

A Method for Synthesizing Low Voltage Noise in Power Lines for Telecommunication Applications

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Abstract

Nonparametric modelling of measured noise characteristics in both time and frequency domains shows long tailed characteristics of the measured noise models. This finding has been proven through the application of alpha stable models for the power line noise. Thus, the noise process in PLC systems can be considered a Levy (alpha) stable stochastic process that is clearly non-Gaussian. From the alpha stable noise models parameters and the models developed thereof, in this paper, a noise synthesis stochastic framework is then developed. This is necessary because the outcome of such a synthesis process will eliminate the need to perform noise measurements in practical PLC systems in future. In this paper, a mathematical framework that is applicable in the synthesis of the noise process in practical PLC systems as a Levy stable process is developed and an appropriate algorithm for PLC noise synthesis as a Levy stable process is presented. Further, synthesizing of the noise process for a PLC system using the alpha stable noise parameters obtained in earlier research works by the same author using the proposed algorithm for a random number of noise samples is done. The synthesis results obtained are validated appropriately using error analysis and Chi-square tests.

1. Background mathematical concepts for noise synthesis

The importance of stable distributions, otherwise known as Levy stable distributions cannot be overemphasized. They are fundamentally justified by the central limit theorem, as an approximation for normality. Actually, these distributions are the only limiting laws of normalized independent, identically distributed variables. These distributions are excellent for modelling phenomena that are characterized by high variability, like the one witnessed in powerline noise, where the impulsive noise can be as high as 50 dB above the background noise. With the confirmation that power line noise in indoor low voltage networks is actually alpha stable, there is need for an urgent development of a synthesizer of such noise. However, except for a few special classes of limiting distributions that include the Gaussian, Cauchy and Levy distributions, the closed form expressions for the cumulative distribution inverse do not exist, and the

inverse transform method cannot be used as well. A major breakthrough towards the development of a generator of stable random variables was proposed by Chambers *et al* [1], even though the journey towards the same was started by Kanter in 1975 [2]. The proposals in both papers have proven to be very useful and have also been applied in the generation of discrete stable and Linnik's random variables, see for example [3, 4]. More recently, this method was revisited in [5, 6], where the equality in law of a skewed stable variable was proven together with a nonlinear transformation of an independent exponential variable and an independent uniform variable. The Chamber *et al.* method is based on the proofs. The power line noise synthesis framework developed is based on the equality in law of a skewed stable variable and the nonlinear transformation of an independent exponential variable and an independent uniform variable. This is because, the method proposed in [2] has been proven to be the most accurate and the fastest as well, even though other proposals have been found in literature [7]. It has also been widely studied with applications to other fields; see for example [8-13]. The mathematical basis/background for the algorithm proposed is described below.

From Zolotarev [14], through the transformation of both β and σ , a standard random variable N is stable if and only if its log characteristic function is given as:

$$\log \phi(t) = \begin{cases} -\sigma_2^\alpha |t|^\alpha \exp \left[-j\beta_2 \operatorname{sgn}(t) \frac{\pi}{2} K(\alpha) \right] + j\mu t & \alpha \neq 1 \\ -\sigma_2 |t| \left[\frac{\pi}{2} + j\beta_2 \operatorname{sgn}(t) \log|t| \right] + j\mu t & \alpha = 1 \end{cases} \quad (1)$$

Where σ is another notation for dispersion parameter while μ is the location parameter, and:

$$K(\alpha) = \alpha - 1 + \operatorname{sgn}(1 - \alpha) = \begin{cases} \alpha & \alpha < 1 \\ \alpha - 2 & \alpha > 1 \end{cases} \quad (2)$$

The new dispersion and symmetry parameters are related to those of Equation (4.1) by:

$$\tan \left(\beta_2 \frac{\pi K(\alpha)}{2} \right) = \beta \tan \left(\frac{\pi \alpha}{2} \right), \quad \sigma_2 = \sigma \left(1 + \beta^2 \tan^2 \frac{\pi \alpha}{2} \right)^{\frac{1}{2\alpha}}, \quad \text{for } \alpha \neq 1 \quad (3)$$

And for $\alpha = 1$,

$$\beta_2 = \beta, \quad \sigma_2 = \frac{2}{\pi} \sigma \quad (4)$$

Corollary: Any two admissible quadruples of parameters $(\alpha, \beta, \sigma, \mu)$ and $(\alpha, \beta, \sigma', \mu')$ uniquely determine real numbers $a > 0$ and b such that:

$$N((\alpha, \beta, \sigma, \mu)) = aN(\alpha, \beta, \sigma', \mu') + b \quad (5)$$

Where

$$a = \frac{\sigma}{\sigma'}, \quad b = \begin{cases} \mu - \mu' \frac{\sigma}{\sigma'} & \alpha \neq 1 \\ \mu - \mu' \frac{\sigma}{\sigma'} + \gamma \beta \frac{2}{\pi} \ln \frac{\sigma}{\sigma'} & \alpha = 1 \end{cases} \quad (6)$$

