A Method to Model the Maximum Power Output of Photovoltaic Modules Using Statistical Analysis and Matlab-Simulink Simulation

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Abstract- A photovoltaic module has been fabricated with 36 pseudo-square large size mono-crystalline silicon solar cells (125mm X 125mm) with the spacing between cells as 3mm and 2mm in horizontal and vertical directions respectively and its illuminated current-voltage (I-V) characteristics have been measured using a solar simulator. All the cells used in the modules were earlier modeled using the two-diode equivalent circuit model of the solar cell; and the parameters of all the solar cells were individually extracted using the Particle Swarm Optimization (PSO) method through curve-fitting of measured and calculated I-V curves. A Matlab-Simulink model has been developed for the module to simulate the characteristics of the fabricated modules. Simulated I-V curve has been matched to the measured I-V curve of a 36-cell series connected module by adjustment of various parameters and environment factors. Also, output of one random 36-cells module has been simulated 2500 times with parameters of the individual cells randomly picked through the lognormal distribution and the maximum output power from the module has been statistically analyzed.

Keywords—PV Modules; Matlab-Simulink Model; Module Characteristics; Simulated Module Characteristics; PV System; Maximum Power Output

1. INTRODUCTION

Photovoltaic conversion of solar energy is gaining importance since the last three decades due to ever increasing demand for power and limited availability of fossil fuels. Easy availability and pollution-free nature of photovoltaic energy makes it even more useful source of energy. A single photovoltaic (PV) cell or solar cell does not generate enough power to drive any major and useful application. So many solar cells are connected in series and parallel to generate high voltage and high power output. In the first level of interconnection, cells are usually connected in series to form a PV module. In the second level of interconnection, the PV modules are connected in series or parallel to form a PV array. To increase the output voltage, the series connection of the modules is done, whereas to increase the output current, the parallel connection is done.

In a PV module, the solar cells are usually connected in series to increase the voltage output of the module. The current from the PV module with series-connected solar cells is equal to the current through each solar cell and it primarily depends on the size of the solar cells used in the PV module. In a PV module, spacing of few millimeters is kept between solar cells to make connections between cells and to avoid electrical shorting of cells. So, the actual module area is generally higher than the total area of solar cells by about 10% to 20% [1]. The output voltage (V_{module}) and output current (I_{module}) of the module with N_c number of series connected solar cells (all assumed to be equivalent in the ideal case) is usually modeled in literature [2-4] using a one-diode model for each cell (each cell assumed to be identical) and is given by

\[ I = I_{module} = I_{ph} - I_0 \left[ \exp \left( \frac{V_d}{nV_T} \right) - 1 \right] - \frac{V_d}{R_{sh}} \]  \hspace{1cm} (1)

where \( V_d = \) voltage across each diode of the solar cell = \( V_{module}/N_c \) + IR.

Modeling of a p-n junction crystalline silicon solar cell (referred to as solar cell hereafter) and PV array has been studied in references [5-9] to predict the electrical output of a solar cell or PV system under different values of temperature and irradiance. All these authors have simulated the behavior of one solar cell or a combination of solar cells using software such as PSpice, LTSpice and Matlab. One-diode and two-diode equivalent circuit models have been used by most of the researchers for simulating the behavior of a single p-n junction silicon solar cell. Models for estimating the outputs of PV modules and PV panels have been suggested with simplified current equations [2-4, 10-12], most of which demonstrate good performance under varying conditions of temperature and irradiance. Can, et al. [11] suggested a numerical method to find the initial solution of the transcendental model of PV panels. PV panel models have been proposed using single-diode five parameter model [3, 4]; Ma, et al. [4] have even assessed the performance of a PV system at a remote island in Hong-Kong using the proposed model. Models for PV modules have been developed for software such as PSpice, Saber and Matlab/Simulink [6, 13-17], and these models have been simulated under varying conditions of temperature and irradiance. Performance of a PV system installed at New Delhi was assessed to investigate the electrical energy output and power conversion efficiency at different climatic conditions by Sharma and Tiwari [18].

