

A New Efficiency Diagram Based on Normalized Effective Modulus for Design of Perforated Tube-Sheets with Array of Holes of Different Sizes

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Abstract - Tube-sheets with perforations are widely used in heat exchangers, steam generators, and high-temperature components of gas-cooled type nuclear reactors. Conventional ASME/TEMA design procedures are limited to tubesheets with uniformly sized holes arranged in regular square or triangular pitch patterns. However, there are many practical applications where tube-sheets with non-uniform hole sizes (i.e. holes of different sizes and arranged in different patterns) are used. For design of these components, effective elastic modulus or equivalent modulus reduction factors are not available. To address this issue, the normalized effective modulus has been evaluated through finite element analysis of perforated plates with explicit modelling of hole patterns. Both square and triangular pitch types have been considered in the analysis. Initially, perforated plates with same size holes have been analyzed and the data of normalized effective modulus as a function of ligament efficiency has been compared with ASME curves. Later, holes of 2 and 3 different sizes have been analyzed and a new correlation has been developed so that the same can be used in design. It was observed that the normalized effective modulus gets reduced for holes of different sizes compared with the plates with same hole size for a given value of ligament efficiency.

Keywords: Ligament efficiency, Finite element analysis, Perforated plates, ASME code, Pitch pattern, Plate with different hole sizes, Efficiency diagram.

1. INTRODUCTION

Tube-sheets are critical structural components in heat exchangers, steam generators, and high-temperature systems such as gas-cooled type nuclear reactors, where they provide mechanical support to tubes and ensure structural integrity under combined mechanical and thermal loads. Due to the presence of a large number of perforations, tube-sheets exhibit significantly reduced stiffness compared to solid plates, and their effective elastic properties play a key role in design and safety assessment. Current design practices based on TEMA [1] and ASME [2, 3] standards model perforated tube-sheets using an equivalent solid plate approach, with reduced elastic modulus (E^*) expressed as a function of ligament efficiency (η) and thickness-to-pitch (h/p) ratio, for regular square- and triangular-pitch arrangements with uniform hole sizes (i.e. holes of same size ' d '). However, in many practical applications, tube-sheets contain non-uniform hole sizes, for which standard code-based reduction factors are not available.

This limitation necessitates the use of finite element analysis for each configuration. The present work addresses this gap by developing a generalized and conservative design methodology for tube-sheets with non-uniform

hole distributions using FE-based derivation of equivalent normalized effective elastic modulus as a function of ligament efficiency. A new definition of ligament efficiency is also proposed for two- and three-sized hole configurations, enabling practical code-type design without the need for repeated three-dimensional FE analyses.

Perforated tube sheets are widely used in heat exchangers, and their structural behaviour has been studied to account for stiffness reduction due to the presence of holes. Gardner [4, 5] developed the early semi-theoretical framework for tube-sheet design by modelling perforated plates as equivalent solid plates on elastic foundations, introducing the concepts of ligament efficiency and effective stiffness for fixed, floating, and U-tube configurations. Bailey and Hicks [6] investigated the elastic behaviour of perforated plates with square and triangular hole patterns, providing design curves for effective Young's modulus, shear modulus, Poisson's ratio, and stress concentration factors, highlighting the strong influence of hole arrangement and pitch-to-diameter ratio.

O'Donnell and Langer [7] proposed equivalent solid-plate formulations and practical design charts for pressure-loaded perforated plates, improving applicability in engineering design. Slot and O'Donnell [9] later extended these formulations to thick perforated plates under generalised plane-strain conditions and validated the effective elastic constants for square and triangular patterns. Subsequently, the effective elastic constants for the bending of thin perforated plates with triangular and square penetration patterns were derived by O'Donnell [9]. Comprehensive design methodologies integrating these concepts into ASME-based practice are summarised in Miller [10].

The effect of hole-reinforcement on the stiffness of perforated plates has been studied by Abdul-Wahab and Harrap [11]. The evolution of the concept of effective elastic constants for the design of tubesheets and its use in French pressure vessel design code has been discussed extensively by Osweiler [12, 13]. The assessment of performance of perforated plates subject to general loading conditions has been presented in Ref. [14].

An efficient tube-sheet design concept has been presented in Ref. [15]. Finite element analysis has been used in Ref. [16] to calculate stress values in shell-and-tube type heat exchangers. Numerical analyses of perforated steel sheets under uniaxial tensile force and for different end support conditions were presented by Sayed [17, 18] and Cetin [19]. Dai et al. [20] conducted experiments on perforated 6061-T6 aluminium alloy H-section beams under patch loading conditions and compared the results with numerical simulation.

The local buckling load of a perforated plate were evaluated numerically by Naraidoo and Rossi [21]. The dynamic characteristics of perforated plates with elastically restrained edges under impact loading was studied by Wang and Liu [22]. The perforated steel plate shear walls under cyclic loading was analysed using FEM in Ref. [23]. The free vibration behaviour of perforated cylindrical shells of revolution and perforated plates using equivalent elastic properties were analysed using numerical methods in Ref. [24, 25]. The effective elastic constants of perforated plates and shells were predicted using different schemes and for different loading and boundary conditions in Ref. [26-28].