If we then consider the standard stable distribution case, we can also transform it to the general case as:

$$N((\alpha, \beta, \sigma, \mu)) = aN(\alpha, \beta, 1, 0) + b \quad (7)$$

With

$$a = \sigma, \quad b = \begin{cases} \mu & \alpha \neq 1 \\ \mu - + \sigma \beta \frac{2}{\pi} \ln \sigma & \alpha = 1 \end{cases} \quad (8)$$

Next the integral forms of the density and cumulative distribution functions of the parameters α and β are determined. Consider the following three important expressions regarding the probability density function, cumulative density function and the characteristic function of alpha stable random variables respectively:

$$f(-n, \alpha, \beta) = f(n, \alpha, -\beta) \quad (9)$$

$$F(-n, \alpha, \beta) = 1 - F(n, \alpha, -\beta) \quad (10)$$

$$\phi(-t, \alpha, \beta) = \phi(t, \alpha, -\beta) \quad (11)$$

If we assume that $\phi(t, \alpha, \beta)$ and $f(n, \alpha, \beta)$ are the characteristic and density functions of a standard random variable, then, according to the inversion formula of the characteristic function:

$$\begin{aligned} f(n, \alpha, \beta) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itn} \phi(t, \alpha, \beta) dt \\ &= \frac{1}{2\pi} \left(\int_0^{\infty} e^{-itn} \phi(t, \alpha, \beta) dt + \int_0^{\infty} e^{itn} \phi(-t, \alpha, \beta) dt \right) \end{aligned} \quad (12)$$

And, given that $e^{-itn} \phi(t, \alpha, \beta) = e^{itn} \phi(-t, \alpha, \beta)$, then:

$$\begin{aligned} f(n, \alpha, \beta) &= \frac{1}{\pi} \operatorname{Re} \int_{-\infty}^{\infty} e^{-itn} \phi(t, \alpha, \beta) dt \\ &= \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} e^{-itn} \phi(-t, \alpha, \beta) dt = \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} e^{itn} \phi(t, \alpha, -\beta) dt \end{aligned} \quad (13)$$

From which the stable density function in integral form, for $\alpha \neq 1$ is given by:

$$f(n, \alpha, \beta) = \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} \exp \left(-itn - t^{\alpha} \exp \left(-i\beta \frac{\pi K(\alpha)}{2} \right) \right) dt \quad (14)$$

And for $\alpha = 1$:

$$f(n, 1, \beta) = \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} \exp \left(-itn - \frac{\pi}{2} t - i\beta t \ln t \right) dt \quad (15)$$

The cumulative function of a standard stable random variable for $\alpha = 1, \beta_2 > 0$ is then given by:

$$F(n, 1, \beta_2) = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp \left(-\exp \left(-\frac{n}{\beta_2} \right) U_1(\gamma, \beta_2) \right) d\gamma \quad (16)$$

And for $\alpha \neq 1, n > 1$,

$$F(n, \alpha, \beta_2) = C(\alpha, \beta_2) + \frac{\epsilon(\alpha)}{\pi} \int_{\gamma_o}^{\frac{\pi}{2}} \exp \left(-N^{\frac{\alpha}{1-\alpha}} U_{\alpha}(\gamma, \gamma_o) \right) d\gamma \quad (17)$$

Where:

$$\epsilon(\alpha) = \operatorname{sgn}(1 - \alpha), \quad \gamma_o = -\beta_2 \frac{\pi K(\alpha)}{2} \quad (18)$$

$$C(\alpha, \beta_2) = 1 - \frac{1}{4} \left(1 + \beta \frac{K(\alpha)}{\alpha} \right) (1 + \epsilon(\alpha)) \quad (19)$$

$$U_{\alpha}(\gamma, \gamma_o) = \left(\frac{\sin(\gamma - \gamma_o)}{\cos \gamma} \right)^{\frac{\alpha}{1-\alpha}} \frac{\cos(\gamma - \alpha(\gamma - \gamma_o))}{\cos \gamma} \quad (20)$$

$$U_1(\gamma, \beta_2) = \frac{\frac{\pi}{2} + \beta_2 \gamma}{\cos \gamma} \exp \left(\frac{1}{\beta_2} \left(\frac{\pi}{2} + \beta_2 \gamma \right) \tan \gamma \right) \quad (21)$$

From the above definitions, the random variable N is said to be a $S_{\alpha}(1, \beta_2, 0)$ random variable if and only if for $\gamma_o < \gamma < \frac{\pi}{2}$ and $n > 0$:

$$\frac{1}{\pi} \int_{\gamma_o}^{\frac{\pi}{2}} \exp \left(-N^{\frac{\alpha}{1-\alpha}} U_{\alpha}(\gamma, \gamma_o) \right) d\gamma = \begin{cases} P(0 < N \leq n) & \alpha < 1 \\ P(N \geq n) & \alpha > 1 \end{cases} \quad (22)$$

