

# A New Particle Swarm with Center of Mass Optimization

Razan A. Jamous

Department of Mathematics - Faculty of Science –  
Ain Shams University.  
Cairo- Egypt

Essam El. Seidy

Assistant Professor of Pure Mathematics –  
Department of Mathematics - Faculty of Science -  
Ain Shams University.  
Cairo- Egypt

Assem A. Tharwat

Professor of Operations Research –  
Department of Operations Research and Decision Support –  
Faculty of Computer and Information –  
Cairo University. Cairo- Egypt

Bayoumi Ibrahim Bayoumi

Professor of Pure Mathematics –  
Department of Mathematics - Faculty of Science –  
Ain Shams University.  
Cairo- Egypt

**Abstract—** This work deals with a new modification of Particle Swarm with Center of Mass Optimization, Which We denoted by (PSOCM). This modification gives a new efficient search technique. It gets benefit from the physical principle “center of mass” to move the particles to the new best-predicted position. The new proposed technique improves the performance of current PSO technique. To evaluate the proposed algorithm (PSOCM) we compare it with two existed versions of PSO techniques, Center Particle Swarm Optimization (Center PSO) and Linear Decreasing Weight particle swarm optimization (LDWPSO) algorithms, the experimental results show that the PSOCM overcome Center PSO and LDWPSO in term of convergence rate, complexity, and scalability.

**Keywords—** Computational intelligence; Particle Swarm Optimization; Local; Global, and Center of Mass.

## I. INTRODUCTION

Kennedy and Eberhart introduced particle Swarm Optimization (PSO) in 1995 as a stochastic optimization algorithm based on social simulation model [1].

The research in PSO has resulted in a large number of new PSO algorithms that improves the performance of the original PSO and enables application of PSO to different optimization problem types (e.g., unconstrained optimization, optimization in dynamic environments, constrained optimization, multi-objective optimization and finding multiple solutions). Elaborate theoretical studies of PSO dynamics have been done, and PSO parameter sensitivity analyses have resulted in a better understanding of the influence of PSO control parameters. PSO applications vary in complexity and cover a wide range of application areas. The PSO algorithm simulates the behaviors of bird flocking, the flight of a bird flock can be simulated with relative accuracy by simply maintaining a target distance between each bird and its immediate neighbors. This distance may depend on its size and desirable behavior. Therefore in PSO, each single solution

(a bird in the search space) is called a particle, and each particle has fitness value which is evaluated by the objective function to be optimized, and has a velocity which directs the flying of the particle. All particles fly through the problem space by following the current optimum particle.

Many research papers have appeared in the literature using particle swarm optimization (PSO). A number of basic modifications to the basic PSO have been developed to improve speed of convergence and the quality of solutions found by the PSO. These modifications include the introduction of an inertia weight, velocity clamping, and velocity constriction.

The following description of the PSO algorithm is adapted from [2]. In PSO, a swarm consists of  $N$  particles moving around in a  $D$ -dimensional searching space. Let  $X_i(t) = (x_{i1}, x_{i2}, \dots, x_{id})$  denote the position of particle  $i$  in the search space at time step  $t$ ,  $V_i(t) = (v_{i1}, v_{i2}, \dots, v_{id})$  denote the velocity particle  $i$  in the search space at time step  $t$ ,  $P_i = (p_{i1}, p_{i2}, \dots, p_{id})$  denote the best solution achieved so far by the particle itself,  $P_g = (p_{g1}, p_{g2}, \dots, p_{gd})$  denote the best solution achieved so far by the whole swarm. Adding a velocity to the current position, as follows, changes the new position of the particle:

$$X_{id}^{(t+1)} = X_{id}^{(t)} + V_{id}^{(t+1)} \quad (1)$$

$$V_{id}^{(t+1)} = w \cdot V_{id}^{(t)} + c_1 r_1 (P_{id} - X_{id}^{(t)}) + c_2 r_2 (P_{gd} - X_{id}^{(t)}) \quad (2)$$

Where  $c_1$  and  $c_2$  are two positive constants,  $r_1$  and  $r_2$  are two random numbers in the range  $[0, 1]$ ;  $w$  is the inertia weight. The velocity vector drives the optimization process, and reflects both the experiential knowledge of the particle and socially exchanged information from the particle's neighborhood. The experiential knowledge of a particle is generally referred to as the cognitive component, which is proportional to the distance of the particle from its own best position (referred to as  $p_{besti}$ ). The socially exchanged information is referred to as the social component of the velocity equation (2), which is

proportional to the distance of the particle from the best position found by the swarm (referred to as gbest).