The effective elastic properties of rhombic mesh structures [19], rhombic dodecahedron structures [30], cellular materials or structures [31], lattice structures [32], triangular grid structures [33, 34], porous structures [35] were predicted based on computational homogenization technique and simplified analytical techniques. The elastic modulus of the porous $\text{La}_{0.6}\text{Sr}_{0.4}\text{Co}_{0.2}\text{Fe}_{0.8}\text{O}_{3-\delta}$ cathodes was predicted accurately from microstructural data using a combined FEM and machine learning algorithm.

In summary, it was observed that a wide variety of computational and analytical techniques were applied by various researchers in literature in order to simplify the analysis of the perforated and porous structures by deriving the effective material properties in terms of effective elastic modulus and other such parameters as applicable so that the structures can be modelled as a continuum and the effects of perforations or holes can be taken care of inherently.

From a detailed survey of the research works as presented in literature, it was observed that the existing design curves are largely based on idealised assumptions and limited parameter ranges, motivating the present finite element study to systematically investigate the influence of hole pattern, ligament efficiency, and thickness ratio (h/p) on the effective elastic modulus of the tube sheets with non-uniform hole sizes.

The primary goal of this research is to develop a generalised and practical design methodology for perforated tubesheets containing non-uniform hole sizes by extending the existing ASME-based normalized effective elastic modulus based approach. Since the current design codes are limited to tubesheets with uniformly sized holes arranged in regular square and triangular pitch patterns, this study aims to overcome this limitation by introducing a new definition of ligament efficiency suitable for tubesheets with two and three different hole sizes.

Using finite element analysis, the data for normalized effective elastic modulus (E^*/E) relationships (as a function of ligament efficiency) are derived. The results have been validated initially for plates with uniform hole configurations and later, the same method has been extended to derive correlation for plates with non-uniform hole sizes. The proposed correlations are intended to provide conservative, code-compatible design equations that can eliminate the need for complex three-dimensional finite element analysis in routine engineering design, thereby enabling safe, efficient, and practical design of tubesheets used in advanced industrial and high-temperature applications.

The paper is divided into four sections. The details of FE analysis of the perforated plate is presented in Section 2 and the results of analysis for different cases are elaborated in Section 3. The discussion of the results are embedded in Section 3 and some of the major conclusions from this work are highlighted in Section 4.

2. 3D FINITE ELEMENT ANALYSIS OF THE PERFORATED TUBE-SHEET

In the present study, perforated tube-sheets with square triangular type pitch arrangements are considered for analysis. The material elastic properties are modelled using Young's modulus E (210 MPa) and Poisson's ratio μ (0.3). The tube-sheet is modeled as a flat plate with different thickness-to-pitch (h/p) ratios, which range from 0.1 to 2. The 3D FE model (along with the mesh) of the tube-sheet with arrangement of holes in square and triangular pitch patterns are presented in Fig.1. The 3D solid model has been modelled with 20-noded 3D solid brick type elements.

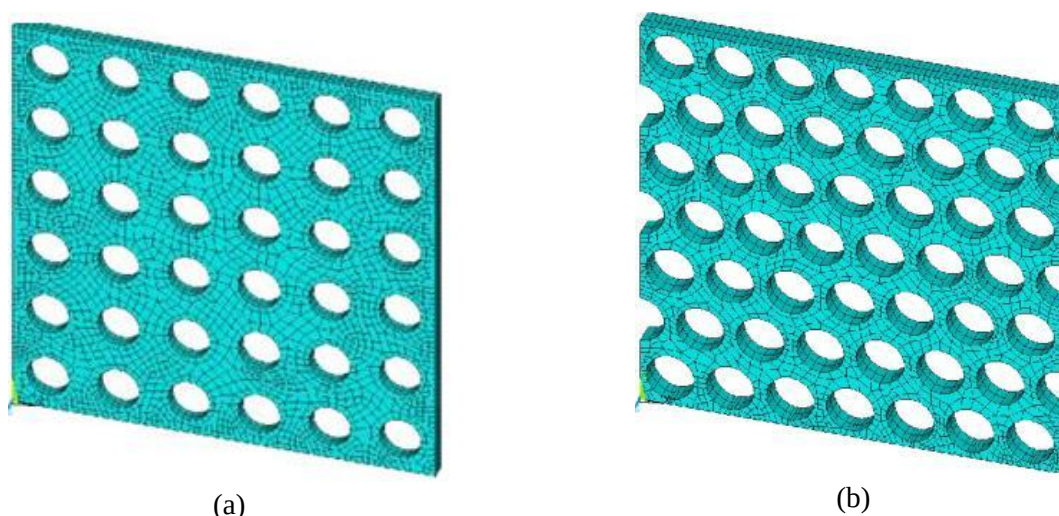


Fig. 1: FE model and 3D solid mesh of the plates with holes arranged in (a) square pitch; and (b) triangular pitch patterns.

The plate is simply supported at the left edge and the right edge is modelled as a symmetric boundary and hence, the effective length of the plate is twice the length shown in Fig. 1. Similarly, the bottom side of the plate has been modelled as a symmetric boundary and hence, the effective width is double the width as presented in the model. The plate has been subjected to uniformly-distributed transverse load and the normalized effective elastic modulus has been calculated using the results of deflection efficiency as summarized in detail in Gardner [3, 4].