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Huld, et al. [19] have suggested a model for PV modules that takes into consideration temperature and irradiance conditions; moreover, it incorporates certain coefficients found by fitting of measured data for various indoor and outdoor conditions. The authors [19] validated the model by taking data in different seasons from three different PV modules in outdoor conditions and data of 18 different PV module types in indoor conditions.

Following gaps in understanding output characteristics of modules were there.

1. No work has been reported where each of the solar cells constituting the module was studied before the module was fabricated and outputs of each cell and the module were correlated. Most assumed that the cells in the module are identical.

2. Most simulations of modules have been done using the one-diode model for constituent cells. Only a few reports [10] of use of two-diode model (a better model than one-diode model) for the cells are available for such simulation of modules.

3. No correlation has been established experimentally between the output characteristics of the module and the combined effect of module fabrication process and the environmental factors.

4. There is no method available to predict the power output of modules made from a large batch of solar cells in production line.

In this work, an attempt has been made to address these identified gaps. One solar cell module was custom fabricated using 36 large size pseudo-square (125mm X 125 mm in size) single crystalline silicon solar cells taken from the production line of an Indian PV industry. Earlier, all these cells were modeled using the two-diode equivalent circuits; and the parameters of models of each of these cells were estimated from their measured illuminated current-voltage (I-V) characteristics individually using the Particle Swarm Optimization (PSO) method.

Matlab-Simulink model was created to simulate the 36-cell module; individual cells were fed with the estimated parameters as inputs. Also, the I-V characteristics of the fabricated module were measured using a calibrated solar simulator. The differences in the measured and simulated I-V curves of the module have been observed in this paper, which in turn helped in discovering the effects of environmental factors and the module fabrication process on the output characteristics of the module. The variation in parameters of the equivalent circuit model could be noticed when well characterized individual PV cells became part of the PV module.

In addition, the 36-cell modules have been simulated 2500 times for the statistical analysis of the maximum output power obtained from the modules, with the parameters of each cell of each module chosen randomly using the lognormal distributions that the parameters follow for a batch production. Through this process, the power outputs of modules made from a large batch of solar cells have been predicted.

The paper is organized as follows. Section 2 presents the overview of the custom module fabricated. Section 3 shows the Simulink model of the module, and the estimated parameter values used for two-diode model of individual solar cells. The mismatch in simulated and measured I-V curves of the fabricated module are discussed in Section 4 followed by a detailed discussion on the parameter values affecting this mismatch. Statistical analysis of the maximum power obtained from 36-cells module fabricated using randomly picking individual cells with random distribution of the two-diode model parameters is discussed in Section 5. Finally, the paper is concluded in Section 6.

2. CUSTOM PV MODULE

A custom module was fabricated using 36 single crystalline pseudo-square silicon solar cells of size 125mm X 125 mm. These cells with n+-p+p+ structure were taken from one of the PV industry in India. Each cell was made from a boron-doped p-type Cz single crystalline silicon wafer. n+-p junction was created by POCI3 dustion, back surface field (BSF) as a p+ layer was created using aluminum alloying. Front and back contacts were created using Ag metal through screen-printing technology. Wafers were randomly textured by dipping in alkaline solution before device fabrication to reduce reflectivity. Also, all the cells used in this study had anti-reflection coating of silicon nitride to further reduce reflection losses from solar cells. More details are available in our earlier work [20].

All solar cells were individually tested for their maximum power outputs and efficiencies and were sorted in different groups or bins. The cells were sorted in such a way that the cells in the one group had efficiency values as close as possible. The cells from the same group were taken for parameter estimation through PSO method (discussed in Section 3) and later these cells were used in the fabrication of the two modules. The average values of Im and conversion efficiency (AM1.5) of this batch of samples were 6A and 18.44% respectively.