Which can be proven as follows for $0 < \alpha < 1$:

$$F(n, \alpha, \beta_2) = P(N \leq n) = \frac{1 - \beta_2}{2} + \frac{1}{\pi} \int_{\gamma_0}^{\frac{\pi}{2}} \exp\left(-N^{\frac{\alpha}{1-\alpha}} U_{\alpha}(\gamma, \gamma_0)\right) d\gamma \quad (23)$$

$$= \frac{1 - \beta_2}{2} + P(0 < N \leq n)$$

Given that for $\alpha < 1$, $\frac{1-\beta_2}{2} = P(N \leq 0)$. Also for the case when $1 < \alpha < 2$, we have:

$$F(n, \alpha, \beta_2) = P(N \leq n) = 1 - \frac{1}{\pi} \int_{\gamma_0}^{\frac{\pi}{2}} \exp\left(-N^{\frac{\alpha}{1-\alpha}} U_{\alpha}(\gamma, \gamma_0)\right) d\gamma \quad (24)$$

$$= 1 - P(N \geq n)$$

Now, for γ_0 as defined above, we define the following theorem: If γ is uniformly distributed on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, and Z is another independent random variable that is exponentially distributed with a mean equal to 1, then, we can write:

$$K = \frac{\sin \alpha (\gamma - \gamma_0)}{(\cos \gamma)^{\frac{1}{\alpha}}} \left(\frac{\cos(\gamma - \alpha(\gamma - \gamma_0))}{Z} \right)^{\frac{1-\alpha}{\alpha}} \quad (25)$$

Where $K \sim S_{\alpha}(1, \beta_2, 0)$ for $\alpha \neq 1$, while for $\alpha = 1$:

$$K = \left(\frac{\pi}{2} + \beta_2 \gamma\right) \tan \gamma - \beta_2 \log \left(\frac{Z \cos \gamma}{\frac{\pi}{2} + \beta_2 \gamma} \right) \quad (26)$$

Where $K \sim S_1(1, \beta_2, 0)$.

To prove the theorems represented by Equations (25) and (26), we proceed as follows: From Equations (25), we can have:

$$K = \left(a(\gamma) / Z \right)^{\frac{1-\alpha}{\alpha}} \quad (27)$$

where

$$a(\gamma) = \frac{\sin \alpha (\gamma - \gamma_0)^{\frac{\alpha}{1-\alpha}} \cos(\gamma - \alpha(\gamma - \gamma_0))}{\cos \gamma} \quad (28)$$

For $0 < \alpha < 1$, Equation (25) has the implication that N is greater than zero if and only if $\gamma > \gamma_o$, and, since $\frac{1-\alpha}{\alpha} > 0$, then:

$$\begin{aligned} P(0 < N \leq n) &= P(0 < N \leq n, \gamma > \gamma_o) \\ &= P\left(0 < \left(a(\gamma)/Z\right)^{\frac{1-\alpha}{\alpha}} \leq n, \gamma > \gamma_o\right) = P\left(Z \geq n^{\frac{\alpha}{\alpha-1}}a(\gamma), \gamma > \gamma_o\right) \end{aligned} \quad (29)$$

Therefore,

$$\begin{aligned} P\left(Z \geq n^{\frac{\alpha}{\alpha-1}}a(\gamma)\right)P(\gamma > \gamma_o) &= E_{\gamma} \exp\left(-n^{\frac{\alpha}{\alpha-1}}a(\gamma)\right)1_{\{\gamma > \gamma_o\}} \\ &= \frac{1}{\pi} \int_{\gamma_o}^{\frac{\pi}{2}} \exp\left(-n^{\frac{\alpha}{\alpha-1}}a(\gamma)\right) d\gamma \end{aligned} \quad (30)$$

Given that $a(\gamma) = U_{\alpha}(\gamma, \gamma_o)$, this completes the proof that $N \sim S_{\alpha}(1, \beta_2, 0)$.

Also, for the case of $1 < \alpha \leq 2$, since $\frac{1-\alpha}{\alpha} > 0$, for $n > 0$, we can write that:

$$P(N \geq n) = P(N \geq n, \gamma > \gamma_o) \quad (31)$$

$$\begin{aligned} P\left(0 < \left(a(\gamma)/Z\right)^{\frac{1-\alpha}{\alpha}} \geq n, \gamma > \gamma_o\right) &= P\left(0 < \left(Z/a(\gamma)\right)^{\frac{\alpha-1}{\alpha}} \geq n, \gamma > \gamma_o\right) \\ &= P\left(Z \geq n^{\frac{\alpha}{\alpha-1}}a(\gamma), \gamma > \gamma_o\right) \\ &= E_{\gamma} \exp\left(-n^{\frac{\alpha}{\alpha-1}}a(\gamma)\right)1_{\{\gamma > \gamma_o\}} \\ &= \frac{1}{\pi} \int_{\gamma_o}^{\frac{\pi}{2}} \exp\left(-n^{\frac{\alpha}{\alpha-1}}a(\gamma)\right) d\gamma \end{aligned} \quad (32)$$

From which we conclude that $N \sim S_{\alpha}(1, \beta_2, 0)$.