The ligament efficiency is defined as the ratio of ligament width between adjacent holes ($p-d$) to the pitch 'p' and it is defined as the ratio ' $\eta = (p-d)/p$ '. To investigate the influence of perforation geometry, different hole diameters and ligament efficiencies are considered while maintaining the same boundary and loading conditions. Plates with both uniform and non-uniform hole size distributions are analyzed to evaluate their effect on the normalized effective elastic modulus of the tube-sheet. Initially, the effect of number of holes on the normalized effective elastic modulus for a given tube-sheet geometry has been studied. Both square and rectangular type pitch patterns are considered in the FE analysis.

Later, the perforated plates with uniform size of holes have been analysed. As the results for these cases are available in ASME curve, the same have been compared against the codal data. Later, the method has been extended to perforated plates with different hole sizes. A new definition of ligament efficiency has been defined and the results have been presented in the form of new normalized effective elastic modulus vs ligament efficiency curves, which can be readily used in design of perforated plates with different hole sizes without resorting to explicit 3D FE analyses.

3. RESULTS AND DISCUSSION

3.1 FE analysis of perforated rectangular plate with holes arranged in square and triangular pitch (study of effect of number of holes in the FE model)

Initially, the effect of number of holes on the normalized effective modulus (E^*/E) of the perforated plate has been studied. For analysis, the h/p ratio of the plate has been taken as 0.5. The E^*/E value has been plotted as function of number of holes in the plate. Both square and triangular pitch type arrangement of holes have been considered in the analysis. The typical 3D FE model and the corresponding FE mesh of the plate are shown in Fig. 1. The loading and boundary conditions of the perforated plates are same as discussed in previous section.

The results of E^*/E are presented in Fig. 2. It can be observed that the value of E^*/E reduces with increasing number of holes and saturates to a constant value for the square pitch arrangement and the trend is opposite for the triangular pitch arrangement. However, the values remain almost unchanged when the number of holes exceed 16. The difference in values of E^*/E for lower number of holes can be explained on the basis of effect of free boundary on the deformation behaviour of perforated plates. The boundary effects diminish once sufficient number of holes are modelled. Hence, for subsequent analysis, number of holes are kept high and more than 40 holes are used. The results are also compared with corresponding data available in ASME code. It can be seen from Fig. 2 that the results of E^*/E as obtained from FE analysis (for number of hole more than 16) are very close to those of ASME

code and hence, this validates the FE model as used in this work for evaluation of the effective modulus of the perforated plates.

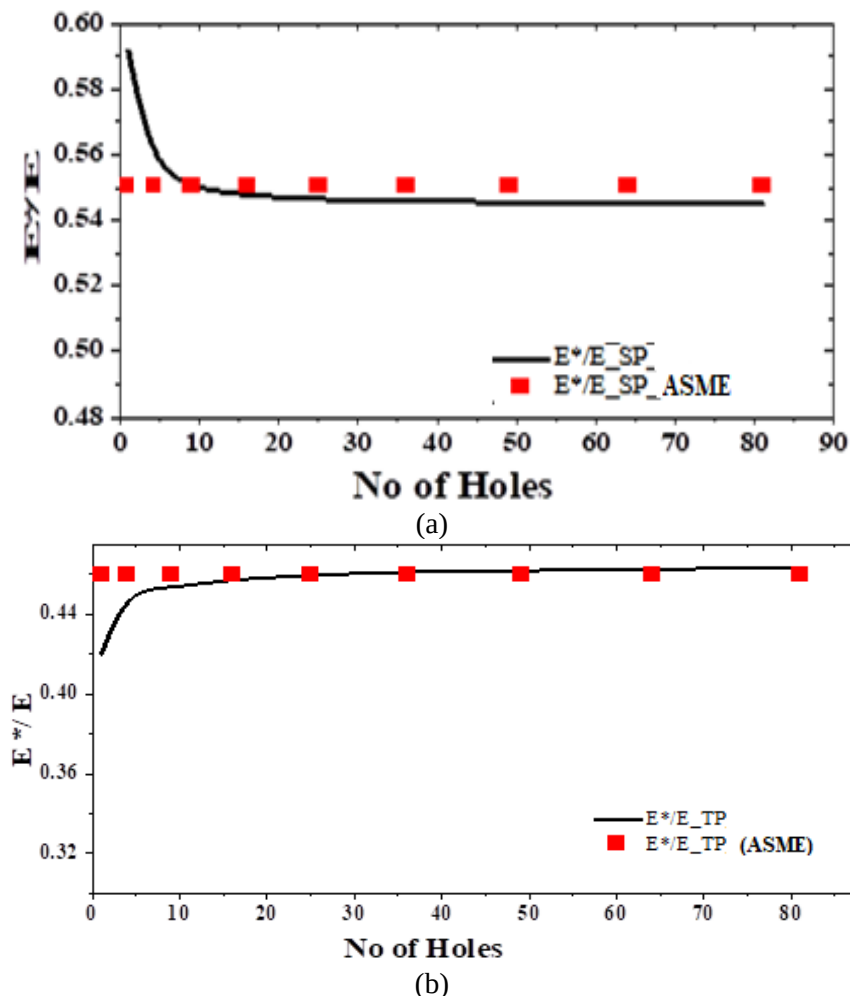


Fig. 2: Convergence of normalized effective modulus (E^*/E) of the plate with number of holes. Results for (a) square pitch (SP) and (b) triangular pitch (TP) and their comparison with ASME data.