Linear Decreasing Weight particle swarm optimization (LDWPSO) algorithm was presented by Shi and Eberhart [3]. The inertia weight  $w$  is decreased linearly over the searching iterations, from an initial value to a final value as follows:

$$w = (w_{\max} - w_{\min}) \times \frac{(\text{Max.Iter} - \text{Iter})}{\text{Max.Iter}} + w_{\max} \quad (3)$$

Where  $w$  is the inertia weight that controls the velocity of particles,  $w_{\max}$  is the initial inertia weight,  $w_{\min}$  is the final inertia weight, Max.Iter is the maximum number of iterations, and Iter is the current iteration. LDWPSO algorithm uses equation (1) to update position, equation (2) to update velocity and equation (3) to update the inertia weight. Center particle swarm optimization algorithm (Center PSO) was presented by [2] based on introducing a center particle to the LDWPSO algorithm proposed where a center particle is incorporated into linearly decreasing weight particle swarm optimization (LDWPSO). Unlike other ordinary particles in LDWPSO, the center particle has no explicit velocity, and is set to the center of the swarm at every iteration. But it is involved in all operations the same as the ordinary particle, such as fitness evaluation, competition for the best particle, except for the velocity calculation. The center particle has opportunities to become the gbest of the swarm. After  $N-1$  particles update their positions as the usual PSO algorithms at every iteration, a center particle is updated according the following formula:

$$X_{cd}^{(t+1)} = \frac{1}{N-1} \sum_{i=1}^{N-1} X_{id}^{(t+1)} \quad (4)$$

We will compare our proposed algorithm with these methods.

Section 2 of this work gives the proposed algorithm. In section 3, the evaluation of the proposed algorithm is presented. Finally, in Section 4 we conclusion this paper by the summary of main points.

## II. THE PROPOSED TECHNIQUE

### A. Definitions

System of point masses is that system consisting of  $N$  individual point masses  $1, \dots, N$ . Their motion is described by specifying their position vectors  $\vec{r}_1, \dots, \vec{r}_N$  as function of time  $t$ :  $\vec{r}_i(t)$ , where  $i = 1, \dots, N$ . Center of Gravity or Center of Mass, is a point in a system of point masses at which the position vector  $\vec{R}$  is calculated using the masses  $m_i$  and position vectors  $\vec{r}_i$  as follows:

$$\vec{R} = \frac{1}{M} \sum_{i=1}^N m_i \times \vec{r}_i \quad (5)$$

$$M = \sum_{i=1}^N m_i \quad (6)$$

Where  $M$  is the summation of masses and  $N$  is the number of masses. The terms "Center of Mass" and "Center of Gravity" are equivalently used in a uniform gravity field to represent a unique point in an object or system, which can be used to describe the system's response to external forces and torques. Center of Mass can also be defined as an average of masses factored by their distances from reference point [4].

