International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 10 Issue 09, September-2021

A Model for Deterministic Inventory with Deteriorating Items with Demand Dependent on Time Fractionally and Constant Holding and Cost Deterioration Rate

Kusum
Research Scholar, Department of Mathematics
School of Applied Sciences,
Om Sterling Global University, Hisar-125001,
Haryana, India

Vinod Kumar
Professor and Head, Department of Mathematics,
School of Applied Sciences,
Om Sterling Global University,
Hisar-125001, Haryana, India

***Abstract* -- The paper presents an inventory model for deteriorating items where demand rate dependent on time raise to power m by n. All other parameters like deterioration rate and holding cost and ordering costs are considered as constant. Shortages are allowed and fully backlogged. This model is numerically verified with the help of numerical illustration. Maple 18 software is used to verify the model graphically.**

Keywords— Inventory model, Deterministic, Deterioration, Time Dependent, Shortages, Fractional Polynomial.component.

I. INTRODUCTION

Inventory is basically a product that needs to be controlled in order to maximize profit. Inventory control includes some tasks like shipping, purchasing, packaging, tracking, and many more services. Inventory includes goods which are particularly of two types: the first type includes goods which deteriorate with time like bread and fruits etc. and the second type includes the goods which do not deteriorate with time if placed in a soothing environment like paper and cotton etc.

Several researchers have considered the demand rate to be constant, and some considered the demand rate to be a linear or quadratic function of time. The demand rate of certain goods was found to be dependent on selling price and some dependent on time.

B. N. Mandal and S. Phaujdhar [1] from Calcutta university constructed an inventory model for deteriorating items where they considered demand rate depends on stock. Mohit Rastogi, Prashant Kushwaha, S.R. Singh, and Shilpy Tayal [18] invented the inventory model with price-sensitive demand. Vinod Kumar Mishra and Lal Sahab Singh [12] developed deteriorating inventory with linear demand and constant deterioration.

A.K. Jalan, R. R. Giri, and K. S. Chaudhuri [7] developed an EOQ model with linear demand and Weibull distribution deterioration, and Y. K. Shah [5] considered linear demand with exponential deterioration. Kuo-Lung Hou [4] constructed a model with linear demand and constant deterioration and is applied to on-hand inventory. Y. K. Shah and RAM B. MISRA [12] invented deterministic inventory with a constant demand rate, and no replacement or repairing is allowed in a given cycle. Mingbao Cheng and Guoqing Wang [9] constructed an inventory model for deteriorating items with trapezoidal type demand rate and constant deterioration rate.

Chaitanya Kumar Tripathy and Umakant Mishra in [11] presented an order level inventory a system with time-dependent Weibull deterioration and ramp type demand rate where production and demand are time-dependent.

S. K. Ghosh AND K. S. Chaudhuri [3] constructed an inventory model for deteriorating items having instantaneous supply, demand was a quadratic time-varying, and a two-parameter Weibull distribution is taken to represent the time deterioration. In 2014, S.R. Singh and Himanshu Rathore in [16] considered deterioration rate to be a controlled variable and demand linearly dependent on time.

Later in 2016, Nita H. Shah with Urmila Chaudhari and Mrudul Y. Jani [17] constructed an integrated production-inventory model with preservation technology investment for the time-varying deteriorating item under time and price-sensitive demand, and Nita Shah in 2018 with Monika K. Naik constructed an EOQ model for the deteriorating item under full advance payment availing of discount when demand is price-sensitive.

Abu Hashan Md Mashud [19] developed an EOQ deteriorating inventory model with different types of demand and fully backlogged shortages where he considered demand rate as a function of time as well as the selling price. Ajanta Roy [8] invented an inventory model for deteriorating items with price-dependent demand and time-varying holding cost.

R. Amuth and Dr. E. Chandrasekaran [13] developed an inventory model for deteriorating products with Weibull Distribution deterioration, partial backlogging, and time-varying demand.