From the case of $\alpha = 1$, the right hand side of Equation (26) reduces to $\frac{\pi}{2} \tan \gamma$ with a Cauchy distribution.

When $\beta_2 \neq 0$, Equation (26) can be written as:

$$\beta_2 \log\left(a_1(\gamma)/Z\right) \quad (33)$$

Where

$$a_1(\gamma) = \frac{\frac{\pi}{2} + \beta_2 \gamma}{\cos \gamma} \exp\left(\frac{1}{\beta_2} \left(\frac{\pi}{2} + \beta_2 \gamma\right) \tan \gamma\right) \quad (34)$$

For $\beta_2 > 0$ we can then write:

$$\begin{aligned} P(N \leq n) &= P\left(\beta_2 \log\left(a_1(\gamma)/Z\right) \leq n\right) \\ &= P\left(Z \geq e^{\frac{-n}{\beta_2}} a_1(\gamma)\right) \\ &= E_\gamma \exp\left(-e^{\frac{-n}{\beta_2}} a_1(\gamma)\right) \\ &= \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp\left(-e^{\frac{-n}{\beta_2}} a_1(\gamma)\right) d\gamma \end{aligned} \quad (35)$$

From which we draw the conclusion that for all admissible values of β_2 , $N \sim S_1(1, \beta_2, 0)$, which completes the proof.

2. Proposed algorithm and noise synthesis results

From Equations (25) and (26) and the proofs thereof, a method for the synthesis of the PLC noise is proposed. The model is tested using the stable non-Gaussian noise models (and their parameters) developed in [15]. The following algorithm is proposed, after the Chambers *et al.* method:

1. For a standard alpha stable noise process with $\alpha \in [0, 2]$ and $\beta \in [-1, 1]$, generate V , which is a randomly distributed uniform variable on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and Z , which is an independent random variable that is exponentially distributed with a mean equal to 1.
2. For $\alpha \neq 1$, compute

$$N = S_{\alpha, \beta} \frac{\sin\left(\alpha(V + B_{\alpha, \beta})\right)}{(\cos V)^{\frac{1}{\alpha}}} \left(\frac{\cos\left(V - \alpha(V + B_{\alpha, \beta})\right)}{Z}\right)^{\frac{1-\alpha}{\alpha}} \quad (36)$$

Where N is the variable that defines the standard stochastic power line noise process, and:

$$S_{\alpha, \beta} = \left(1 + \beta^2 \tan^2 \frac{\pi \alpha}{2}\right)^{\frac{1}{2\alpha}} \quad (37)$$

$$B_{\alpha,\beta} = \frac{\tan^{-1}\left(\beta \tan\left(\frac{\pi\alpha}{2}\right)\right)}{\alpha} \quad (38)$$

3. Else, for $\alpha = 1$, compute:

$$N = \frac{\pi}{2} \left(\frac{\pi}{2} + \beta_2 V \right) \tan V - \beta_2 \log \left(\frac{\frac{\pi}{2} Z \cos V}{\frac{\pi}{2} + \beta_2 V} \right) \quad (39)$$

4. Generalize the stochastic alpha stable power line noise process by transforming the standard stable noise process as follows:

$$N_1 = \begin{cases} \sigma N + \mu & \alpha \neq 1 \\ \sigma N + \frac{2}{\pi} \beta \sigma \log \sigma + \mu & \alpha = 1 \end{cases} \quad (40)$$

Where $N_1 \sim S_\alpha(\sigma, \beta_2, \mu)$ defines the alpha stable powerline noise synthesis process.

From the alpha stable noise model parameters obtained in earlier research, the synthesis of the power line noise process as a stochastic process is carried out for different random number of alpha stable noise samples N . Results for 100, 1000, and 10000 alpha stable noise samples are presented here. The time domain synthesised power line noise series for 100, 1000 and 10000 noise samples are shown in Figures 1 to 3 respectively.

