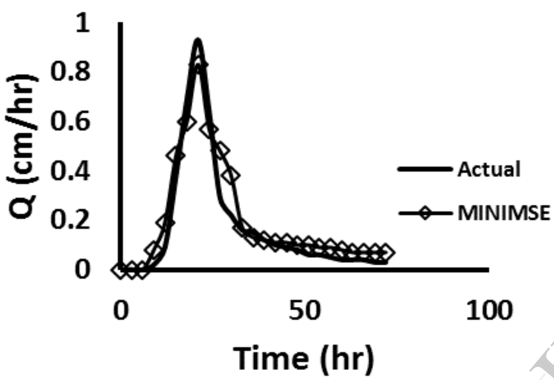

Figure 3. Actual direct runoff and the hydrograph produced by the approach MINIMSE

As it may been observed, from table 4 and figure 3, the proposed methodology works very well with an actual set of data, the peak of the hydrograph is well reproduced and the time to peak, as well.
The application of the proposed approach is restricted to the fact that the computer code for the Rosenbrock's constrained multivariable method must be available, given that performing the required computations for such method without a computer code is just out of the question.

4. Conclusions

A procedure to identify the discrete kernels for the UH was presented, based in a non-linear optimization technique known as the Rosenbrock´s method for multiple constrained variables. The proposed methodology has some nice features like the easiness on problem formulation and computer code design. These characteristics aids the application of the procedure in real-time flood forecasting situations. The proposed procedure has consistency in reaching better results when compared with existing schemes in the literature.

Table 4. Direct runoff hydrographs for the actual hydrograph and for the approach MINIMSE

(1)	(2)	(3)	(4)	(5)
0	0	0	0	0
3	0.046	0	0.03	0
6	0	0	0.08	0
9	2.18	0.02	0.19	0.08
12	0	0.09	0.21	0.19
15	1.42	0.38	0.22	0.46
18	1.08	0.69	0.03	0.6
21	0.18	0.93	0.03	0.83
24	0	0.64	0.03	0.57
27		0.29	0.03	0.48
30		0.22	0.02	0.38
33		0.16	0.02	0.17
36		0.15	0.02	0.13
39		0.12		0.12
42		0.1		0.11
45		0.09		0.11
48		0.08		0.1
51		0.06		0.1
54		0.06		0.09
57		0.05		0.09
60		0.04		0.08
63		0.04		0.07
66		0.04		0.07
69		0.03		0.07
72		0.03		0.07

(1) Time (hr)
(2) Mean effective rainfall (cm/hr)
(3) Actual direct runoff hydrograph (cm/hr)
(4) MINIMSE discrete kernels unit hydrograph
(5) MINIMSE direct runoff hydrograph (cm/hr)

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