Fig. 1 shows the various material layers used in the fabrication of the custom crystalline silicon PV module. The metal contacts were made on each cell using soldering of tinned copper metal strips (this process is called tabbing). All the 36 cells were connected in series using the process called stringing. Tabbing and stringing using tinned copper strips were done manually for fabrication of module used in this work. These electrically connected cells were kept between two layers of encapsulant to protect the solar cells from environment. Ethylene Vinyl Acetate (EVA) was used as encapsulant for silicon PV module. To provide rigidity to the module, glass was used at the front side of the module and a hard polymer material (white in colour), called Tedlar was used at the back side. The sheets of glass, encapsulants, electrically connected cells and Tedlar were arranged together and placed in a laminator and module was heated for lamination to about 80°C to 100°C. Initially the EVA sheet was translucent, but during the lamination process, the EVA
sheets on both sides of the cells melted and surrounded the solar cells forming seals to the front glass and the Tedlar sheet at the back side of the module. After the lamination process, the module was framed in an aluminium frame and a plastic junction box was added at the rear side of the module for electrical connections. Similar steps for the fabrication of a crystalline silicon PV module have been discussed in reference [1].

The spacing between cells in the fabricated module was kept as per industry standard practice (2-3mm), 2mm and 3mm in vertical and horizontal directions respectively (Fig. 2). While carrying out I-V measurements of the panels under the solar simulator, the white space at right side was covered with a black sheet.

3. SIMULINK MODEL

Matlab-Simulink model was generated for the PV module as shown in Fig. 3. The subsystem block had 36 solar cells connected in series. Authors in an earlier work [5] had worked on a similar model for simulation of single solar cell and two solar cells in series using one-diode model. In the present work, the model was created to simulate a module with 36-cells (cells connected in series) to get the estimated I-V characteristics of the module and to predict the output power obtained from the module. Subsystem in Fig. 3 consisted of 36 cells in series for simulation of the module. Voltage sensor and current sensor blocks were used to measure the voltage and current through the module. Blocks for varying irradiance and temperature and to plot I-V and P-V characteristics were also added. A couple of blocks were added as interface blocks between the other major blocks.
Each individual solar cell (modeled using the two-diode model available in Matlab-Simulink) in the PV module model was fed with the two-diode model parameters estimated through the PSO method [21-22]. The two diode equation (Eq. 2) for the $i^{th}$ silicon solar cell (Fig. 4) is given by

$$I = I_{ph} - I_{d1} - I_{d2} - I_{sh}$$

$$= I_{ph} - I_{01} \left[ \exp \left( \frac{V_i + R_{sh}}{n_1 V_t} \right) - 1 \right] - I_{021} \left[ \exp \left( \frac{V_i + R_{sh}}{n_2 V_t} \right) - 1 \right]$$

where $V_i =$ voltage across the $i^{th}$ silicon solar cell in the module of ‘$N_s$’ cells.

3.1 Determination of Two-Diode Model Parameters using Particle Swarm Optimization (PSO)

Extensive work has been done and reported on how to estimate the parameters of the one-diode and two-diode models using analytical [23-26] and evolutionary techniques [21-22, 27-32] from the measured illuminated I-V characteristics of a cell. PSO technique is one of the evolutionary methods, which has been applied for estimating the parameters of one-diode and two-diode models of the PV cells in references [21-22, 31-32].

PSO is an optimization algorithm based on the movement of a flock of birds in search of food. Initially, when birds start searching for food, they move randomly and after some time, all the birds follow the bird that is nearest to the food. This algorithm can be applied to any optimization where a function needs to be minimized (or maximized). Initially some random solutions (called particles or birds) are generated; each particle has its own position and velocity. Fitness value of the fitness function (to be optimized) is calculated for each particle. At every iteration, two fitness values are calculated; one the best fitness value of each particle called particle best or ‘pbest’ and second the best fitness value achieved so far by any particle in the population, called global best or ‘gbest’. After finding ‘pbest’ and ‘gbest’ values, the position and velocity of each particle is updated as per the equations 3(a) and 3(b).

$$v_{[t+1]} = w \ast v_{[t]} + c1 \ast rand \ast (pbest - x_{[t]}) + c2 \ast rand \ast (gbest - x_{[t]})$$  

$$x_{[t+1]} = x_{[t]} + v_{[t+1]}$$

where, $v_{[t]}$ and $x_{[t]}$ represent the velocity and position of particles at time $t$, and $v_{[t+1]}$ and $x_{[t+1]}$ are the updated velocity and position at time $t+1$, which are updated based on the best fitness values pbest and gbest. $w$ is an inertia weight that lies in range (0, 1), and decreases in subsequent iterations to slowly reduce the inertia, and thus solution is diverted towards optimization through local exploitation. $rand$ is a random number generated between 0 and 1. $c1$ and $c2$ are constants called cognitive (local) and social (global) weights respectively; usually $c1=2$ and $c2=2$.

