

# A New Time-Varying Convergence Parameter For The LMS Adaptive Filtering Algorithm

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**Abstract - A number of variable step size LMS algorithms are used to improve the performance of conventional LMS algorithm. This paper proposes a new Time varying LMS (NTVLMS) algorithm by making use of a nonlinear function to get the variable step size parameter  $\mu$ . The proposed algorithm is applied to adaptive noise cancellation system. The simulation results show that the performance is better compared to conventional LMS and TVLMS algorithms.**

**Keywords** - Adaptive filter, Convergence rate, LMS algorithm, New Time-varying LMS algorithm (NTVLMS), Time-varying LMS algorithm (TVLMS), Steady state misadjustment.

## I. Introduction

The least-mean-square (LMS) algorithm was developed by Widrow[1][2]. It is still used in adaptive signal processing for its simplicity, less computation, ease of implementation and good convergence property. The LMS algorithm is described by the equation:

$$e(n) = d(n) - X^T(n)W(n) \quad \dots \dots \dots (1)$$

$$w(n+1) = w(n) + 2\mu(n)e(n)X(n) \quad \dots \dots \dots (2)$$

Where  $w(k)$  is filter coefficient at time  $k$ ,  $\mu(k)$  is step size,  $e(k)$  is adaptation error and  $x(k)$  are the filter input respectively. Equation (1) shows that LMS algorithm uses an adaptation error and a variable step size to update the filter coefficients. In this algorithm, the step size parameter  $\mu(n)$  is constant. The choice of this parameter  $\mu$  is very important to the convergence and stability. As in [2], the stability of the convergence of LMS algorithm requires the step size parameter  $\mu$  satisfy the condition

$$0 \leq \mu \leq \frac{1}{tr(R)} \leq \frac{1}{\lambda_{max}} \quad \dots \dots \dots (3)$$

Where  $tr(R)$  is the trace of the autocorrelation matrix of input  $x$  and  $\lambda_{max}$  is the maximum eigenvalue of  $R$ .

In general a smaller step size leads to a small steady state misadjustment (SSM) but a slower convergence rate. While a larger step size

gives faster convergence but a large SSM so, suitably select  $\mu$  based on eqn(3). But it is still far from optimum trade-off between SSM and convergence rate. Then TVLMS algorithm is proposed to solve this problem. The TVLMS algorithm uses a suitable step size in the initial stage to speed up the convergence and the step size is adjusted to a smaller value gradually during the convergence.

In section 2, the TVLMS is discussed and their performance is introduced. In section 3, the new TVLMS algorithm is proposed. In section 4, the proposed algorithm is applied to noise cancellation system and simulation results are presented. Finally draws the conclusion.

## II. TVLMS algorithm

Many variable step size LMS algorithms are used to achieve the optimum convergence rate and SSM performance. In TVLMS algorithm [4]-[5] the step size parameter is found out by using the equation as

$$\mu(n) = \alpha(n) \times \mu_o \quad \dots \dots \dots (4)$$

where  $\mu_o$  is the value of step size parameter in conventional LMS algorithm. This  $\mu_o$  is used to update  $\mu(n)$  in this algorithm.

$\alpha(n)$  is a decaying factor. We will consider the following decaying law:

$$\alpha(n) = C^{1/(1+an^b)} \quad \dots \dots \dots (5)$$

Where  $C$ ,  $a$ ,  $b$  are positive constants that will determine the magnitude and the rate of decrease for  $\alpha(n)$ . According to the above law,  $C$  has to be a positive number larger than 1. When  $C = 1$ ,  $\alpha(n)$  will be equal to 1 and the TVLMS algorithm will be the same as that of conventional LMS algorithm. As  $\alpha(n)$  decreases with respect to time, the convergence parameter  $\mu(n)$  decreases and rate of convergence increases compared to LMS algorithm, but still convergence rate is high and the computational complexity of this algorithm is large.

