# A Modeling Approach of Robust Regression for Developing Thermal Error Compensation Model for CNC Turning Centre

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*Abstract-* The demand for high-speed/high-precision machine tools is rapidly increasing in response to the development of production technology that requires high-precision parts and high productivity. The impact of thermally induced errors in machine tools is more on the dimensional tolerances of a product produced as compared to geometric and cutting force errors. In order to achieve sustain accuracy of machine tool; it requires an accurate and robust thermal error model. Thetraditional thermal error compensation models mostly

focus on the fitting accuracy without considering the robustness of the models, it makes the research results into practice is difficult. This paper is a study of the application of regularization regression algorithms to solve the problems of multi-collinearity of linear thermal error models. The primary focus is on optimization of thermal sensors for eliminating the redundant sensors in the machine tool. Based on analyzing the existing approaches of thermal error modeling for machine tools, robust regression technique is proposed to improve the accuracy as well as robustness of thermal error estimates with respect to techniques least squares regression, partial least square regression previously used in the literature. More distinctively, the performance of the model obtained from robust regression has superior results and it can be applied for real-time error compensation in CNC machine tools effectively.

*Keywords* - Robust regression- Thermal error compensation, Robustness, CNC machine tool, Standard Deviation (SD).

#### I. INTRODUCTION

Due to the thermal deformation of machine tools when machining, the relative distance between the cutting tool and the part being machined, by which the machine accuracy is defined, is changed, so the machining error was made. And the thermally induced error is the biggest contributor to the whole machine errors. The solutions to the thermal errors of machine tools include reduction in the heat sources, design of a thermally robust structure and compensation of thermal error [3]. Among these solutions, the reduction in heat sources is not possible beyond a certain limit as friction between parts in motion would certainly generate some heat. The design of thermally robust structure has a limit to the accuracy that could be achieved. Errors like thermal and cutting force deformation cannot be completely accounted for in design. It is usually time consuming and costly and it often ends in over design of machine structure. The use of alternative materials for machine tool applications is popular amongst machine tool builders, but these methods are still incapable of catering to changes that take place in the shop floor environment on a day to day basis. Compensation after thermal deformation gains success these days both on account of its implementation as well as its cost-effectiveness. Recently, by the help of the development of sensing, modeling, and computer techniques, real-time error compensation based on software approach has received wide attention to further improve the machine accuracy cost effectively [4–7]. One of the main difficult issues in thermal error compensation is to select appropriate temperature variables as well as to obtain an accurate thermal model. For solving this problem, the Kandell's tau-b Bivariant Correlation and Grey correlation analysis are put forward to determine the best combination of temperature variables [8-10], so that the high accuracy model of thermal error could be obtained and the computational time is reduced greatly.

In this paper, testing set-up for collecting the data of temperature field and thermal deformation or errors of a turning center is used. Then, the optimal temperature variable selected using the Kandell's tau-b Bivariant correlation and grey correlation analysis for thermal error modeling. Finally, the thermal error optimal model was set by the multiple regression analysis (MRA) in the general linear form using robust regression approach which is alternative to Least squares regression. More distinctively, the performance of the model obtained from robust regression has superior results and it can be applied for real-time error compensation in CNC machine tools effectively.

#### II EXPERIMENTATION

The sensor points for temperature measurement were selected by accounting for the key heat sources in the bed CNC turning center which have influence on the spindle deformation [7]. The locations for temperature measurement include the spindle bearings, chucking cylinder and motor. The temperature sensors (PT100) were attached to the structure by heat flow paste and are insulated from the environment by foam. The details of the temperature points are given below.

 $T_1$  = Chucking Cylinder,  $T_2$  = hydraulic pack of the Chucking Cylinder,  $T_3$  = Spindle Motor,  $T_4$  = Spindle rear bearing,  $T_5$  = Spindle front bearing,  $T_6$  = Lubricant cover of the spindle,  $T_7$  = Headstock Temperature,  $T_8$  =

Bed underneath the spindle,  $T_9 = Oil$  box, machine side,  $T_{10} = Coolant$  input close to the spindle,  $T_{11}$ = Bed close to the transformer,  $T_{12}$  = Bracket of the transformer,  $T_{13}$ =Bearing of the spindle motor.