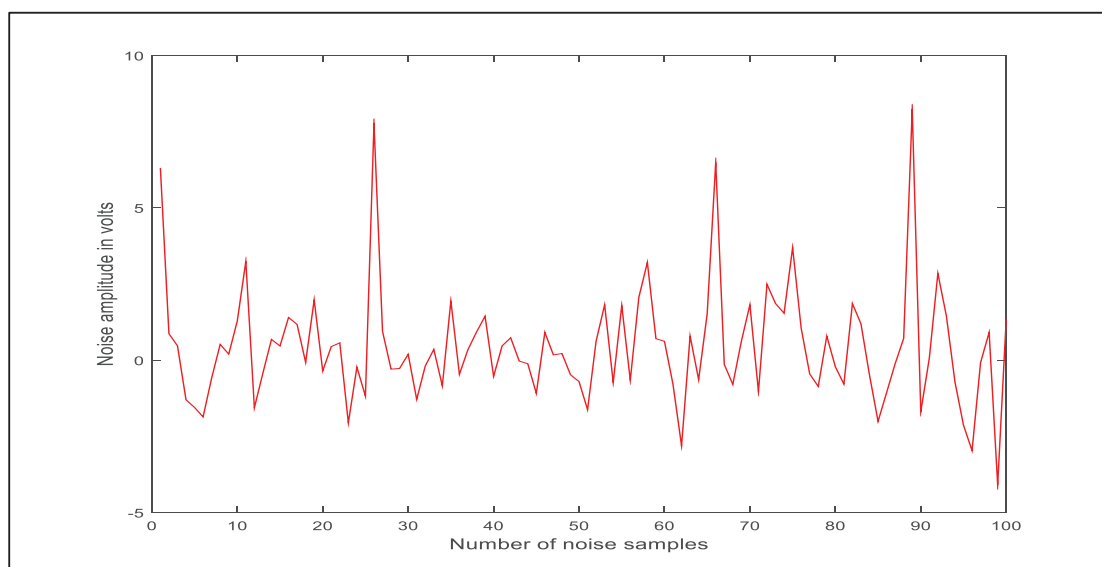


Figure 1: Sythesised time domain alpha stable power line noise for 100 noise samples

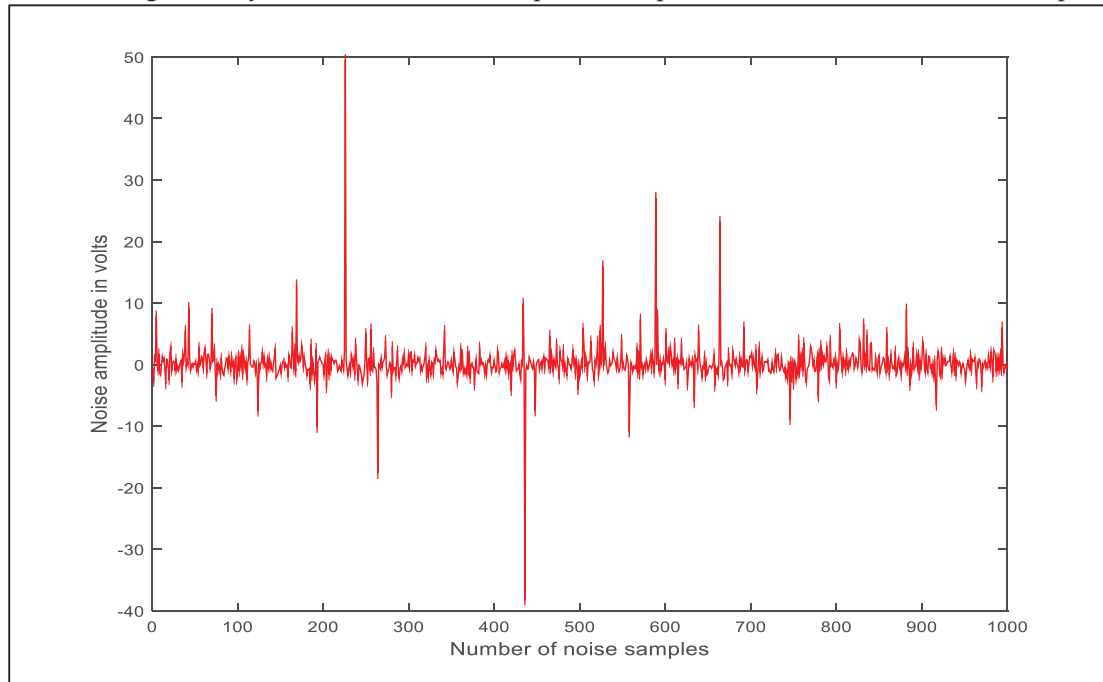


Figure 2: Sythesised time domain alpha stable power line noise for 1000 noise samples