### B. Assumptions

Back to the particle swarm optimization algorithm and in particular to the equation of velocity, which controls the movement of the particles using the main parameters (gbest, lbest, acceleration coefficients and inertia weight), a new effective center of mass particle  $X_{cm}$  is proposed. This particle will contribute to accelerating the convergence of the algorithm in quite less number of iterations. It can also help to enhance the quality of the solution (by finding closer solution to the optimal). Unlike the center particle in Center PSO algorithm [2], the center of mass particle is virtual particle represent to the swarm at every iteration, and weighted by the fitness values of the particles that form the swarm, it has no velocity and doesn't involved in all operations of the ordinary particle, such as fitness evaluation, competition for the best particle. By considering the particle swarm as a system of point masses, and make the value of the objective function of each particle meets the mass, then the weighted center of swarm can be calculated as follows:

$$X_{cm}(t) = \frac{1}{F_{cm}} \sum_{i=1}^N F(x_i) \times x_i(t) \quad (7)$$

Where  $F(x_i)$  is the fitness values at position  $x_i(t)$  of particle  $i$ , and  $F_{cm}$  is the summation of particles' fitness values. By analogy with the summation of masses  $M$  in equation (8),  $F_{cm}$  can be calculated using equation (10). The fitness value  $F(x_i)$  is considered to be the same value as the used objective function  $f(x_i)$  when the algorithm searches for maximum optima, and to be the inverse of the objective function value searching for minimum optima as in equations (11) and (12).

$$F_{cm} = \sum_{i=1}^N F(x_i) \quad (8)$$

$$F(x_i) = f(x_i), \text{ when maximizing an objective function} \quad (9)$$

$$F(x_i) = \frac{1}{f(x_i) + \epsilon}, \text{ when minimizing an objective function,} \\ \epsilon \cong 0 \quad (10)$$

The range of the objective function assumed to be non-negative in this case

### C. Velocity Update Function

By calculating the center of swarm  $X_{cm}$ , the best position over all the swarm  $P_g$  (gbest), and the best position discovered by each particle  $P_i$  (pbest), the new velocity update equation of each particle can be produced using the following formula:

$$v_{id}^{(t+1)} = wv_{id}^{(t)} + c_1 r_1 \left( \frac{P_{id}^{(t)} + X_{cm}^{(t)}}{2} - X_{id}^{(t)} \right) + c_2 r_2 \left( \frac{P_{gd}^{(t)} - X_{cm}^{(t)}}{2} - X_{id}^{(t)} \right) \quad (11)$$

$$X_{id}^{(t+1)} = v_{id}^{(t+1)} + X_{id}^{(t)} \quad (12)$$

Consequently, the new position of each particle is updated using equation (14), and the searching process continues in progress until a stopping criterion is met. It must be noted that, if the inertia weight  $w$  in equation (13) was decreasing linearly, the resulted technique will be PSOCM, and if the inertia weight was fixed, the resulted technique will be PSOCM1. A roughly comparative visualization between the movement of particles in both PSOCM technique and SPSO algorithm is shown in Figure1.

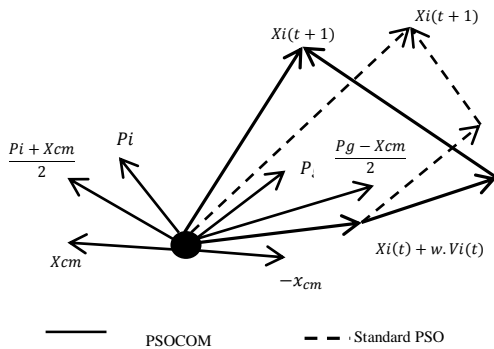


Fig 1. Comparative movement of a particle in SPSO and PSOCM.

#### D. Description of Particle Swarm with Center of Mass Technique

As said previously, the main difference between our proposed technique PSOCM and Center PSO algorithm is that the Center PSO used average of particles' positions to calculate the center particle. All particles except center update their velocities and positions using equations of the standard PSO. On the other hand, in our PSOCM, center of mass particle is calculated using weighted average of particles' positions as given in equation (3), and doesn't involved in all operations of the ordinary particle. The philosophy behind that is to make such a center of mass particle X<sub>cm</sub> effects on the attraction of particles towards the global optima, and helps them to avoid the local optima. This can be explained by clarifying the role of the second and third terms in equation (7). The second term  $\left(\frac{P_{id}^{(t)} + X_{cm}^{(t)}}{2} - X_{id}^{(t)}\right)$  is responsible for the attraction of the particle's current position towards the mean of the positive direction of its own best position (pbest) and the positive direction of the center of mass particle's position (X<sub>cm</sub>), which helps the cognitive behavior component to avoid the local optima. On the other hand, the third term  $\left(\frac{P_{gd}^{(t)} - X_{cm}^{(t)}}{2} - X_{id}^{(t)}\right)$  is responsible for the attraction of the particle's current position towards the mean of the positive direction of the global best position (gbest) and the negative direction of the center of mass particle's position (-X<sub>cm</sub>), which helps maintaining the diversity of the swarm during the searching process. This increases the opportunity of fast convergence to global (or near global optima), where the center of mass particle will attract particles to the region of best found solutions, that gives particles the best chance to occupy the position of global best found solution during the search process, all previous movement are supported by linearly decreasing weight which give the balance between exploration and exploitation during the search process.