3.2 Effect of h/p ratio on the normalized effective modulus of perforated plate

Finite element analyses (considering symmetry about x and y axes as discussed earlier) were performed on a 80×80 mm perforated plate containing 64 holes arranged in both square and triangular pitch patterns. The ligament efficiency (η), the thickness (h) and the (h/p) ratios were varied. The results as obtained from FE analysis are presented in **Fig. 3**. As can be seen from Fig. 3, the E^*/E ratio decreases with increasing thickness of the plate. As the plate becomes thick, the stiffness of the plates increases, however, due to presence of holes, the reduction in stiffness becomes significant compared to the thin plates. Hence, the effective modulus goes on reducing with increasing thickness. This effect is significant for intermediate values of ligament efficiency. As the ligament efficiency becomes close to 1 or 0, the effect of thickness is insignificant. In order to understand this effect, the results of E^*/E are plotted with respect to normalized thickness, i.e. h/p ratio is shown in **Fig. 4(a)** for different h/p ratios ranging from 0.1 to 2 (similar to that of ASME data). The corresponding results of ASME are also plotted in **Fig. 4(b)** so that a comparative analysis can be carried out.

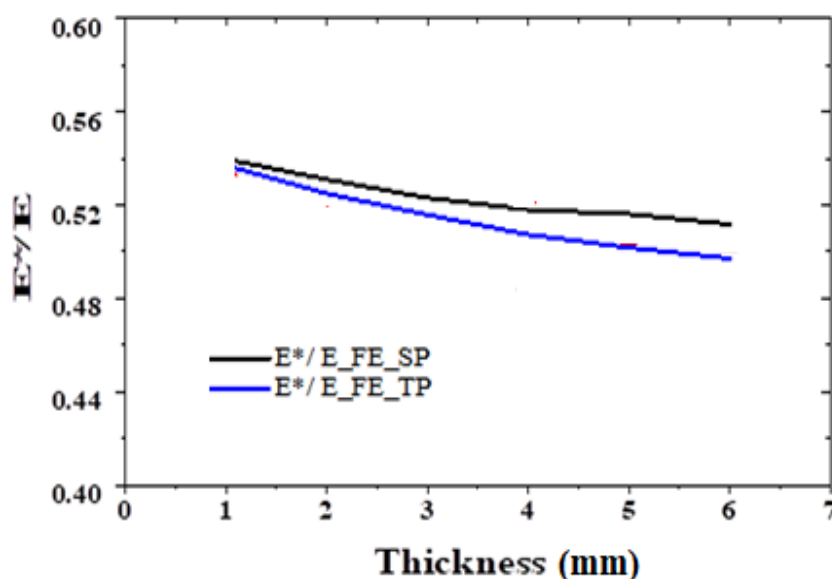
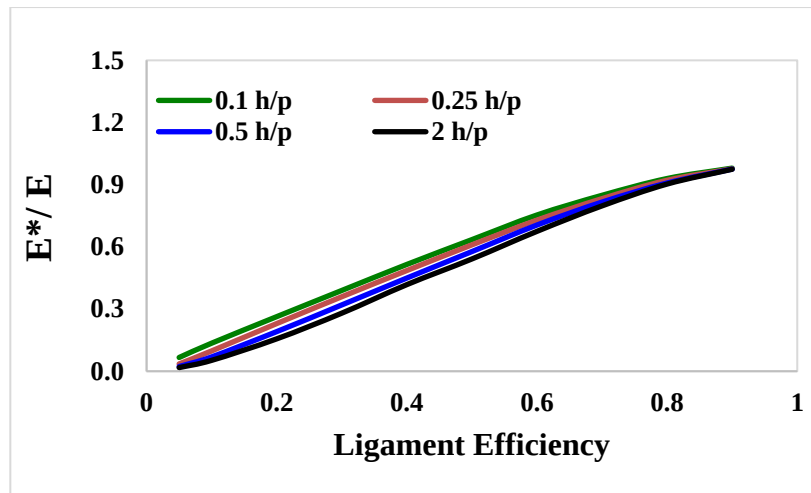


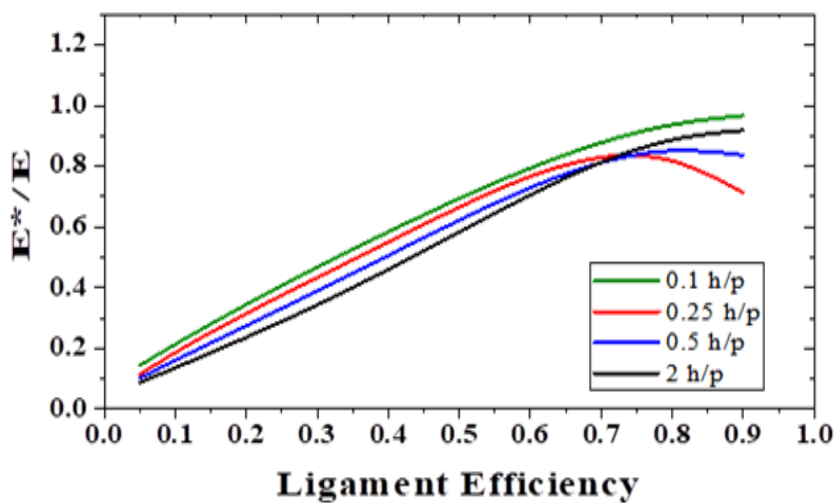
Fig. 3: Effect of plate thickness on the normalized effective modulus of perforated plate for both square pitch and triangular pitch patterns as obtained from FE analysis.