In this paper, a deterministic inventory model is developed for deteriorating items which deals with a single item with demand rate as a function of time raise to power m by n and deterioration rate is constant. The holding cost is considered to be constant, and the shortages are allowed and fully backlogged.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 10 Issue 09, September-2021

II. FORMULATION AND SOLUTION OF MATHEMATICAL MODEL

Assumptions and Notations:

Notations:

- D - Demand Rate.
- C_D - Deterioration Cost.
- A - The ordering cost.
- C_H –Holding Cost.
- Q - Order Quantity
- θ - Deterioration Rate.
- C - Purchase Cost per unit
- L – Order Quantity
- Ω is the time at which the level of inventory reaches to zero
- T - The length of a cycle time.
- C_S -Shortage cost per unit time.
- TIC -Total inventory cost.
- I(t) – Inventory level.

Assumptions:

The inventory Model uses the following assumptions: -

- The demand rate is fractionally dependent on time which is of type m by n

$$D(t) = \alpha + \beta t^{\frac{m}{n}} \quad \text{and} \quad \alpha \geq 0,\ \beta \geq 0.$$

- Holding cost per unit time is constant

H(t) = C_H , where $C_H \geq 0$.

- The Order quantity per cycle is Q.
- Shortages are allowed and are fully backlogged.
- The lead time is zero.
- The deterioration rate is

$$\theta(t) = \gamma, 0 < \gamma < 1$$

- Ordering cost per item is A.
- The shortages cost per unit time is C_s
- This inventory model deals with single item.
- Ω is the time at which inventory level reaches zero and Ω≥0.
- TIC is assumed to be the total inventory cost per unit time.
- I(t) is assumed as the inventory level.
- C is assumed as the purchasing cost per unit.

Mathematical Formulation

The figure 1 shows the behavior of inventory at any time. This shows the optimal order quantity Q verses time.

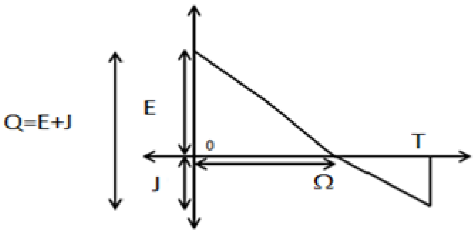

Figure: 1 Inventory Level (Q) verses Time

In the period (0,Ω) inventory level is positive and decreasing and at time t= Ω, the inventory level reaches to zero and shortage starts and inventory became negative in the period (Ω,T) .The differential equation for instantaneous inventory level at time t is as follows :-

$$\frac{dI_1(t)}{dt} + \gamma I(t) = -(\alpha + \beta t^{\frac{m}{n}}); 0 \leq t \leq \Omega \qquad (1)$$

$$\frac{dI_2(t)}{dt} = -(\alpha + \beta t^{\frac{m}{n}}); \Omega \leq t \leq T \qquad (2)$$

where m is any natural number
with boundary conditions

$I_1(t) = I_2(t) = 0$ at t= Ω

$I_2(t) = J$ at $t = T$ and $I_1(t) = E$ at t=0

On solving equation (1) and neglecting higher powers of t, we get

$$I_1(t) = \left[\begin{array}{l} \alpha(\Omega - t) + \frac{n\beta}{m+n}(\Omega^{\frac{m+n}{n}} - t^{\frac{m+n}{n}}) \\ + \frac{\alpha\gamma}{2}(\Omega^2 - t^2) + \frac{n\beta\gamma}{m+2n}(\Omega^{\frac{m+2n}{n}} - t^{\frac{m+2n}{n}}) \\ + \frac{\alpha\gamma^2}{6}(\Omega^3 - t^3) + \frac{n\beta\gamma^2}{2(m+3n)}(\Omega^{\frac{m+3n}{n}} - t^{\frac{m+3n}{n}}) \\ - \alpha\gamma(\Omega t - t^2) - \frac{n\beta\gamma}{m+n}(\Omega^{\frac{m+n}{n}} t - t^{\frac{m+2n}{n}}) \\ - \frac{\alpha\gamma^2}{2}(\Omega^2 t - t^3) - \frac{n\beta\gamma^2}{m+2n}(\Omega^{\frac{m+2n}{n}} t - t^{\frac{m+3n}{n}}) \end{array} \right]$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 10 Issue 09, September-2021

