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A New Approach on Tensor Norms and Its Classification

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ABSTRACT - In this paper we are going to establish a new approach on Tensor Norms and its classification with basic properties we discuss five norms on algebric tensor product which are mutually distinct But in general there are several distinct (usually in complete) C*- norms on algebric tensor product A⊗B-we also begin with the dual norms and this leads naturally to the Vital Concept of accessibility, which Can be thought of as an analogue for tensor norms of the approximation property for spaces- Next we have to attempt to the identification of the duals of the chevet - saphar tensor norms in terms of The Classes of p-integral operations.

In final section we conclude with Grothendicks classification of the natural tensor norms.

Keyword : Banach space, Algebric Tensor product, Approximation property. Isometric lonbeding finite dimensional space, C*- Algebra, W*- Algebra,

INTRODUCTION

The Tensors are classified according to their type (n,m) where n is the number of contra variant indices , m is the number of covariant indices and n + m gives the total order of the tensor . Whereas a norm is a function from a real or complex vector space to the non – negative real numbers that behaves in certain ways like the distance from the origin it commutes with scaling obeys a from of the triangle in equality and is zero only at the origin ,

In particular the Euclidean distance in a Euclidean space is defined by a norms on the associated Euclidean vector space called Euclidean norm , the 2 – norm or some times the magnitudes of the vector . This norm can be defined as the square root of the inner product of a vector with it self. As dual norm. If A and B are finite dimensonal normed spaces and α be a tensor norm then A⊗ B is algebrically the dual space of (A* $\otimes_{\alpha}$B*) and we may define α^1 to be a dual norm

A $\otimes_{\alpha 1}$B = (A* $\otimes_{\alpha}$B*)*

In other words if U ∈A ⊗ B

then π^1 (u) = Sup $\{|< u, v|: V \in A\otimes B, \alpha\ (V) < 1\}$

Here we discuss the five norms α, v_1 ,v_r,β and V on A⨀A Latter , we will find that all five norms are mutually distinet .

Let A and B be C*- algebra b with algebric tensor product A⨀B. In general there are serveral distinct C*- norms on A⨀B. Two such norms are of particular interest. The maximal norm Vand the minimal norm α.

If $\pi_1$and π_2 are representaves of A and B respectively , on the Hilbort space H { π_1 , π_2 } is said to be a commuting pair of representatoins of A, B if $\pi_{1\,(a)}\pi_{2\,(b)} = \pi_{2\,(b)}\pi_{1\,(a)}$, (a ∈ A, b ∈ B) The norms v is defined by V($\sum a_i$ ⊗b_i) = Sup$\|\sum_i(a_i)\ \pi(b_i)\|$

Proposition 1 :-

Let A and B be Banach space with the metric approximation property , then $\alpha^s = \alpha^1$ on A⊗ B . This result does not explain the fact that $\pi^s = \pi^1 = \epsilon$. This coincidence can be explained by the possession by the injective norm of a prpoery that is deal to finite generation

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Proposition 2:-

Let A = M⊗N , then the five norms α, v_1 , v_r,β and v on A⨀A are mutually disfimet More over π is normal if and only if $\pi_1$and π_2 are, and for $\sum x_i \otimes b_i \in M_1 \otimes B$, $\sum y_j \otimes C_j \in M_2 \otimes B \left\|\sum \pi(x_i)\pi^I(b_i) + \sum \pi(y_i)\pi^I(c_j)\right\|$ = max $(\left\|\sum \pi_i(x_i)\pi^I{}_j(b_i)\right\|, \left\|\sum \pi_2(y_i)\pi^I{}_2(c_j)\right\|)$

The lemma follows easily from this relation and the definitions of the various norms.

PROOF OF PROPOSITION

In view of the lemma, it is Sufficient to check any two of the norms , v_1 , r , β and V differ on at least one of the tensor products

M⨀M ,M⨀N , N⨀M and N⨀N

(i) On M⨀M , $\alpha = v_1 = \beta$

In the notation of homomorphism's.

X → $\emptyset$ (x) , (x ∈ M)

And Y → R ($\tilde{y}$) ,(y∈ N)

Constitute a commuting pair of representative of M ,N on H (N), The second representation being normal . Thus the homomorphism $\sum x^c \otimes y^c \rightarrow \sum \emptyset(x^c)\, R(\tilde{y}^i)j$ M⨀N→ LH(N) is lemma.

Let M_1 , M_2 and B be w^*Algebra then the canonical isomorphism

$(M_1 \otimes M_2) \odot B$ $(M_1 \odot B) \otimes (M_2 \odot B)$extends to an isomorphism of $(M_1 \otimes M_2) \otimes_n B$ on to

$(M_1 \otimes_n B) \otimes (M_1 \otimes_n B)$

When n is any of the above five norms.

PROOF OF LEMMA

Let e and f be the identity Projections of M_1 and M_2respectively , then e + f = 1 ,

Let { π ,π^I }be commuting pair of representations of ($M_1 \otimes M_2$), B on the Hilbert space H. π (e) andπ(f) commute with($M_1 \otimes M_2$)and π^I(B) so that $H_I = \pi$ (e)H and $H_2 = \pi$ (f)H are in varient subspaces for π and π^I

Let $\pi_1 = \pi/H_i$,$\pi^I = H^I/H_i$ (i = 1,2)

Then { π_1 , $\pi^l{}_1$ } and { π_2 , $\pi^l{}_2$ } are commuting pairs of representations of $M_1 \otimes M_2$, B on H_1 and H_2 respectively.

(i) Continuous relative to the norm V_r on M⨀N and also if it is not continious relative to α , so that $\alpha \neq v_r \leq$ v on M⨀N.

(ii) Exactly the same process $\alpha = v_r = \beta \neq v_1$ on N⨀M

(iii) The representation$\sum x_i \otimes y_i \rightarrow x_i$ R ($\tilde{y}$) of N⨀N on H (H) is clearly continuous relative to the norm B on N⨀N .

Again by the other relevant proposition, this representation is not a continous relative to .

Thus $\alpha \neq \beta$ on N⨀N

Thus the proposition is now complrted Hence the result.

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