It can be observed from Fig. 4(a) that the difference in the E^*/E values are higher for the range of η in the range of 0.1 to 0.6. After $\eta = 0.6$, all the curves come close to each other and hence, the effect of thickness gets reduced. As the ligament efficiency becomes more than 0.6, the perforated plates behaves almost like an unperforated plate and hence, the effect of holes on the stiffness reduction is negligible. This is the reason why the normalized effective modulus E^*/E becomes almost independent of h/p ratio of the plate.

However, as can be seen from the ASME curve presented in Fig. 4(b), the order of curves change after $\eta = 0.6$. It may be noted that the ASME data for E^*/E are valid strictly in the range of $\eta = 0.2$ to 0.6 and hence, the extrapolated curve beyond $\eta = 0.6$ cannot be compared with corresponding data of FE analysis. Nevertheless, the results of FE analysis for single size holes are almost similar to the data presented in ASME code for the range of ligament efficiency between 0.2 and 0.6. Hence, the results of FE analysis are validated with ASME data for perforated plates with single-sized holes. The effect of number of holes on the results of E^*/E for different h/p values are also presented in Fig. 5. The results of 20*20 and 40*40 holes arrangement (square-pitch pattern) are presented in Fig. 5. It can be observed that the results are independent of number of holes and only these are functions of ligament efficiency and h/p ratio. The results are also close of ASME data as a single sized holes are considered in analysis.

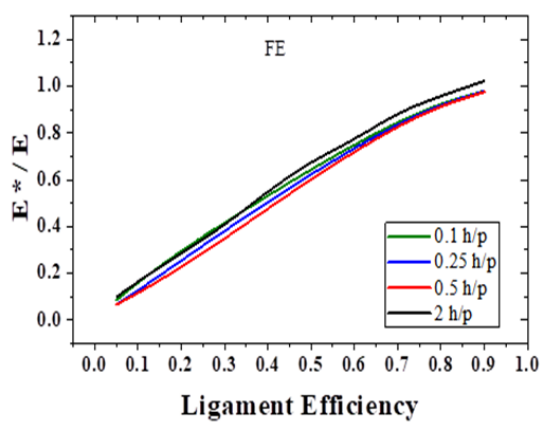


(a)

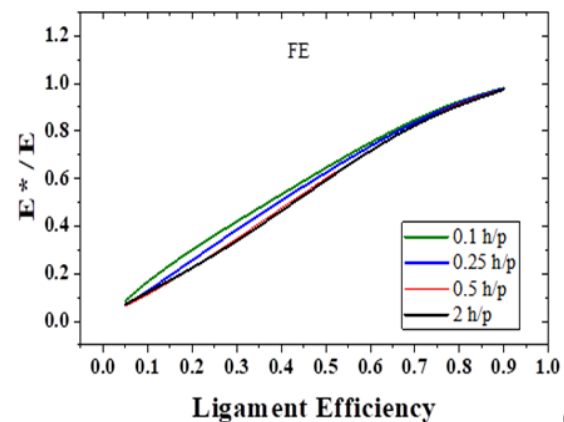


(b)

Fig. 4: Variation of normalized effective modulus of perforated plate with square pitch pattern of holes as a function of ligament efficiency as obtained from (a) FE analysis and (b) ASME equations; for 4 different h/p ratios of the plate as per ASME code.



(a)



(b)

Fig. 5: Effect of number of holes on the normalized effective modulus vs ligament efficiency curves for different h/p ratios. The number of holes in the plates has been taken as (a) 20*20 and (b) 40*40 in the plate. The holes are arranged in square pitch pattern.