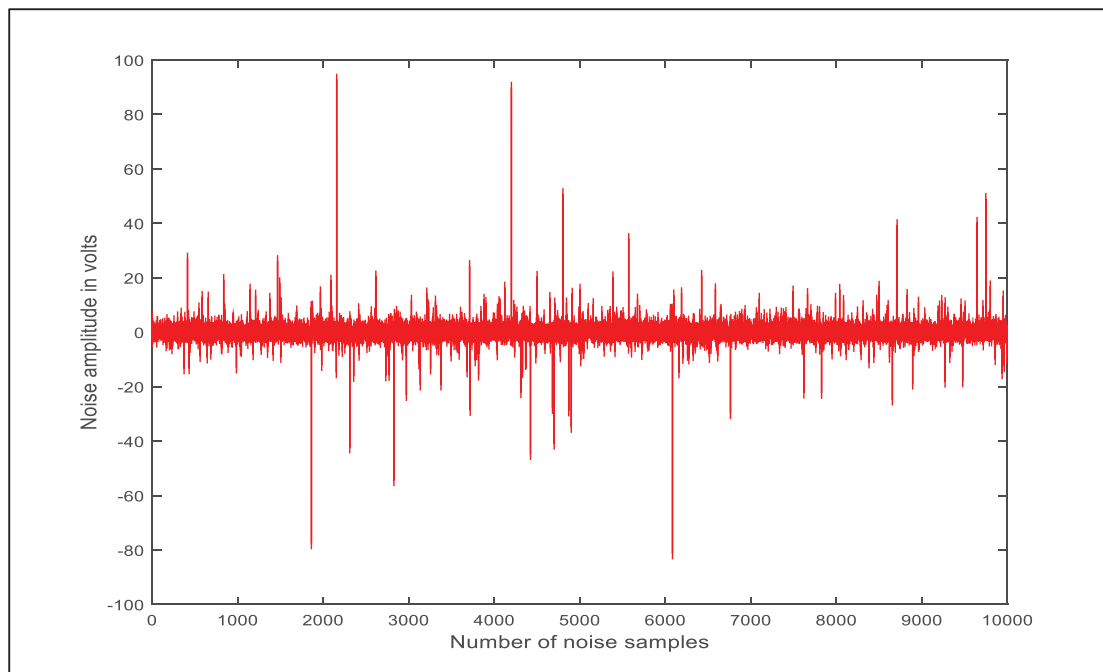


Figure 3: Sythesised time domain alpha stable power line noise for 10000 noise samples

Similarly, the synthesised frequency domain power line noise series for 100, 1000 and 10000 noise samples are shown in Figs. 4 to 6 respectively.

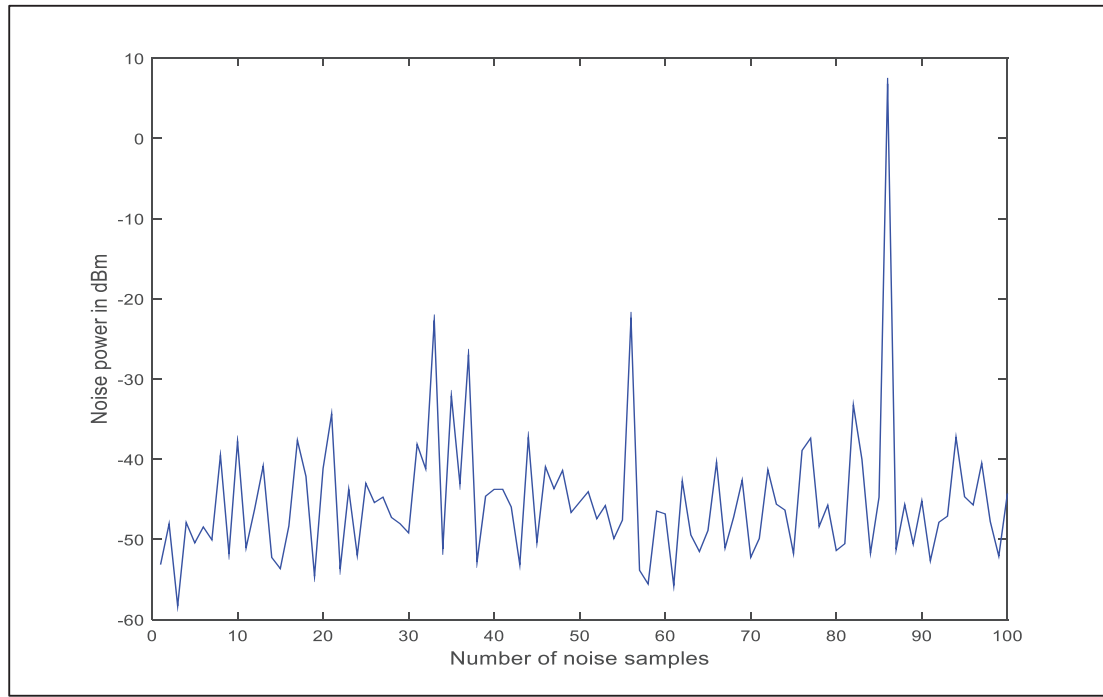


Figure 4: Synthesised frequency domain alpha stable power line noise for 100 noise samples