$$+\begin{bmatrix} -\frac{\alpha\gamma^3}{6}(\Omega^3 t - t^4) - \frac{n\beta\gamma^3}{2(m+3n)}(\Omega^{\frac{m+3n}{n}} t - t^{\frac{m+4n}{n}}) \\ +\frac{\alpha\gamma^2}{2}(\Omega t^2 - t^3) + \frac{n\beta\gamma^2}{2(m+n)}(\Omega^{\frac{m+n}{n}} t^2 - t^{\frac{m+3n}{n}}) \\ +\frac{\alpha\gamma^3}{4}(\Omega^2 t^2 - t^4) + \frac{n\beta\gamma^3}{2(m+2n)}(\Omega^{\frac{m+2n}{n}} t^2 - t^{\frac{m+4n}{n}}) \\ +\frac{\alpha\gamma^4}{12}(\Omega^3 t^2 - t^5) + \frac{n\beta\gamma^4}{4(m+3n)}(\Omega^{\frac{m+3n}{n}} t^2 - t^{\frac{m+5n}{n}}) \end{bmatrix} \quad (3)$$

On solving (2), we get

$$I_2(t) = \alpha(\Omega - t) + \frac{n\beta}{m+n}(\Omega^{\frac{m+n}{n}} - t^{\frac{m+n}{n}}) \quad (4)$$

Now using condition $I_1(t) = E$ at t=0, we get

$$E = \begin{bmatrix} \alpha\Omega + \frac{n\beta}{m+n}\Omega^{\frac{m+n}{n}} + \frac{\alpha\gamma}{2}\Omega^2 + \frac{n\beta\gamma}{m+2n}\Omega^{\frac{m+2n}{n}} \\ +\frac{\alpha\gamma^2}{6}\Omega^3 + \frac{n\beta\gamma^2}{2(m+3n)}\Omega^{\frac{m+3n}{n}} \end{bmatrix}$$

Now using the condition $I_2(t) = J$ at t=T, we get

$$J = -\alpha(\Omega - T) - \frac{n\beta}{m+n}(\Omega^{\frac{m+n}{n}} - T^{\frac{m+n}{n}})$$

Now the order quantity per cycle is given by:

Q = E +J

$$Q = \begin{bmatrix} \alpha\Omega + \frac{n\beta}{m+n}\Omega^{\frac{m+n}{n}} + \frac{\alpha\gamma}{2}\Omega^2 \\ +\frac{n\beta\gamma}{m+2n}\Omega^{\frac{m+2n}{n}} + \frac{\alpha\gamma^2}{6}\Omega^3 \\ +\frac{n\beta\gamma^2}{2(m+3n)}\Omega^{\frac{m+3n}{n}} - \alpha(\Omega - T) \\ -\frac{n\beta}{m+n}(\Omega^{\frac{m+n}{n}} - T^{\frac{m+n}{n}}) \end{bmatrix} \quad (5)$$

Now the inventory holding cost per cycle is given by:

$$IHC = C_H \int_0^{\Omega} I_1(t)dt$$

$$IHC = C_H \begin{bmatrix} \frac{\alpha}{2}\Omega^2 + \frac{n^2\beta}{(m+n)(m+2n)}\Omega^{\frac{m+2n}{n}} \\ +\frac{\alpha\gamma}{6}\Omega^3 + \frac{2n^2\beta\gamma}{(m+2n)(m+3n)}\Omega^{\frac{m+3n}{n}} \\ +\frac{5\alpha\gamma^2}{12}\Omega^4 + \frac{3n^2\beta\gamma^2}{2(m+3n)(m+4n)}\Omega^{\frac{m+4n}{n}} \\ -\frac{n^2\beta\gamma}{(m+n)(m+3n)}\Omega^{\frac{m+3n}{n}} \\ -\frac{2n^2\beta\gamma^2}{(m+2n)(m+4n)}\Omega^{\frac{m+4n}{n}} \\ -\frac{1}{15}\alpha\gamma^3\Omega^5 - \frac{3n^2\beta\gamma^3}{2(m+3n)(m+5n)}\Omega^{\frac{m+5n}{n}} \\ +\frac{n^2\beta\gamma^2}{2(m+n)(m+4n)}\Omega^{\frac{m+4n}{n}} \\ +\frac{n^2\beta\gamma^3}{(m+2n)(m+5n)}\Omega^{\frac{m+5n}{n}} \\ +\frac{\alpha\gamma^4}{72}\Omega^6 + \frac{3n^2\beta\gamma^4}{4(m+3n)(m+6n)}\Omega^{\frac{m+6n}{n}} \end{bmatrix} \quad (6)$$