3.3 Normalized effective modulus for holes of two sizes (d_1 & d_2) arranged in square-pitch pattern

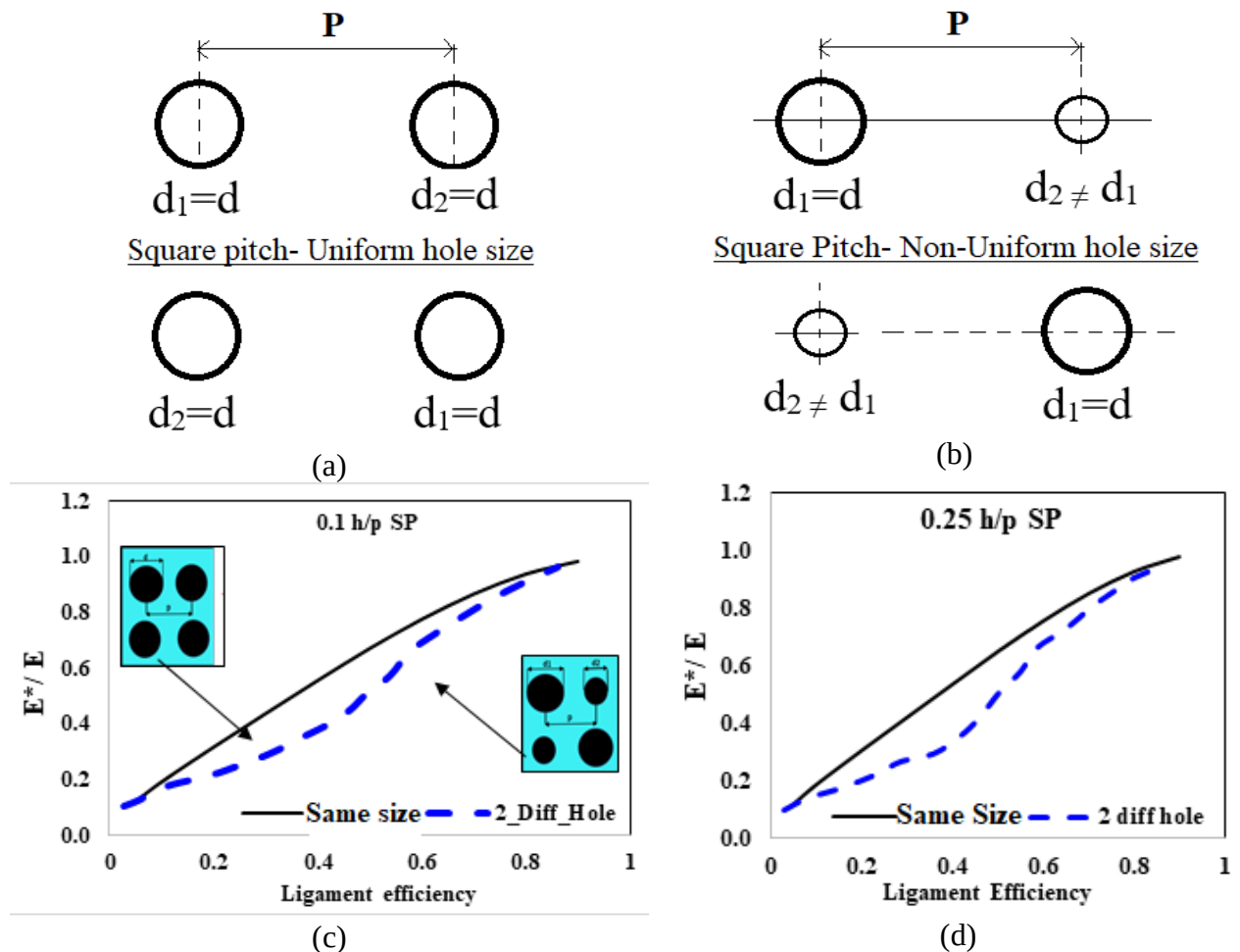
Finite element analysis was performed on a rectangular perforated plate of size 38 mm × 10 mm containing two different hole diameters (d_1 & d_2) arranged within the same pitch (p). The hole arrangement was designed using multiple combinations of hole diameters while maintaining different ' h/p ' ratios. The schematic representation of arrangement of holes in a square-pitch pattern in a plate is shown in Fig. 6(a) for uniform hole sizes and Fig. 6(b) for holes of two different sizes (i.e., non-uniform hole arrangement).

In this work, a new expression for ligament efficiency (η) is defined as two different hole sizes are present in the plate. The expression is given in Eq. (1).

$$\eta = \frac{p - 0.5(d_1 + d_2)}{p} \quad (1)$$

This efficiency becomes same as the conventional efficiency, i.e. ' $(p-d)/p$ ' when the two holes sizes become same (i.e. $d_1=d_2$). Hence, Eq. (1) can be considered as a modified form of ligament efficiency, which is valid for both single-sized holes and holes of two different sizes. The results of E^*/E for different values of ligament efficiency and different h/p ratios (0.1 to 1) are shown in Fig. 6 (c-f).

It can be observed that the E^*/E values for perforated plates with two different hole sizes become lower than that of the perforated plate with single hole size. Two factors must be considered to explain this behavior. First, the definition of ligament efficiency affects the E^*/E value.



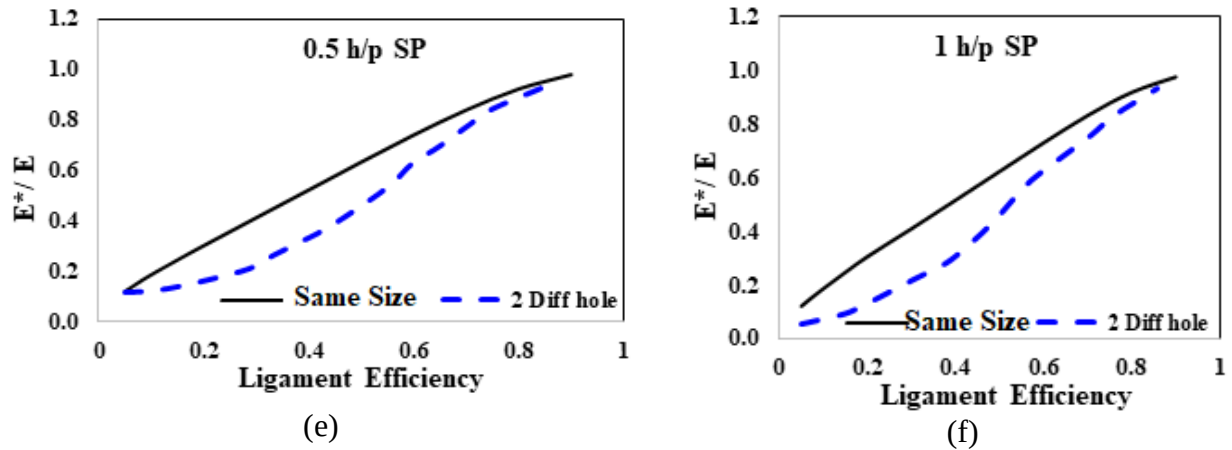


Fig. 6: Schematic representation of arrangement of holes in a square-pitch pattern in a plate; (a) uniform hole size; (b) holes of two different sizes. Variation of normalized effective modulus of the plate as a function of ligament efficiency for square pitch arrangement of holes for different h/p ratios, (a) $h/p=0.1$; (b) $h/p=0.25$; $h/p=0.5$; $h/p=1$. Results for single-sized holes are compared with results of holes of two different sizes.