PSO algorithm was implemented on the measured I-V characteristics to estimate the two-diode equivalent circuit model parameters of all the solar cells, which were later used in fabrication of the custom module. The I-V characteristics of the individual solar cells were measured at 2500 points between short circuit and open circuit conditions using a solar simulator and the error between the measured and the calculated I-V characteristics was minimized to estimate the parameters of two-diode equivalent circuit model (Fig. 4) of the solar cells. The parameters to be extracted for two-diode equivalent circuit model were following: $I_{ph}$, $I_{01}$, $I_{02}$, $R_{s}$, $R_{sh}$, $n_1$ and $n_2$. Matlab coding was done to implement the PSO algorithm following through the following steps:

i) Random solutions were initially generated (within pre-defined ranges) for all the parameters to be estimated.

ii) Objective (fitness) function was defined as the Mean Absolute Error (MAE) as in Eq. 4, which is a measure of mean of the absolute errors calculated at each point of the I-V curve. Coding was done to minimize the objective function.

$$MAE = \frac{1}{n_{ph}} \sum_{i=1}^{n_{ph}} \left| \frac{I_{calculated}-I_{measured}}{I_{calculated}} \right|$$

iii) Roots of the Eq. 2 were found by using Newton Raphson method. Fitness values of all the particles were calculated at velocity and position for time $t$.

iv) Each particle’s velocity and position were updated for time $t+1$ (equations 3(a) and 3(b)).
Steps iii) and iv) were repeated till a reasonably low value of MAE was achieved or till a particular number of iterations were done.

Estimated parameters of solar cells used in fabrication of the custom module are tabulated in Table 1.

4. SIMULATED VS. MEASURED I-V CHARACTERISTICS OF THE MODULE

The I-V characteristics of the fabricated module were measured using a solar simulator installed at Central Electronics Ltd., Sahibabad. Simulated I-V characteristics were generated by carrying out simulation of the module under standard test conditions (STC) using the models discussed in Section 3. Fig. 5 shows the plot of experimental/measured and simulated I-V characteristics of the module. There was a mismatch in the I-V characteristics; the measured open-circuit voltage of the module, $V_{oc_{module}}$, was less as compared to the simulated value. Measured value of short-circuit current of the module, $I_{sc_{module}}$, was slightly lower than the corresponding simulated value.

Fig. 5 - Measured and simulated (at standard test conditions (STC)) I-V characteristics of the 36-cell module.

4.1 Irradiance and Temperature Effects on PV Module

The combined effect of module fabrication process and the environmental factors of temperature and irradiance affect the I-V characteristics of a module and hence the power output in the following ways:

- Packing density of a module is defined as the percentage of cell area in the entire module area, this is generally defined for different shapes of solar cells used on the module.

Less the packing density of the module, more would be the light transmitted up to the rear sheet through the free space between cells, and this light from the white rear sheet would be reflected back to the solar cells causing light trapping in the module. This in-turn would slightly increase the $I_{sc_{module}}$ of the module. However, there will be absorption of solar radiation in the front glass cover and the EVA layer that would cause the $I_{sc_{module}}$ to decrease.

- The local cell temperature inside the PV module would be higher than the ambient temperature, depending on the conditions such as solar irradiance, wind circulation. The higher cell temperature is due to the glass cover of the module that would trap the infrared radiation and increase the cell temperature. With the increased cell temperature, the decrease in $V_{oc_{module}}$ is more prominent than increase in $I_{sc_{module}}$. Solanki [1] has given the following values of temperature coefficients of a typical PV module: -0.075 to -0.085 V/°C for $V_{oc_{module}}$ and +0.06%/°C to +0.1%/°C for $I_{sc_{module}}$.