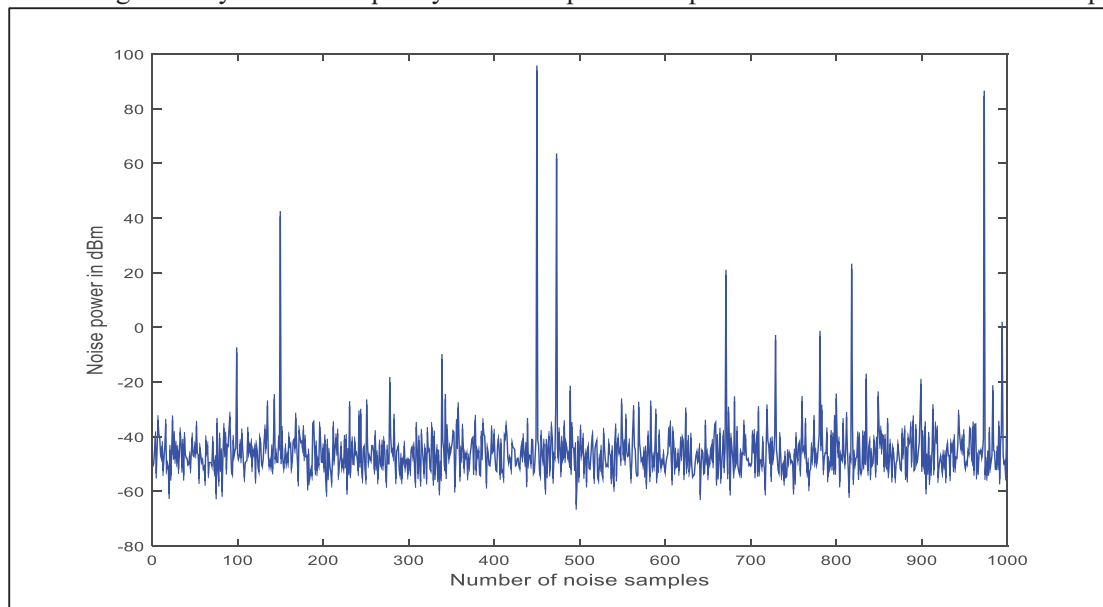


Figure 5: Synthesised frequency domain alpha stable power line noise for 1000 noise samples

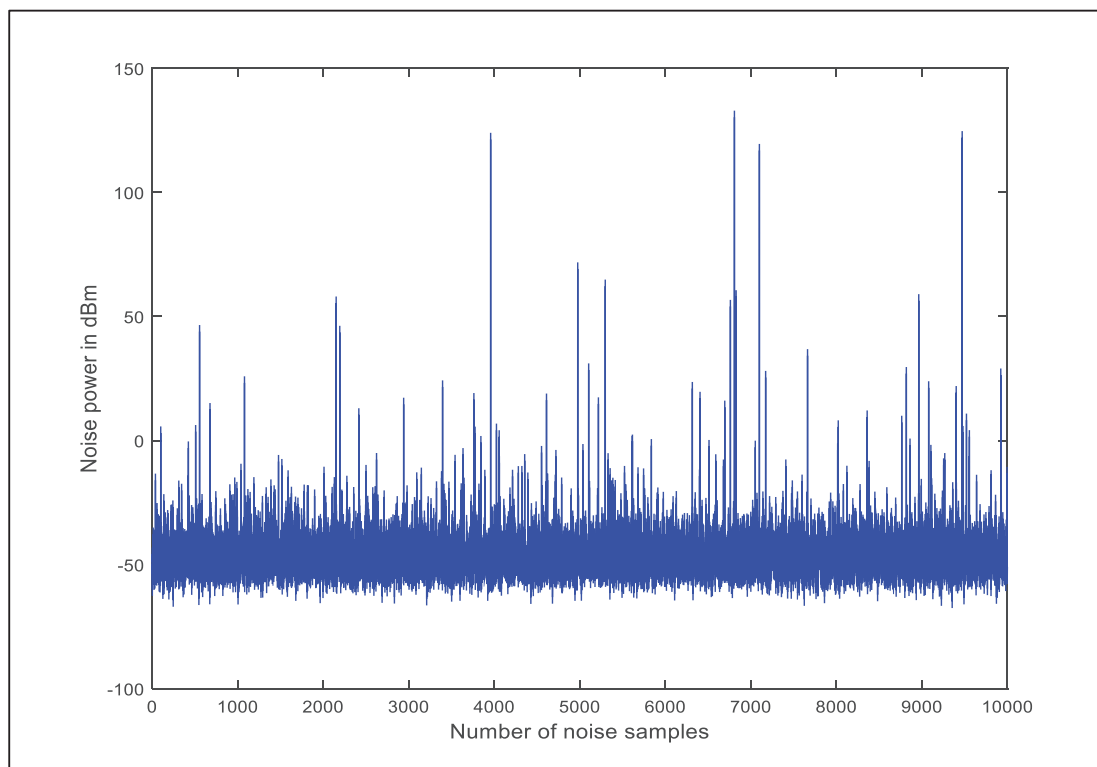


Figure 6: Synthesised frequency domain alpha stable power line noise for 10000 noise samples

From both the time and frequency domain power line noise series, we see that high variability is observed in the synthesised noise series for the noise parameters that were obtained from our noise measurements. This kind of high variability is an indication of the impulsive nature of power line noise. Moreover, most of the noise power appears to be constrained to a small range, which indicates the background noise component in PLC systems. Also, we observe that the high power impulses are fewer compared to the background noise that is more prevalent. The background noise is known to be Gaussian distributed, characterized by a fairly flat spectrum, while impulsive noise is characterized by high variability, which leads to a long-tailed characteristic of its marginal distribution. The fact that the synthesised noise for all random samples in both domains clearly shows both background and impulsive noise events confirms that the framework developed is robust. The algorithm proposed is applicable for all admissible values of the alpha stable noise parameters; meaning it can be used for the study of highly impulsive to weakly impulsive PLC systems. Thus, using this algorithm, it is now possible to generate impulsive noise for the study and development of PLC systems without necessarily resorting to noise measurements. Overall, we observe that the higher the number of noise samples considered, the higher the variability (range) in the overall noise power or voltage. Also, a higher number of noise samples results in more powerful noise

power/voltage spikes, an indication that the framework developed produces meaningful results. Thus, the algorithm works very well for our noise results. Also, the similarities between the synthesised noise and those obtained in the measurements cannot go unnoticed.