Pseudo Code of the PSO with Center of Mass Algorithm

The following pseudo-code explains the whole process of PSOCM:

Algorithm	The proposed (PSOCM) algorithm
01:	begin
02:	Randomly initialize particles swarm
03:	while (stopping criterion is not met)
04:	for i=1 the swarm size
05:	Compute fitness of the particle swarm
06:	Find local best P <sub>i</sub> and global best P <sub>g</sub>
07:	Calculate center of swarm X <sub>cm</sub> by equation (3)
08:	for d=1 the problem dimensionality
09:	Update particle velocity using equation (7)
10:	Update particle position using equation (8)
11:	end
12:	end
13:	update the inertia weight value by equation (3)
14:	end-while
15:	end-algorithm

#### Evaluation the performance of PSOCM Technique

The benchmark test functions are problems with varying difficulty levels and problem size. Those problems will be solved by the proposed PSOCM technique and the other versions of particle swarm optimization algorithms, namely, SPSO, LDWPSO, Center PSO and Mean PSO. Problem Set consists of four scalable problems, namely, Rosenbrock, Rastrigrin, Griewank and Ackely function, the dimension of those problems can be increased/decreased, so the complexity of those problem increases as the problem size is increased [5].

**Rosenbrock:** A uni-modal function, with significant interaction between the variables. Its global minimum equal to zero located at (1, 1, ..., 1), so there are n design variables with lower and upper limits of [-100, 100].

$$f_1(x) = \sum_{i=1}^{n-1} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2) \quad (13)$$

**Rastrigrin:** A multi-modal version of Spherical function, characterized by deep local minima arranged as sinusoidal bumps, there are n design variables with lower and upper limits of [-10, 10], its global minimum equal to zero at (0, 0, ..., 0).

$$f_2(x) = \sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i) + 10) \quad (14)$$

**Griewank:** A multi-modal function with significant interaction between its variables caused by the product term, there are n design variables with lower and upper limits of [-600, 600], its global minimum equal to zero at (0, 0, ..., 0).

$$f_3(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \quad (15)$$

**Ackley:** A multi-modal function with deep local minima, there are n design variables with the lower and upper limits of [-30, 30], its global minimum equal to zero at (0, 0, ..., 0).

$$f_4(x) = -20 e^{-0.02 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}} - e^{\left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)\right)} + 20 + e \quad (16)$$

#### • Parameter Settings

The evaluation of proposed PSOCM performance against the performance of PSO versions was performed by three comparisons. Firstly, the same set of parameters was

applied to the three tested algorithms, namely, LDWPSO, Center PSO and the proposed PSOCM to solve the first four benchmark test functions. Inertia weight  $w$  was linearly decreased from 0.9 to 0.4; acceleration coefficients were set to  $c1 = c2 = 2$ ; the maximum velocity was set to  $V_{max} = X_{max}$ . The complexity was investigated for three cases of the first four problems dimensionality ( $D = 10, 20, 30$ ), correspondingly, the maximum number of iterations was set to ( $Iter_{max} = 1000, 1500, 2000$ ). Initialization range for particles' position was set to  $15 \leq x_i \leq 30$ ,  $2.56 \leq x_i \leq 5.12$ ,  $300 \leq x_i \leq 600$  and  $15 \leq x_i \leq 30$  for  $f1(x)$ ,  $f2(x)$ ,  $f3(x)$  and  $f4(x)$  respectively. The scalability of algorithms was investigated using four population sizes ( $N = 20, 40, 80, 160$ ) for each function with different dimensionality; all solutions were computed over 100 runs.