<table>
<thead>
<tr>
<th>Cell no.</th>
<th>$I_0$ (nA)</th>
<th>$I_C$ (μA)</th>
<th>$R_s$ (mΩ)</th>
<th>$R_{sh}$ (Ω)</th>
<th>$n_1$</th>
<th>$n_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3.483</td>
<td>21.076</td>
<td>7.171</td>
<td>10.269</td>
<td>1.188</td>
<td>2.544</td>
</tr>
<tr>
<td>5</td>
<td>2.731</td>
<td>81.351</td>
<td>11.239</td>
<td>20.438</td>
<td>1.168</td>
<td>2.970</td>
</tr>
<tr>
<td>6</td>
<td>4.311</td>
<td>63.324</td>
<td>13.293</td>
<td>16.183</td>
<td>1.195</td>
<td>2.794</td>
</tr>
<tr>
<td>8</td>
<td>6.501</td>
<td>2.705</td>
<td>9.963</td>
<td>105.766</td>
<td>1.223</td>
<td>2.245</td>
</tr>
<tr>
<td>9</td>
<td>4.445</td>
<td>10.960</td>
<td>11.130</td>
<td>15.187</td>
<td>1.195</td>
<td>2.493</td>
</tr>
<tr>
<td>10</td>
<td>8.399</td>
<td>15.286</td>
<td>15.152</td>
<td>33.704</td>
<td>1.233</td>
<td>2.570</td>
</tr>
<tr>
<td>11</td>
<td>8.388</td>
<td>14.674</td>
<td>9.835</td>
<td>11.743</td>
<td>1.228</td>
<td>2.654</td>
</tr>
<tr>
<td>12</td>
<td>5.296</td>
<td>18.677</td>
<td>13.636</td>
<td>8.825</td>
<td>1.204</td>
<td>2.671</td>
</tr>
</tbody>
</table>
Based on the discussion above, the simulations of the I-V characteristics of the module were carried out at varying values of irradiance (I_r) and cell temperature (T) to match the I_{sc module} and V_{oc module} values. Since there was negligible space between cells, no light was reflected back from white Tedlar sheet at the back. In fact, the I_{sc module} slightly decreased perhaps mainly due to the absorption of solar irradiance in the front glass cover and EVA sheet (Please see Fig. 1). Thus, the measured and simulated I-V characteristics for the module matched at solar radiation of 975W/m² and at a cell temperature equal to 43°C as can be seen in Fig. 6.

![](image)

**Fig. 6 - Measured and simulated I-V characteristics of the 36-cell module at different values of irradiance, I_r, and temperature, T.**

### 4.1 Effect of Series and Shunt Resistances on PV Module Characteristics

Despite of match at I_{sc module} and V_{oc module} points of the I-V characteristics due to change in irradiance and temperature, as seen in Fig. 6, the measured and simulated I-V curves were
not matching well elsewhere for the module. Observations from the Fig. 6 showed that \( R_{smodule} \) and \( R_{shmodule} \) of the measured characteristics of the module had low values as compared to the simulated characteristics. The slopes of the I-V curves near \( I_{scmodule} \) showed that \( R_{sh} \) values needed adjustments. Similarly, \( R_s \) values needed adjustment to match the simulated characteristics to the measured characteristics of the module. \( R_{si} \) and \( R_{shi} \) (series and shunt resistances of \( i^{th} \) cell, where \( i=1 \) to \( 36 \) for a 36-cells module) for each cell in module could have changed during moduleing.

- Equivalent circuit of a photovoltaic module consisting of ‘\( N_s \)’ solar cells in series, can be modeled as given in Fig. 7.
- As all solar cells are connected in series in the module, \( R_s \)'s are connected in series and the series resistance of the module (\( R_{smodule} \)) should be \( \sum R_s \). But the effective \( R_{smodule} \) was found to be less, i.e., equal to \( \sum R_s/3.5 \) for the 36-cell module. The effective reduction in \( R_{smodule} \) could be averaged for each cell giving the reduction in the effective \( R_s \) of each cell. While connecting solar cells in series for module fabrication, the effective series resistance of individual cells might have reduced due to tabbing that reduced the front grid mid-rib contribution to \( R_s \) of each cell.
- Since total \( R_{sh} \) was also decreasing. It could not be predicted if \( R_{shi} \)'s of each cell had decreased or not, since the connections amongst \( R_{sh} \)'s are not in series or parallel (Fig. 7). It was assumed that there was an equivalent shunt resistance, \( R_{shmodule} \), appearing at the terminals of the module, during the process of module fabrication.