It may be noted that the hole with larger diameter reduces stiffness drastically as compared to the smaller hole, even though the average diameter may be same as that of perforated plate with uniform hole size. Hence, the E^*/E value always reduces when compared to that of perforated plate with uniform hole size if one keeps the ligament efficiency same. Again, this effect is more dominant in the mid-ligament-efficiency region (i.e. between $\eta = 0.2$ to 0.8). For higher values of ligament efficiency, the effect of hole on E^*/E diminishes as explained earlier.

In order to derive an expression of E^*/E as a function of ligament efficiency, a fourth order polynomial expression has been fitted (similar to the ASME expression) and it is presented in Eq. (2). This equation represents the lower bound curve corresponding to normalized effective modulus of a perforated plate with two different sizes of holes arranged in a square pitch pattern. The corresponding constants α_0 to α_4 are presented in Table-1 for different h/p ratios. Hence, Eq. (2) should be used for design for tube-sheets with square pitch arrangement of holes of two different sizes in place of the standard ASME equation.

$$E^*/E = \alpha_0 + \alpha_1\eta + \alpha_2\eta^2 + \alpha_3\eta^3 + \alpha_4\eta^4 \quad (2)$$

Table-1: Constants α_0 to α_4 for different h/p values to be used for perforated plate with two hole sizes arranged in square pitch pattern

h/p	α_0	α_1	α_2	α_3	α_4
0.1	0.0829	1.0294	-3.2781	8.8528	-5.8914
0.25	0.0740	0.9550	-3.4178	9.5756	-6.4393
0.5	0.1059	0.1896	-0.3232	4.6004	-3.6820
1	0.0388	0.2203	0.5591	2.5977	-2.4923

3.4 Normalized effective modulus for holes of two different sizes, second size located at the center of square-pitch pattern

In this analysis, the pattern of arrangement of holes is shown in Fig. 7. As can be seen from this arrangement, the third hole is located at the center of the square pitch arrangement and the diameter of four holes in the square pitch are same. First, a new expression for ligament efficiency needs to be defined. We have the following information.

$$d_1 = d_2 = d$$

$$d_3 = \text{diameter of third hole} \neq d_1 \text{ or } d_2$$

$$d_{max} = \max (d_1, d_2)$$

As $d_1 = d_2$, we get

$$d_{max} = d$$

The effective diameter representing the combined influence of the two identical holes and the third different hole is defined as

$$d_e = 0.5(d_{max} + d_3) = 0.5(d + d_3)$$

This effective diameter is used to determine the ligament efficiency of the perforated plate. For a square pitch arrangement, the diagonal ligament length between holes is given by $0.7071p$. Therefore, the ligament efficiency is defined as

$$\eta = (0.7071p - d_e)/0.7071p = [0.7071p - 0.5(d + d_3)]/0.7071p \quad (3)$$

where η = ligament efficiency, p = hole pitch, d = diameter of identical holes, d_3 = diameter of the center hole as shown in Fig. 7.

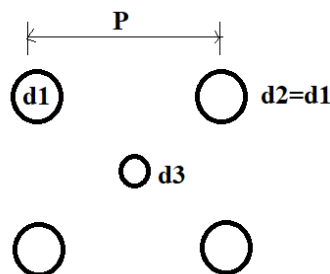


Fig. 7: Arrangement of holes of two different sizes. The third hole has a different diameter compared to other two holes arranged in square pitch pattern and the location of third hole is the center of the square grid.

From FE analysis, the results of normalized effective modulus is presented in **Fig. 8** for four different values of h/p ratio (in the range of 0.1 to 1). The results of two different cases are presented, i.e. first one with two different hole sizes arranged in square pitch (as shown in **Fig. 6b** and discussed in detail in section 3.3) and the other with two different hole sizes and arranged in square-pitch pattern as shown in **Fig. 7**. For comparison the results of perforated plate with uniform holes (schematic arrangement shown in **Fig. 6a**) are also presented.

For the case of plate with uniform or single-sized hole, both data obtained from FE analysis and the corresponding ASME data are presented for the purpose of comparison. For the first two cases, the lower bound data of E^*/E is plotted as a function of ligament efficiency as defined earlier in Eq. (3). It can be observed that the results (i.e. normalized elastic modulus) for hole arrangement as shown in **Fig. 7** are lower as compared to the results of plate with uniform hole size.

However, the results for case with third hole at centre of square pitch is almost similar to the results of two different hole sizes arranged in square pitch. This is due to introduction of a new definition of ligament efficiency as presented in Eq. (3). The results are also presented in the form of correlations (i.e. normalized elastic modulus vs ligament efficiency) similar to Eq. (2) and the corresponding constants are presented in **Table-2** for different h/p ratios of the perforated plate. It is recommend that these constants along with Eq. (2) and Eq. (3) should be used in order to evaluate the effective elastic modulus of the perforated tube-sheet with arrangement of holes similar to that shown in Fig. 7.