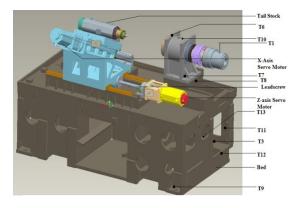


Fig 1 Locations of temperature sensors

In order to investigate the thermal behavior of the CNC turning centre, the following load pattern of spindle has been formulated based on the operations performed on the machine tool. The selected load cycle is of fluctuating type where spindle speed varies between 0 and 2800 rpm.

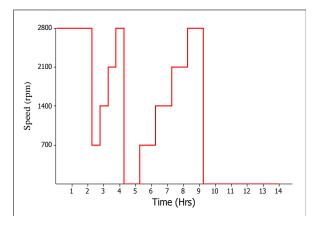


Fig 2 Load pattern of Spindle

In order to facilitate the measurement of spindle deformation, an invar rod was mounted on the spindle and the eddy current displacement sensors were mounted in a fixture connected to the turret as shown in Fig 4. The temperature and spindle thermal deformation were measured and recorded at a sampling interval of 5 min.

Among the components of displacement, x-component of thermal error (Fig. 5) due to the spindle tilt in the lateral direction assumes greater significance in turning centre and hence it is considered in the present investigation [8].

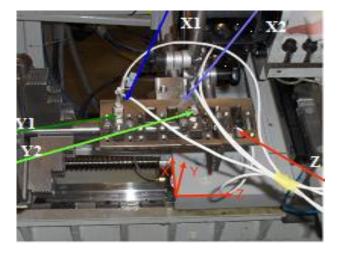


Fig 4 Displacement sensors mounted in the fixture

The transient variation of temperature and the resulting xcomponent of thermal error corresponding to load cycle-I are depicted in Figures 6, 7 and 8.

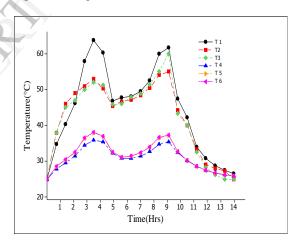


Fig 6 Temperature Variation (T<sub>1</sub> to T<sub>6</sub>)

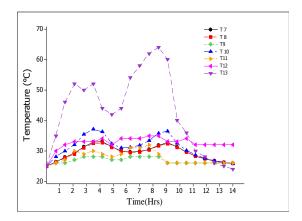


Fig 7 Temperature Variation (T<sub>7</sub> to T<sub>13</sub>)

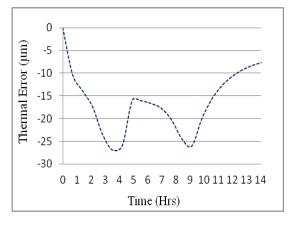


Fig 8 Transient variation of thermal error

From the figures 6, 7 and 8, it is conceded that the transient temperature and corresponding thermal error is varying according to load pattern shown in figure 2.

# III SELECTION OF SIGNIFICANT THERMAL SENSORS IN MACHINE TOOL

In any system for thermal error compensation, thermal sensor location is critical. The selection of the number and locations of thermal sensors becomes the first step to carry out error compensation. Grey correlation [12], ANN, Thermal Mode Analysis [2,11], Fussy logic, etc., are the other methods used for optimizing temperature key points but they relatively complex and time consuming.

In present study, Kandell's tau-b Bivariant correlation analysis, Grey correlation analysis are performed to select the significant thermal sensors. Correlation analysis is also undertaken to determine the dependency of temperature key points. The value of a correlation coefficient can vary from minus one to plus one. A minus one indicates a perfect negative correlation, while a plus one indicates a perfect positive correlation. A correlation of zero means there is no relationship between the two variables. When there is a negative correlation between two variables, as the value of one variable increases, the value of the other variable decreases, and vise versa. In other words, for a negative correlation, the variables work opposite each other. When there is a positive correlation between two variables, as the value of one variable increases, the value of the other variables, as the value of one variables work opposite each other. When there is a positive correlation between two variables, as the value of one variable increases, the value of the other variables as the value of one variable increases, the value of the other variables as the value of one variable increases, the value of the other variables as the value of one variable also increases.