Standard deviation (SD) is a widely used measurement of variability or diversity used in statistics and probability theory. A low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data is spread out over a large range of values. Technically, the standard deviation of a statistical population, data set, or probability distribution is the square root of its variance and calculated as follows:

$$SD = \sqrt{\sigma} = \sqrt{\frac{\sum_{i=1}^N (x_i - \mu)^2}{N}} \quad (17)$$

Where  $\sigma$  is the variance,  $N$  is the number of data points  $x_i$ , and  $\mu$  is the mean (average) of data points  $x_i$ .

#### Analysis of the Results

To evaluate the performance of the proposed technique, the comparisons of the proposed PSOCM technique with LDWPSO and Center PSO are performed in this experiment. All experiments performed on Intel core-i3 1.8 GHz laptop with 4 GB of RAM under WIN7 platform.

#### Comparison of PSOCM to LDWPSO and Center PSO

It should be noted that, according to the used precision of numeric data types, the numeric value is considered equals zero if it was less than  $5.0 \times 10^{-324}$ , and it will be displayed to  $0.0000 \pm 0.0000$  [6]. Tables 1, 2, 3 and 4 list the mean fitness value and standard deviation of the solutions averaged over 100 runs for Rosenbrock, Rastrigin, Griewank and Ackley functions respectively. As a result from these tables, it is observed that the proposed PSOCM superiors the LDWPSO and Center PSO for all the four test benchmark problems by influence of swarm size scalability, dimension complexity and the convergence rate (speed), it gives the optimal solution accurately for Rastrigin and Griewank functions, and more close to optimal for Ackley function. For scalability, it can be seen that as the swarm size increases the average minimum value by LDWPSO and Center PSO decreases and become close to the optimal, with opportunity for Center PSO to overcome LDWPSO, but they both are bigger than that of PSOCM. Noticeable, average by PSOCM is the

smallest one and fixed for the test functions except for Rosenbrock, it decreases slowly.

Table I Mean fitness value for Rosenbrock function

Size	D.	Max. I.	LDWPSO	Center PSO	PSOCM
20	10	1000	9.1437±4.9662	9.0801±4.6473	0.0000±0.0000
	20	1500	47.7768±17.6409	47.4496±15.9583	0.0000±0.0000
	30	2000	87.9939±25.6677	126.5994±36.3899	0.0000±0.0000
40	10	1000	5.5618±3.2518	5.8603±3.3086	0.0000±0.0000
	20	1500	26.9677±9.1263	30.9730±9.8311	0.0000±0.0000
	30	2000	52.0860±16.7072	80.9298±24.1957	0.0000±0.0000
80	10	1000	3.7311±1.9312	3.8704±2.3116	0.0000±0.0000
	20	1500	20.0285±7.7322	22.1876±8.4571	0.0000±0.0000
	30	2000	33.2141±9.1025	61.0605±19.7476	0.0000±0.0000
160	10	1000	1.9369±1.1581	2.3282±1.3219	0.0000±0.0000
	20	1500	10.5770±3.9736	14.6657±4.8183	0.0000±0.0000
	30	2000	36.7956±11.5786	39.7013±10.0061	0.0000±0.0000

Table II. Mean fitness value for Rastrigin function.