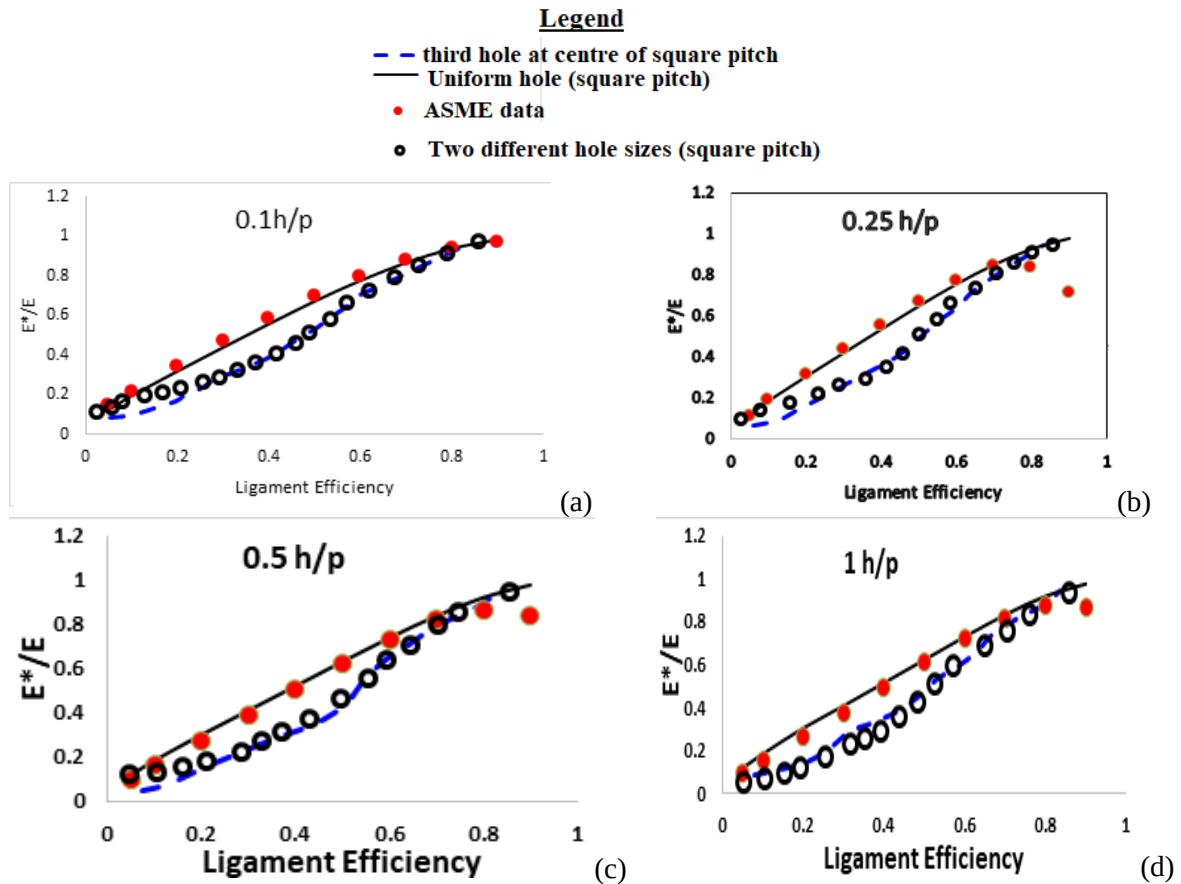


Fig. 8: Variation of normalized effective modulus of perforated plate as a function of ligament efficiency for different arrangement of holes with h/p ratio of (a) 0.1; (b) 0.25; (c) 0.5 and (d) 1.

Table-2: Constants α_0 to α_4 for different h/p values to be used for perforated plate with two hole sizes arranged in square pitch pattern and the third of different size and located at the centre square pitch

h/p	α_0	α_1	α_2	α_3	α_4
0.1	0.245	0.886	-4.02	9.921	-6.2
0.25	-0.05	2.823	-9.98	18.54	-10.8
0.5	0.553	-3.68	12.11	-12.7	5.1775
1	-0.36	3.715	-8.48	12.25	-6.19

4. CONCLUSIONS

This study presents a new method for designing tube-sheets with non-uniform hole sizes. New correlations have been defined so that the design can be carried out without resorting to cumbersome and computationally expensive 3D finite element analysis. It may be noted that conventional efficiency equations (E^*/E) provided by the American Society of Mechanical Engineers (ASME) and Tubular Heat Exchanger Manufacturers Association (TEMA) codes are applicable only to tube-sheets with uniform hole sizes. The following major conclusions can be derived from this work.

- The normalized elastic modulus of the perforated plate only depends upon the ligament efficiency and h/p ratio in addition to the arrangement of holes.
- A proper definition of ligament efficiency is very important in order to derive an expression for E^*/E . This is crucial with perforated plates with two or more different hole sizes.

- The normalized elastic modulus reduces as compared to the corresponding data for uniform distribution of holes. This is due to dominant effect of larger size hole in reduction of stiffness of plate as an average hole size is used in definition of ligament efficiency.
- The new expressions for E^*/E as proposed in this work corresponds to the lower bound data and hence, the design shall be conservative, which is desirable for a safe tube-sheet design.

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