Based on the above discussion, simulations were carried out with reduced values of effective \( R_s \)'s, and \( R_{shmodule} \) connected in simulink model. Simulated characteristics matched measured characteristics by reducing \( R_s \) of each cell by a factor of 3.5 and taking \( R_{shmodule} \) as 100 ohm. Fig. 8 shows the matched simulated and measured I-V characteristics of the custom module.

Fig. 7 - Equivalent circuit of a PV module (with ‘\( N_s \)’ cells connected in series) giving output current of \( I_{module} \) and output voltage of \( V_{module} \) for the module.

Fig. 8 - Matched simulated vs Measured I-V characteristics for module at high values of solar radiation and temperature, reduced series resistance of each cell and one shunt resistance added in parallel to all series connected solar cells.

5. STATISTICAL ANALYSIS OF MAXIMUM POWER OUTPUT OF A MODULE

5.1 Lognormal Distribution of Parameters
PSO algorithm was used to estimate the parameters of two diode equivalent circuit model of about 82 solar cells. All the parameters, \( I_{01} \), \( I_{02} \), \( n_1 \), \( n_2 \), \( R_s \), and \( R_{sh} \) estimated through PSO for two diode equivalent circuit model, showed the lognormal distribution, \( N(x) \), given by.

\[
N(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(\ln x - \mu)^2}{2\sigma^2} \right]
\]

where \( \mu \) and \( \sigma \) are two parameters of the distribution.

The \( \mu \) (or \( \mu_0 \)), and \( \sigma \) (or \( \sigma_0 \)), for lognormal distributions of cell parameters, \( I_{01} \), \( I_{02} \), \( n_1 \), \( n_2 \), \( R_s \), and \( R_{sh} \), were calculated through Matlab and are shown in Table 2. \( I_{ph} \) was assumed to follow a normal distribution and its \( \mu \) and \( \sigma \) were found using ‘excel’; their values came out to be 6.004A and 0.065A respectively. As examples, the lognormal distribution plots of two of the parameters, i.e., \( R_s \) and \( I_{01} \) in form of histograms are shown in Fig. 9 based on their values estimated for ~82 cells.

Table 2 - Values of lognormal \( \mu \) and \( \sigma \) calculated for different two-diode model parameters obtained for all silicon solar cells.
that for the custom 36-cell module, the temperature of the production technique as the one used for the samples. The I-V characteristics in each simulation. It has been seen in section 4 that the parameters of each solar cell fed randomly using lognormal distributions in each simulation. The distributions for the lognormal distribution of R_s are found and the corresponding random values generated were fed as R_s to the individual solar cells for simulations. R_s of 43°C reached the cells of the module. Also, the series resistance R_s of individual cells decreased by a factor of 3.5. They μ and σ for the lognormal distribution of R_s were found and the corresponding random values generated were fed as R_s to the individual solar cells for simulations. R_s of 100 ohm was also added in parallel (as in Fig. 7) for simulation purpose. For n_1 and n_2, the lognormal distributions of (n_1-1) and (n_2-2) were considered, as n_1 and n_2 for all samples were >1 and >2 respectively. Random numbers were generated for (n_1-1) and (n_2-2) distributions, and these values were then fed for simulations of a module. The simulations for 36-cells modules using the simulink model were then carried out 2500 times with the same conditions of temperature (43°C) and irradiance (975W/m²).

In our earlier work [20], the correlation between I_{ph} and n_1 was found by using the measured data of 82 solar cells, as given in Eq. 6.

$$I_{01} = I_0 \exp \left[b_1 \left(1 - \frac{1}{n_1} \right) \right]$$

where I_0 = 62.87 ± 11.38 pA and b_1 = 24.84 ± 0.91

In the present work, for the statistical analysis of simulated maximum power P_{max} of the 36-cell module, simulation of the module was carried out for two cases.

**Case 1: All parameters taken independently**

Random values of all parameters (I_{01}, I_{02}, n_1, n_2, R_s, and R_{sh}) were generated as stated above. Random values of I_{ph} were generated through Matlab command: randn. The simulink model for 36-cells module (Fig. 3) was simulated 2500 times, each time with newly generated random values of all parameters. P_{max} was found for each run.