Total amount of inventory shortage cost per unit time during the period (Ω,T) is given by:

$$ISC = -C_S \int_{\Omega}^{T} I_2(t)dt$$

$$ISC = -C_S \int_{\Omega}^{T} [\alpha(\Omega - t) + \frac{n\beta}{m+m}(\Omega^{\frac{m+n}{n}} - t^{\frac{m+n}{n}})]dt$$

$$ISC = -C_S \begin{bmatrix} \alpha\left(\Omega T - \frac{T^2}{2} - \Omega^2 + \frac{\Omega^2}{2}\right) \\ +\frac{n\beta}{m+n}\left(\Omega^{\frac{m+n}{n}} T - \frac{n}{m+2n} T^{\frac{m+2n}{2}} - \Omega^{\frac{m+2n}{n}} + \frac{m}{2n}\Omega^{\frac{m+2n}{n}}\right) \end{bmatrix}$$

$$ISC = -C_S \begin{bmatrix} \alpha\left(\Omega T - \frac{T^2}{2} - \frac{\Omega^2}{2}\right) \\ +\frac{n\beta}{m+n}\left(\Omega^{\frac{m+n}{n}} T - \frac{n}{m+2n} T^{\frac{m+2n}{n}} - \frac{(m+n)}{(m+2n)}\Omega^{\frac{m+2n}{n}}\right) \end{bmatrix} \quad (7)$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 10 Issue 09, September-2021

Now, Inventory deterioration cost per item is given by:

$$IDC = C_D[E - \int_0^{\Omega}(\alpha + \beta t^{\frac{m}{n}})dt]$$

$$IDC = C_D\left[\frac{\alpha\gamma\Omega^2}{2} + \frac{n\beta\gamma\Omega^{\frac{m+2n}{n}}}{m+2n} + \frac{\alpha\gamma^2\Omega^3}{2\times 3} + \frac{n\beta\gamma^2\Omega^{\frac{m+3n}{n}}}{2(m+3n)}\right] \quad (8)$$

The inventory ordering cost per order during (0,Ω) is given by:

IOC = A (9)

Now, the total cost per unit time for the cycle is given by:

$$TIC = \frac{1}{T}\left(C_H\left(\begin{array}{l}\frac{\alpha}{2}\Omega^2 + \frac{n^2\beta}{(m+n)(m+2n)}\Omega^{\frac{m+2n}{n}} + \frac{\alpha\gamma}{6}\Omega^3 \\ + \frac{2n^2\beta\gamma}{(m+2n)(m+3n)}\Omega^{\frac{m+3n}{n}} + \frac{5\alpha\gamma^2}{12}\Omega^4 \\ + \frac{3n^2\beta\gamma^2}{2(m+3n)(m+4n)}\Omega^{\frac{m+4n}{n}} \\ - \frac{n^2\beta\gamma}{(m+n)(m+3n)}\Omega^{\frac{m+3n}{n}} \\ - \frac{2n^2\beta\gamma^2}{(m+2n)(m+4n)}\Omega^{\frac{m+4n}{n}} \\ - \frac{1}{15}\alpha\gamma^3\Omega^5 - \frac{3n^2\beta\gamma^3}{2(m+3n)(m+5n)}\Omega^{\frac{m+5n}{n}} \\ + \frac{n^2\beta\gamma^2}{2(m+n)(m+4n)}\Omega^{\frac{m+4n}{n}} \\ + \frac{n^2\beta\gamma^3}{(m+2n)(m+5n)}\Omega^{\frac{m+5n}{n}} \\ + \frac{\alpha\gamma^4}{72}\Omega^6 + \frac{3n^2\beta\gamma^4}{4(m+3n)(m+6n)}\Omega^{\frac{m+6n}{n}}\end{array}\right)\right)$$

$$+\frac{1}{T}\left(\begin{array}{l}-C_S\left[\begin{array}{l}\alpha\left(\Omega T - \frac{T^2}{2} - \frac{\Omega^2}{2}\right) \\ + \frac{n\beta}{m+n}\left(\Omega^{\frac{m+n}{n}}T - \frac{n}{m+2n}T^{\frac{m+2n}{2}} - \frac{(m+n)}{(m+2n)}\Omega^{\frac{m+2n}{n}}\right)\end{array}\right] \\ + A + C_D\left[\frac{\alpha\gamma\Omega^2}{2} + \frac{n\beta\gamma\Omega^{\frac{m+2n}{n}}}{m+2n} + \frac{\alpha\gamma^2\Omega^3}{2\times 3} + \frac{n\beta\gamma^2\Omega^{\frac{m+3n}{n}}}{2(m+3n)}\right]\end{array}\right) \quad (10)$$