Size	D.	Max. I.	LDWPSO	Center PSO	PSOCM
20	10	1000	67.3562±118.3036	37.0061±73.9258	8.1585±0.1818
	20	1500	79.3558±100.6168	66.2748±74.3958	18.1835±0.2072
	30	2000	175.8825±322.3206	82.3127±98.1343	28.2233±0.2581
40	10	1000	34.0945±76.6419	23.6052±48.5820	8.1023±0.0279
	20	1500	51.9005±124.0668	43.7853±73.6872	18.1208±0.1053
	30	2000	87.4123±145.7247	66.5193±73.1078	28.1548±0.1729
80	10	1000	14.3379±36.3526	10.5008±27.3195	8.0956±0.0352
	20	1500	47.6202±66.7143	29.4490±42.4971	18.0989±0.0136
	30	2000	59.5880±69.6433	57.5455±67.5122	28.1076±0.0632
160	10	1000	12.3286±35.5062	10.0699±23.3492	8.0911±0.0466
	20	1500	33.5459±51.3636	20.8180±30.9508	18.0928±0.0483
	30	2000	57.2036±83.5046	50.2963±50.2122	28.0932±0.0497



Table III. Mean fitness value for Griewank function.

Size	D.	Max. I.	LDWPSO	Center PSO	PSOCM
20	10	1000	0.5064±0.1804	0.4576±0.1552	4.44E-16±0.0000
	20	1500	0.9905±0.2373	0.8677±0.1954	2.04E-15±1.77E-15
	30	2000	1.3294±0.2671	1.0353±0.1547	3.71E-15±9.64E-16
40	10	1000	0.3759±0.1253	0.3470±0.1114	4.44E-16±0.0000
	20	1500	0.7634±0.2029	0.6615±0.1379	2.26E-15±1.78E-15
	30	2000	1.0762±0.1997	0.7688±0.1189	3.46E-15±1.27E-15
80	10	1000	0.2797±0.0956	0.2513±0.1039	4.44E-16±0.0000
	20	1500	0.5798±0.1479	0.4374±0.0887	1.79E-15±1.72E-15
	30	2000	0.8852±0.1720	0.5764±0.1095	3.36E-15±1.36E-15
160	10	1000	0.1855±0.0849	0.1645±0.0857	4.44E-16±0.0000
	20	1500	0.4945±0.1349	0.3227±0.0551	1.47E-15±1.61E-15
	30	2000	0.7022±0.0727	0.4369±0.0727	3.22E-15±1.47E-15

Table IV. Mean fitness value for Griewank function.

Size	D.	Max. I.	LDWPSO	Center PSO	PSOCM
20	10	1000	0.0891±0.0419	0.0815±0.0493	0.0000±0.0000
	20	1500	0.0663±0.2332	0.0588±0.1314	0.0000±0.0000
	30	2000	0.0331±0.0668	0.0111±0.1959	0.0000±0.0000
40	10	1000	0.0867±0.0423	0.0776±0.0395	0.0000±0.0000
	20	1500	0.0222±0.0223	0.0226±0.0410	0.0000±0.0000
	30	2000	0.0158±0.0175	0.0248±0.0355	0.0000±0.0000
80	10	1000	0.0688±0.0281	0.0659±0.0279	0.0000±0.0000
	20	1500	0.0221±0.0205	0.0215±0.0211	0.0000±0.0000
	30	2000	0.0119±0.0146	0.0192±0.0233	0.0000±0.0000
160	10	1000	0.0577±0.0236	0.0552±0.0221	0.0000±0.0000
	20	1500	0.0233±0.0196	0.0217±0.0195	0.0000±0.0000
	30	2000	0.0099±0.0115	0.0108±0.0109	0.0000±0.0000

Figure 1 :Rosenbrock function convergence.

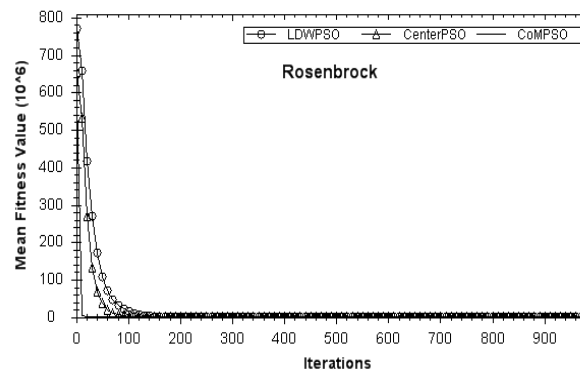


Fig 2.Rastrigin function convergence.