In Kandell's tau-b Bivariant correlation analysis, it is necessary to calculate the correlation among the independent variables. The thermal sensors are grouped according to their correlation value since the temperature variables with high correlation value represent a dependency on each other. The correlation coefficient matrix for thirteen temperature key points is shown in the Table 1.

It is seen from Table 1, that the number of temperature points which are relatively more sensitive are identified as  $T_4$ ,  $T_5$ ,  $T_6$ ,  $T_7$  and  $T_8$  since the group constituted by the above temperatures has the highest correlation value (above 0.97).

## Table1 Kandell's tau-b Bivariant correlation Matrix

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	T1	T2	Т3	T4	Т5	<b>T6</b>	<b>T7</b>	T8	Т9	T10	T11	T12	T13
T1	1	.867	.887	.914	.895	.886	.800	.800	.692	.905	.557	.575	.734
T2	.867	1	.945	.819	.838	.848	.686	.686	.668	.829	.578	.543	.830
Т3	.887	.945	1	.820	.820	.830	.705	.705	.697	.811	.615	.579	.845
T4	.914	.819	.820	1	.981*	.971*	.848	.848	.622	.910	.493	.500	.667
Т5	.895	.838	.820	.981*	1	.990*	.829	.829	.610	.950	.493	.479	.667
T6	.886	.848	.830	.971*	.990*	1	.819	.819	.610	.961	.503	.490	.676
<b>T7</b>	.800	.686	.705	.848	.829	.819	1	$1.000^{*}$	.564	.838	.407	.532	.552
T8	.800	.686	.705	.848	.829	.819	1.000*	1	.564	.838	.407	.532	.552
Т9	.692	.668	.697	.622	.610	.610	.564	.564	1	.610	.856	.676	.644
T10	.905	.829	.811	.910	.950	.961	.838	.838	.610	1	.482	.490	.657
T11	.557	.578	.615	.493	.493	.503	.407	.407	.856	.482	1	.623	.696
T12	.575	.543	.579	.500	.479	.490	.532	.532	.676	.490	.623	1	.606
T13	.734	.830	.845	.667	.667	.676	.552	.552	.644	.657	.696	.606	1

# 3.1 GREY CORRELATION ANALYSIS USING INITIAL VALUE TRANSFORM METHOD

Grey correlation analysis is a principle theory of grey system theory, which can be applied in grey system analysis and random variables processing. The correlation between factors is represented by the similarity level of geometry which is called grey correlation degree, and the correlation degree between reference sequences and comparison sequences can be quantitatively estimated. Grey correlation degree describes the relative change between different factors in the process of system evolution, and the larger the correlation degree is, the higher the similarity level is [12]. Thus, the correlation degree can represent the impact of different sensors on the spindle thermal error behavior. By the calculation of improved correlation degree between thermal error and different sensors, the sensors with a relatively large correlation degree are selected since they have a larger impact on the thermal error. The steps involved in grey correlation degree calculation are given below.

1. Conversion of input data into dimensionless form using initialization value transform method.

$$X_{i}(\mathbf{K}) = \frac{X_{i}(\mathbf{K})}{X_{i}(1)}$$
 (3)

$$\Delta_{i}(K) = |\delta_{x}(K) - X_{i}(K)|$$
(4)

 $\delta_x(K) =$  Thermal Error  $X_i(K) =$  Temperature data

2. Grey Correlation coefficient between thermal error and sensors

$$\xi_{i}(K) = \frac{\min \Delta_{i}(K) + \rho \max \Delta_{i}(K)}{\Delta_{i}(K) + \rho \max \Delta_{i}(K)}$$
(5)

3. Mean of the correlation coefficient

$$r_i = \frac{1}{n} \sum_{k=1}^{n} \xi_i(K), K = 1, 2, \dots, n$$
 (6)

4. Correlation degree with consideration of the diversity of correlation coefficients at different point

$$S(r_{i}) = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (\xi_{i}(K) - r_{i})^{2}}$$
(7)

5. Adjusted Grey Correlation Degree

$$\rho_{i} = \frac{r_{i}}{1 + S(r_{i})} \tag{8}$$

The temperature key points are ranked according to the relative value of grey correlation degree. The temperature key points are selected by setting the threshold value in the range of 0.97 to 1. From the results of grey correlation analysis as shown in Table 3, it is inferred that the four temperature key points, viz,  $T_4$ ,  $T_5$ ,  $T_7$  and  $T_8$  have the highest impact on the overall spindle thermal error.