In order to minimize the total inventory cost, we find the optimal values of T and Ω. The optimal values of T and Ω can be obtained by equating the partial derivatives of total inventory cost with respect to T and Ω respectively to zero.

For optimal value of Ω and T we have

$$\frac{\partial(TIC)}{\partial\Omega} = 0 \quad (11)$$

and

$$\frac{\partial(TIC)}{\partial T} = 0 \quad (12)$$

The total minimum cost per unit time TIC (T,Ω) satisfy by sufficient condition

$$\frac{\partial^2(TIC)}{\partial T^2} > 0 \quad (13)$$

and

$$\frac{\partial^2(TIC)}{\partial\Omega^2} > 0 \quad (14)$$

and

$$\frac{\partial^2(TIC)}{\partial T^2} * \frac{\partial^2(TIC)}{\partial\Omega^2} - \frac{\partial^2(TIC)}{\partial\Omega\partial T} > 0 \quad (15)$$

We get the value of Ω by solving equation (11) and we can get the value of T by solving equation (12) and putting these values in equation (10), we obtain the minimum cost per unit time for the values which satisfy the necessary condition (13), (14) and (15).

III. NUMERICAL ILLUSTRATION

Now, we consider a numerical example to examine the optimization of the solution. We used maple 18 Mathematical software to solve example.

To explain the model numerically, the following parameters of the inventory system are: α= 14, β= 10, ϒ= 12, C_H = 28, C_S = 15, A = 500, C_D =10, m=3,n=4.

We get the optimal shortage value by the above given parameters by using Maple 18 , Ω=0.2437609067 , T = 10.63600040. Finally, the total optimal cost obtained is TIC=64702.11352.

The graph is as shown in figure 2 and its three-dimensional representation is shown in figure 3.

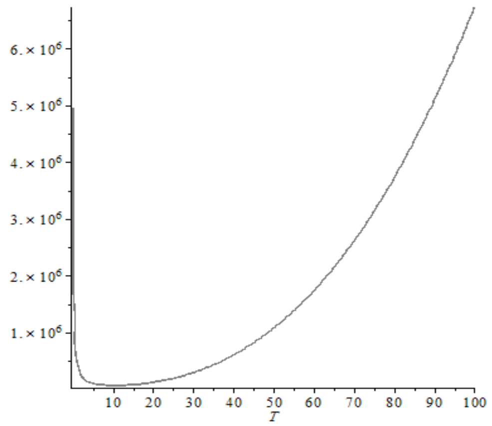

Figure 2. shows total cost function verse

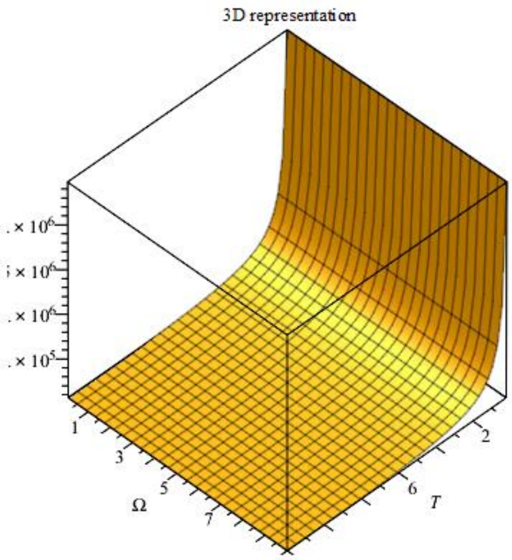

Figure 3. shows total cost function verses Ω and T

IV. CONCLUDING REMARKS

A deterministic inventory model-based demand dependent on time raised to the power m by n, constant decline rate, and constant holding cost is built in this paper. The total optimal cost has been calculated for the values of Ω and T, satisfying the necessary condition. The model has been tested using numerical and graphical diagrams. This model refers to the model in which demand changes fractionally over time as commodity prices increase day by day. This model is more realistic to the environment. As a result, future cities and businesses will benefit greatly from the paradigm.