**Case 2: I_{01} related to n_1 through Eq. 6**

Random values of all parameters except I_{01} were generated as in case 1. I_{01} was calculated using value of n_1 and Eq. 6. Random values of I_{ph} were generated as in case 1. Simulink model for one 36-cell module was simulated 2500 times for this case also and P_{max} was found for each run.

7. **STATISTICAL DISTRIBUTION OF P_{max} FOR SIMULATED MODULES**

Histogram plots for P_{max} found through simulation of randomly chosen 2500 solar cell modules for each of the two cases, gave normal distribution. Fig. 10(a) gives the histogram plot for the P_{max} values obtained from case 1 and Fig. 10(b) shows the plot for P_{max} obtained in case 2. Most parameters of an electronic device follow a lognormal distribution for their occurrence. Since in the present case, the mode of P_{max} was far from zero and its spread was small (Figs. 10(a) & (b)), the lognormal distribution should approximate to a normal distribution for P_{max}. Both histogram plots were fitted with the normal distributions and their mean and standard deviation were calculated and shown in respective figures. When I_{01} and n_1 were taken to vary independently

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**Table 1: Statistical parameter of lognormal distribution of P_{max} for 36-cell module**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \mu ) (lognormal)</th>
<th>( \sigma ) (lognormal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I_{01} (A)</td>
<td>-19.3728</td>
<td>0.6564</td>
</tr>
<tr>
<td>I_{02} (A)</td>
<td>-10.0675</td>
<td>0.8782</td>
</tr>
<tr>
<td>R_s (Ω)</td>
<td>-4.5582</td>
<td>0.1983</td>
</tr>
<tr>
<td>R_{shmodule} = R_s/3.5 (Ω)</td>
<td>-5.811</td>
<td>0.1983</td>
</tr>
<tr>
<td>R_{sh} (Ω)</td>
<td>2.8125</td>
<td>1.0869</td>
</tr>
<tr>
<td>n_1</td>
<td>0.1784</td>
<td>0.03</td>
</tr>
<tr>
<td>n_2</td>
<td>1.0021</td>
<td>0.0673</td>
</tr>
<tr>
<td>n_1-1</td>
<td>-1.6497</td>
<td>0.207</td>
</tr>
<tr>
<td>n_2-2</td>
<td>-0.3526</td>
<td>0.2984</td>
</tr>
</tbody>
</table>

![Chart A](image1.png)

![Chart B](image2.png)

**Fig. 9** - Distribution (histogram) plots of parameters a) R_s and b) I_{01}, estimated from the measured illuminated I-V characteristics of 82 solar cell samples. Lognormal fit to the data is also shown superimposed.

6. **SIMULINK MODEL SIMULATION FOR P_{max}**

It can be assumed that the lognormal distributions for the parameters determined from measured illuminated I-V data of a sample size of 82 cells (as given in Table 2) will be valid for a large production batch of cells made using the same production technique as the one used for the samples. The I-V characteristics of a randomly chosen 36-cell module have been simulated using the simulink model to find the maximum power output, P_{max}, of the module each time, with parameters of each solar cell fed randomly using lognormal distribution in each simulation. It has been seen in section 4 that for the custom 36-cell module, the temperature of the module increased to 43°C and an effective irradiance of 975W/m² reached the cells of the module. Also, the series resistance R_s of individual cells decreased by a factor of 3.5. The μ and σ for the lognormal distribution of R_s were found and the corresponding random values generated were fed as R_s to the individual solar cells for simulations. R_s of 100 ohm was also added in parallel (as in Fig. 7) for simulation purpose. For n_1 and n_2, the lognormal distributions of (n_1-1) and (n_2-2) were considered, as n_1 and n_2 for all samples were >1 and >2 respectively. Random numbers were generated for (n_1-1) and (n_2-2) distributions, and these values were added with 1 and 2 respectively for calculating n_1 and n_2 which were then fed for simulations of a module. The simulations for 36-cells modules using the simulink model were then carried out 2500 times with the same conditions of temperature (43°C) and irradiance (975W/m²).