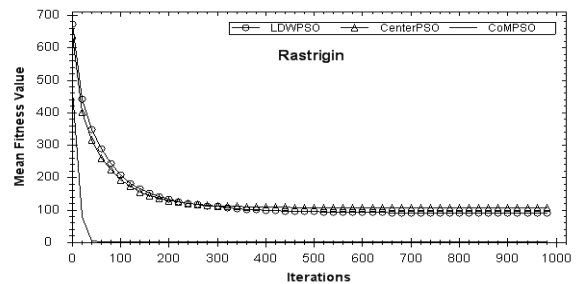


Fig 3. Griewank function convergence.

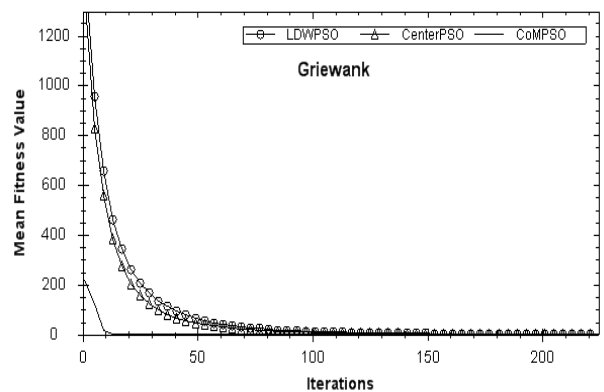
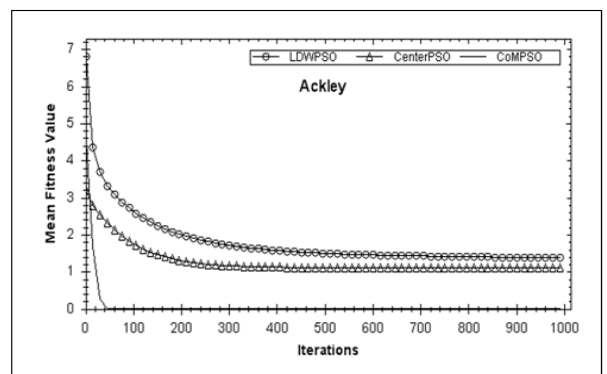


Fig 4. Ackley function convergence.



According to complexity, when the problem dimension increases (more complexity) the average minimum value obtained by LDWPSO, CenterPSO and PSOCM increases, but the average obtained by PSOCM is still the smallest one, it increases slowly while dimension rises for all tests except Rastrigin and Griewank (the optimal is reached). Up to convergence rate, the convergence of PSOCM is faster than that of LDWPSO and CenterPSO.

### III. CONCLUSION AND FUTURE WORK

In this paper, a new variation of PSO, called Particle Swarm Optimization with Center of Mass (PSOCM), is brought forward. A virtual particle called center of mass is inserted to the formula of velocity to help the cognitive behavior component to avoid local optima, and to help maintaining the diversity of the swarm during the searching process. This increases the opportunity of fast convergence to global (or near global optima), where the center of mass particle will attract particles to the region of best found solutions, and this gives particles the best chance to occupy the position of global best found solution during the search process. Two versions of particle swarm optimization, namely, LDWPSO, and CenterPSO, were considered with the proposed PSOCM to be compared. A set of well-known optimization benchmark test problems with varying difficulty levels and problem size are considered to evaluate the compared algorithms. This set of problems consists of four scalable problems, namely, Rosenbrock, Rastrigin, Griewank and Ackely function, the dimension of those problems can be increased/decreased at will, so the complexity of those problem increases as the problem size is increased.

In the future, more theoretical work on PSOCM will be performed, and some real examples from industry and other fields will be applied to PSOCM to evaluate its performance. More physical rules may be used to enhance the performance of the particle swarm optimization, which is still a hot topic for researchers to explore and examine. New enhanced particle swarm optimization algorithm may be investigated to be used in forecasting PSO based models.

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