REFERENCES

[1] B. N. Mandal and S. Phaujdar, An inventory model for deteriorating items and stock-dependent consumption rate, Journal of the operational Research Society, vol. 40, no. 5, pp. 483 488, 1989.
[2] A. Jalan and K. Chaudhuri, Structural properties of an inventory system with deterioration and trended demand, International Journal of Systems Science, vol. 30, no. 6, pp. 627 633, 1999.
[3] S. Ghosh and K. Chaudhuri, An order-level inventory model for a deteriorating item with Weibull distribution deterioration, time-quadratic demand and shortages, Advanced Modelling and Optimization, vol. 6, no. 1, pp. 21 35, 2004.
[4] K.L. Hou, An inventory model for deteriorating items with stock- dependent consumption rate and shortages under inflation and time discounting, European Journal of Operational Research, vol. 168, no. 2, pp. 463 474, 2006.
[5] Y. Shah, An order-level lot-size inventory model for deteriorating items, AIIE transactions, vol. 9, no. 1, pp. 108 112, 1977.
[6] R. B. Misra, Optimum production lot size model for a system with deteriorating inventory, The International Journal of Production Research, vol. 13, no. 5, pp. 495 505, 1975.
[7] A. Jalan, R. Giri, and K. Chaudhuri, EOQ model for items with Weibull distribution deterioration, shortages and trended demand, International Journal of Systems Science, vol. 27, no. 9, pp. 851 855, 1996.
[8] A. Roy, An inventory model for deteriorating items with price dependent demand and time varying holding cost, Advanced modelling and optimization, vol. 10, no. 1, pp. 25 37, 2008.
[9] M. Cheng and G. Wang, A note on the inventory model for deteriorating items with trapezoidal type demand rate, Computers & Industrial Engineering, vol. 56, no. 4, pp. 1296 1300, 2009.
[10] J. Min, Y.-W. Zhou, and J. Zhao, An inventory model for deteriorating items under stock-dependent demand and two-level trade credit, Applied Mathematical Modelling, vol. 34, no. 11, pp. 3273 3285, 2010.
[11] C. Tripathy and U. Mishra, An EOQ model with time dependent Weibull deterioration and ramp type demand, International Journal of Indus- trial Engineering Computations, vol. 2, no. 2, pp. 307 318, 2011.
[12] V. K. Mishra and L. Sahab Singh, deteriorating inventory model for time dependent demand and holding cost with partial backlogging, International Journal of Management Science and Engineering Management, vol. 6, no. 4, pp. 267 271, 2011.
[13] R. Amutha and D. E. Chandrasekaran, An inventory model for deteriorating products with Weibull distribution deterioration, time-varying demand and partial backlogging, International Journal of Scientific & Engineering Research, vol. 3, no. 10, pp. 1 4, 2012.
[14] R. Venkateswarlu and R. Mohan, An inventory model with quadratic demand, constant deterioration and salvage value, Research Journal of Mathematical and Statistical Sciences, Vol. 2320, p. 6047, 2014.
[15] V. K. Mishra, Deteriorating inventory model with quadratically time varying demand and partial backlogging, WPOM, vol. 4, no. 2, pp. 16 28, 2013.
[16] S. Singh and H. Rathore, Optimal payment policy for deteriorating item with preservation technology investment under trade credit and inflation, in International Conference on Recent Advances and Innovations in Engineering (ICRAIE-2014). IEEE, 2014, pp. 1 6.
[17] N. H. Shah, U. Chaudhari, and M. Y. Jani, An integrated production- inventory model with preservation technology investment for time- varying deteriorating item under time and price sensitive demand, International Journal of Inventory Research, vol. 3, no. 1, pp. 81 98, 2016.
[18] M. Rastogi, S. Singh, P. Kushwaha, and S. Tayal, Uncertain supply chain management.
[19] A. H. M. Mashud, An EOQ deteriorating inventory model with different types of demand and fully backlogged shortages, International Journal of Logistics Systems and Management, vol. 36, no. 1, pp. 16 45, 2020.