 Table 2

 Grey Correlation Analysis results

Sensor	$\rho_i$	Relative Value of $\rho_i$	Rank
1	0.70286	0.712	7
2	0.69762	0.437	12
3	0.69871	0.492	10
4	0.70822	1.000	1
5	0.70799	0.988	2
6	0.70731	0.956	5
7	0.70791	0.984	3
8	0.7077	0.976	4
9	0.70123	0.625	8
10	0.70713	0.945	6
11	0.6895	0.000	13
12	0.69841	0.4841	11
13	0.70002	0.5625	9

From the grey correlation and Kandell's tau-b Bivariant correlation analysis, the common four temperature key points selected for thermal error modeling.

#### IV THERMAL ERROR MODELLING

The linear thermal error model will be of the form

$$\delta = \beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3 + \dots + \beta_n T_n + \varepsilon \qquad (1)$$

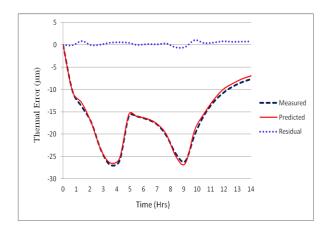
Linear least squares regression is by far the most widely used modeling method. Not only is linear least squares regression the most widely used modeling method, but it has been adapted to a broad range of situations. It plays a strong underlying role in many other modeling methods. The method of least squares assumes that the best-fit curve of a given type is the curve that has the minimal sum of the deviations squared (least square error) from a given set of data.

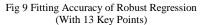
$$\sum_{i=0}^{n} [D_i - f(T_i)]^2 = Minimum$$

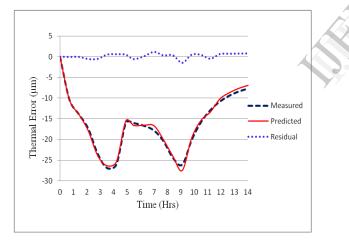
Robust regression works by assigning a weight to each data point. Weighting is done automatically and iteratively using a process called iteratively reweighted least squares. In the first iteration, each point is assigned equal weight and model coefficients are estimated using ordinary least squares. At subsequent iterations, weights are recomputed so that points farther from model predictions in the previous iteration are given lower weight. Model coefficients are then recomputed using weighted least squares. The process continues until the values of the coefficient estimates converge within a specified tolerance.

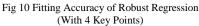
The model developed by the robust regression through the use of MATLAB software is given below.

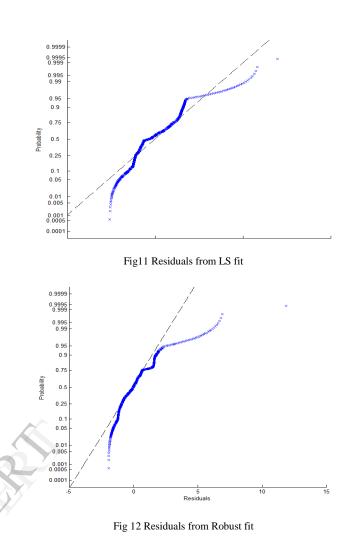
 $\Delta_x = 25.116 - 0.047773 \ T4 - 0.9292 \ T5 - 16.312 \ T7 + 16.008 \ T8$ 











A comparison between the measured error data and the predicted values using the model is represented in Fig. 9 and 10. It can be observed that the model with 13 key points (SD= 0.218  $\mu$ m) and 4 key points (SD = 0.63 $\mu$ m) have the 0.412  $\mu$ m difference of standard deviation. There is no that much difference in residuals of thermal error model with 13 and 4 thermal key points. So, the developed model greatly reduces the working load, computation time as well as the experimental cost.

### V CONCLUSION

- The thermal error range for lateral direction (X-axis) on this precision turning center is approximately 26 μm, which is much larger than we expected.
- 2. Grey correlation and Kandell's tau-b Bivariant correlation analysis for selecting the best combination of temperature variables, by which the thermal error model can be set accurately and quickly with fewer temperature variables.
- 3. More distinctively, the performance of the model obtained from robust regression has superior results and it can be applied for real-time error compensation in CNC machine tools effectively.

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