### III. New TV LMS algorithm

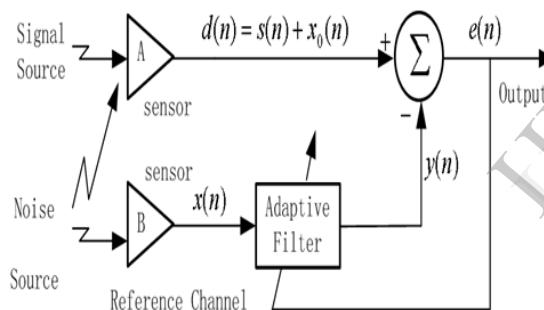
Based on the performance of LMS and TVLMS algorithm, a New TVLMS algorithm is proposed. This algorithm can improve the convergence rate with respect to LMS and TVLMS algorithms and computational complexity is large compared to LMS algorithm and less compared to TVLMS algorithm. The proposed algorithm updates the step size by

$$\mu(n) = d + \mu_0 \tan^{-1}(f n + g) \quad \text{---(6)}$$

where  $f$  and  $g$  are positive constants select suitably.  $\mu_0$  is the value of step size parameter in conventional LMS algorithm. This  $\mu_0$  is used to update  $\mu(n)$  in this algorithm. The proposed algorithm gives good result for additive white Gaussian noise with variable input power as well as variable noise power.

### IV. Application to the adaptive noise cancellation system

The basic adaptive noise cancelling system is shown in fig (1) below



**fig(1) Block diagram of adaptive noise cancellation system**

A signal is transmitted over a channel is received by the receiver with uncorrelated noise  $x_0(n)$ . The signal  $s(n)$  and noise  $x_0(n)$  combined to form the desired signal  $d(n) = s(n) + x_0(n)$ . A second signal input to the adaptive filter is noise  $x(n)$ , which is uncorrelated with the signal but correlated in some unknown way with noise  $x_0(n)$ . The noise  $x(n)$  applied as an input to the adaptive filter to produce an output  $y(n)$  is close enough to the replica of  $x_0(n)$ .

In this system the signal  $x(n)$  is processed by the filter that automatically adjust its weights through the above mentioned algorithms with respect to the error signal  $e(n)$ .

$$y(n) = \sum_{i=0}^{N-1} w(i)x(n-i) = W^T(n)X(n) \quad \text{---(7)}$$

Here,

$$X(n) = [x(n), x(n-1), \dots, x(n-N+1)]^T$$

$$W(n) = [w_0(n), w_1(n), \dots, w_{N-1}(n)]^T$$

Where  $N$  is the order of the filter,  $w_i$  is the filter coefficient.

To implement LMS based adaptive noise cancellation system, it is very important to choose the following parameters, filter order, initial values of the filter coefficients, and value of the step size parameter, etc.

### V. Simulation Results

The simulation results of the three algorithms are given as follows.

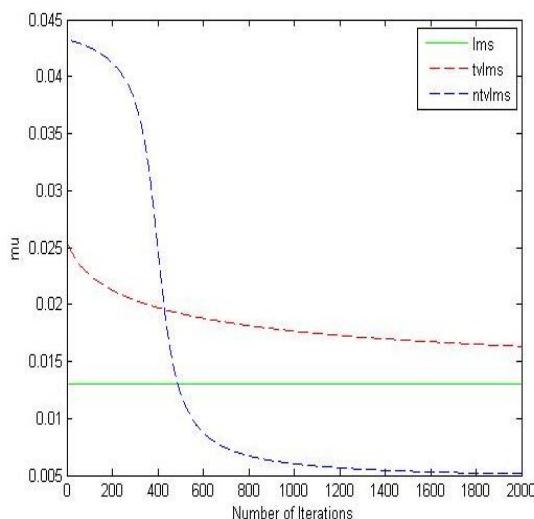
The condition of simulation is:

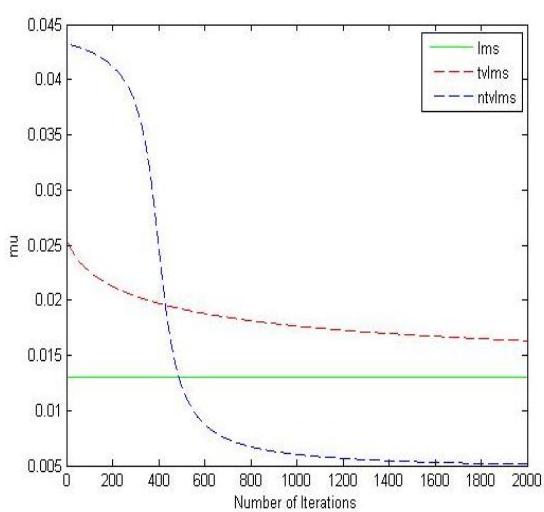
1. The order of the adaptive filter is 2
2. The input signal is continuous wave of sin signal.
3. The noise is a white Gaussian noise
4. The amplitude of the sin signal is 0.01 and 0.2, corresponding to SNR -43 dB and -17 dB.
5. The algorithm was implemented for 200 iterations and 2000 iterations for each case.

#### a. Dynamic range comparison with LMS and TVLMS algorithm

From fig (2), the proposed algorithm has faster convergence rate in case of both high and low signal-to-noise ratio (SNR).

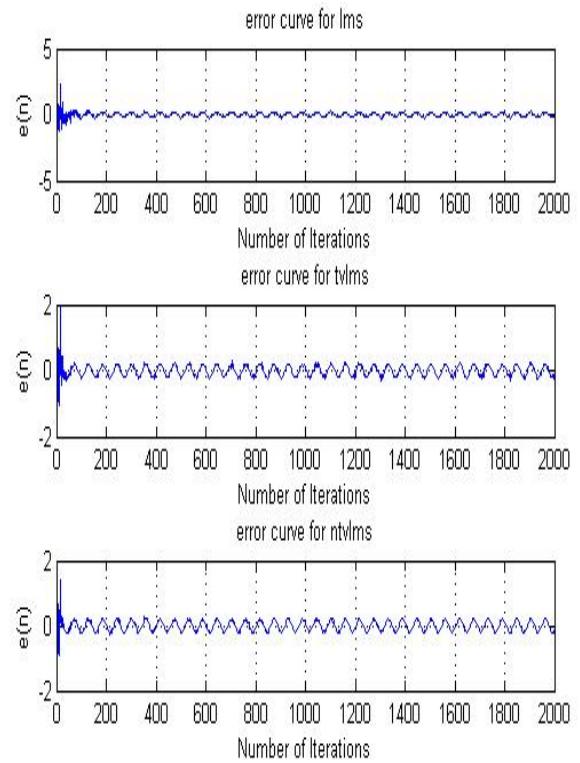
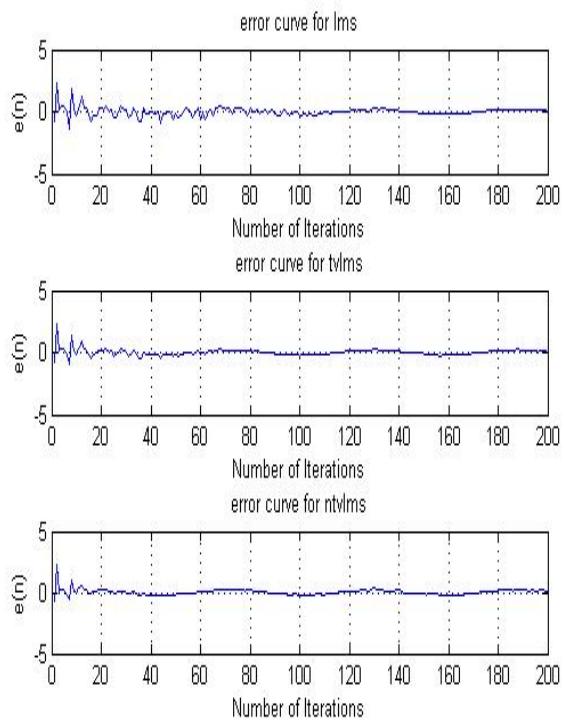
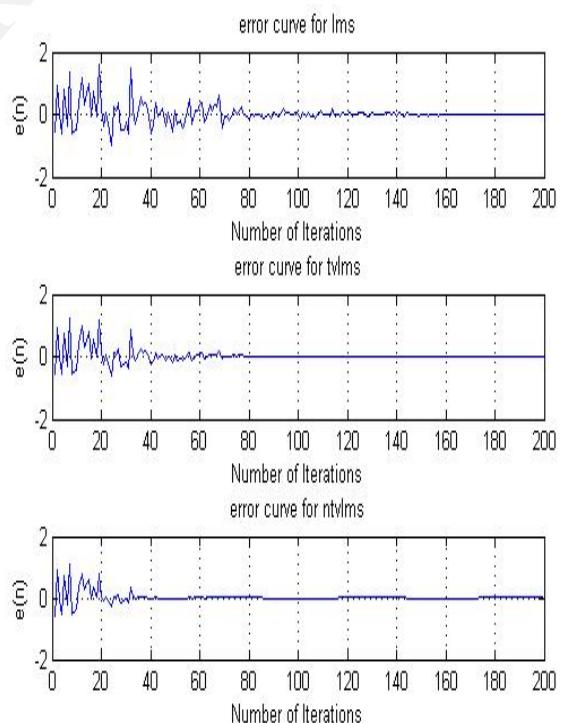
#### Signal amplitude of 0.2