In our earlier work [20], the correlation between I_{01} and n_1 was found by using the measured data of 82 solar cells, as given in Eq. 6.

$$I_{01} = I_0 \exp \left[b_1 \left(1 - \frac{1}{n_1} \right) \right]$$

where I_0 = 62.87 ± 11.38 pA and b_1 = 24.84 ± 0.91

In the present work, for the statistical analysis of simulated maximum power P_{max} of the 36-cell module, simulation of the module was carried out for two cases.

**Case 1: All parameters taken independently**

Random values of all parameters (I_{01}, I_{02}, n_1, n_2, R_s, and R_{sh}) were generated as stated above. Random values of I_{ph} were generated through Matlab command: randn. The simulink model for 36-cells module (Fig. 3) was simulated 2500 times, each time with newly generated random values of all parameters. P_{max} was found for each run.

**Case 2: I_{01} related to n_1 through Eq. 6**

Random values of all parameters except I_{01} were generated as in case 1. I_{01} was calculated using value of n_1 and Eq. 6. Random values of I_{ph} were generated as in case 1. Simulink model for one 36-cell module was simulated 2500 times for this case also and P_{max} was found for each run.
(i.e., case 1), the normal distribution had more spread than the case when the established relationship between $I_{01}$ and $n_1$ [20] was used (i.e., case 2). This can also be judged by the values of standard deviation of 1.02W and 0.77W in case 1 and case 2 respectively. The standard deviation in case 2 was about 25% lower than in case 1. On the contrary, the mean $P_{max}$ value in case 2 was slightly higher (93.72W) as compared to the mean $P_{max}$ value in case 1 (93.31W).

$P_{max}$ value of the matched simulated characteristics to the measured characteristics of the fabricated 36-cell module (Fig. 8), came out to be 94.08W. Custom module was fabricated from the solar cells whose two-diode model parameters were estimated through the PSO algorithm; and multiple(2500) simulations of the 36-cells module were carried out with random values of the parameters fed for all the solar cells determined through the lognormal distribution of estimated parameters of solar cells. In our earlier work [20], 82 sets of parameters were checked for correlation among them, and $I_{01}$ and $n_1$ were found to be highly correlated through an exponential relation (Eq. 6). So, case 2 results become more relevant and are even better than case 1 results, in terms of both mean and standard deviation. In case 2, the minimum and maximum values of $P_{max}$ came out to be 89.41W and 96.01W respectively, with the mean and standard deviation as 93.72W and 0.77W respectively (Fig. 10(b)). The value of $P_{max}$ obtained for the matched simulated characteristics (Fig. 8) as 94.08W, is one typical example that validates our analysis with 2500 modules through simulation of I-V characteristics for both cases 1 and 2.

Thus, the statistical analysis done for the $P_{max}$ of the PV modules, gave us a method to estimate the power output spread that would be obtained from different modules fabricated from solar cells taken from the same production batch that is fabricated with controlled parameters for the starting silicon wafer in a production line. These parameters should be estimated from a sample of appropriate size taken from the production line.

8. CONCLUSION

Comparing the measured I-V characteristics of one series connected PV module and its simulated I-V characteristics, obtained through simulink model of the module using PV parameters of each cell in the module, we have found that there were mismatches between the measured and the simulated I-V characteristics of the module. An increase in the local cell temperature to 43°C would be required to explain the decrease in $V_{oc}$ for the measured I-V characteristics. We have also found that $R_s$ of each solar cell decreased, when connected in module using tabbing process; and an additional shunt resistance $R_{shmodule}$ came into picture, during the module fabrication process. The present work gives a thorough picture of the changes occurring in a module due to module fabrication process. From statistical analysis of simulated maximum power outputs obtained from 2500 such 36-cell modules, simulated each time by randomly picking the parameters of the two-diode model for each solar cell from lognormal distributions of the parameters, we have found that the maximum power outputs of modules followed a normal distribution. If the interdependence of $I_{01}$ and $n_1$ was considered [20], the results of maximum power output were better with higher mean value and lower standard deviation, as compared to the case when $I_{01}$ and $n_1$ were considered independent of each other. This work gives a method to predict the spread in the maximum power output of modules made in a production line if the two diode parameters of silicon solar cells used can be statistically analyzed.
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