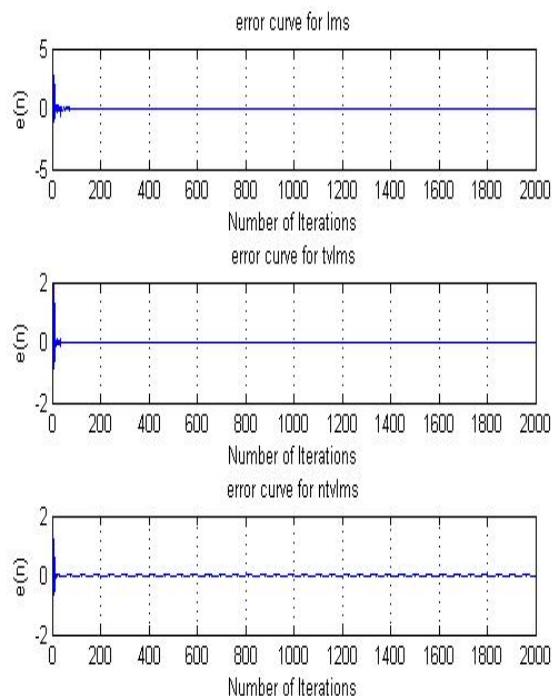


**Signal amplitude of 0.01****Fig(2) The variable step size curve****b. Performance comparison with LMS and TVLMS algorithm**

From fig (3) and fig (4),

1. The proposed algorithm has faster convergence rate
2. The steady error of the proposed algorithm is less compared to LMS and TVLMS algorithms in case of both low signal-to-noise ratios (SNR) and high signal-to-noise ratios (SNR).

**Output waveform****Fig(3) sine signal of amplitude 0.2**



**Fig(4) sine signal of amplitude 0.01**

## VI. Conclusions

The proposed New TVLMS algorithm shows improved performance over the conventional LMS and TVLMS algorithms under white Gaussian noise environment. This algorithm is optimum for specified values of the constant parameters  $d$ ,  $f$ ,  $g$  and  $\mu_0$ . The concept of this paper is useful to the implementation of Radar communication system for the cancellation of jamming signal, interference or noise cancellation in case of Digital communication and satellite communication system and it is useful to other automation fields.

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