To prove that the algorithm works well for the noise results, the determination of the alpha stable parameters for the noise results shown in Figures 1 to 6 was done, using the alpha stable parameter estimation techniques. The alpha stable noise parameters obtained from the synthesised noise in Figures. 1 to 6 are shown in Tables 1 and 2 below.

Table 1: Sythesised time domain noise alpha stable parameters

Time domain Levy synthesis parameters				
Number of noise samples	α	β	γ	δ
100	1.58	0.531	0.928	0.478
1000	1.48	0.208	0.828	0.163
10000	1.47	0.254	0.822	0.231

Table 2: Synthesized frequency domain noise alpha stable parameters

Frequency domain Levy synthesis parameters				
Number of noise samples	α	β	γ	δ
100	1.848	1	4.167	-45.80
1000	1.90	1	4.334	-46.41
10000	1.733	1	4.248	-45.86

From these tables, we observe that the parameters obtained are close to those of alpha stable modelling. Also we observe that a higher number of noise samples produces parameters that are closer to those in obtained from alpha stable processes; which in itself is a more practical scenario. To confirm that the synthesised noise has the same distribution as the one obtained from the measured noise, the pdfs and the cdfs of the synthesised noise parameters and those from measurements are plotted together. These are shown in Figs. 7 to 10.

From these figures, we observe that there is a very close match between the synthesised plots and the measured ones. Also, from the time domain plots, we see that the plot for the 100 noise samples provides the least match to the measured one. The frequency domain plots seem to all fit very well. Overall, the same shape and long-tailed behaviour of the pdfs as well as the cdfs is retained. However, to ascertain the

accuracy of the overall noise synthesis process, RMSE analysis and Chi-square tests were done. These results present a comparison between the alpha stable models and those developed from the noise synthesis in this paper. The results of the tests are shown in Tables .3 and 4.

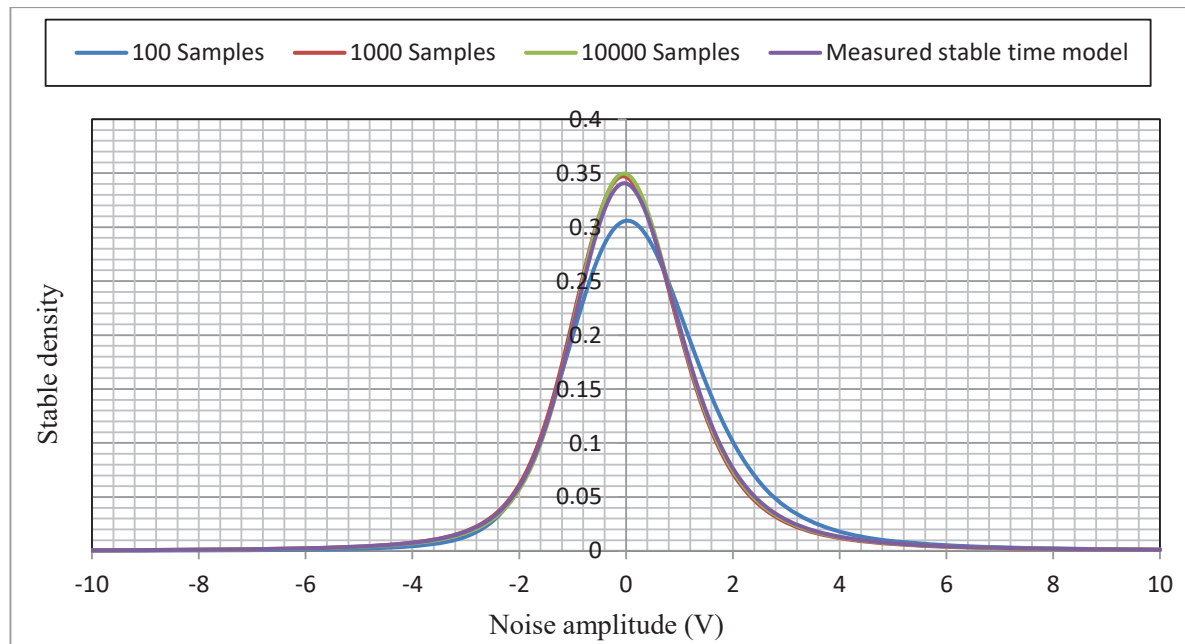


Figure 7: Time domain alpha stable power line noise models

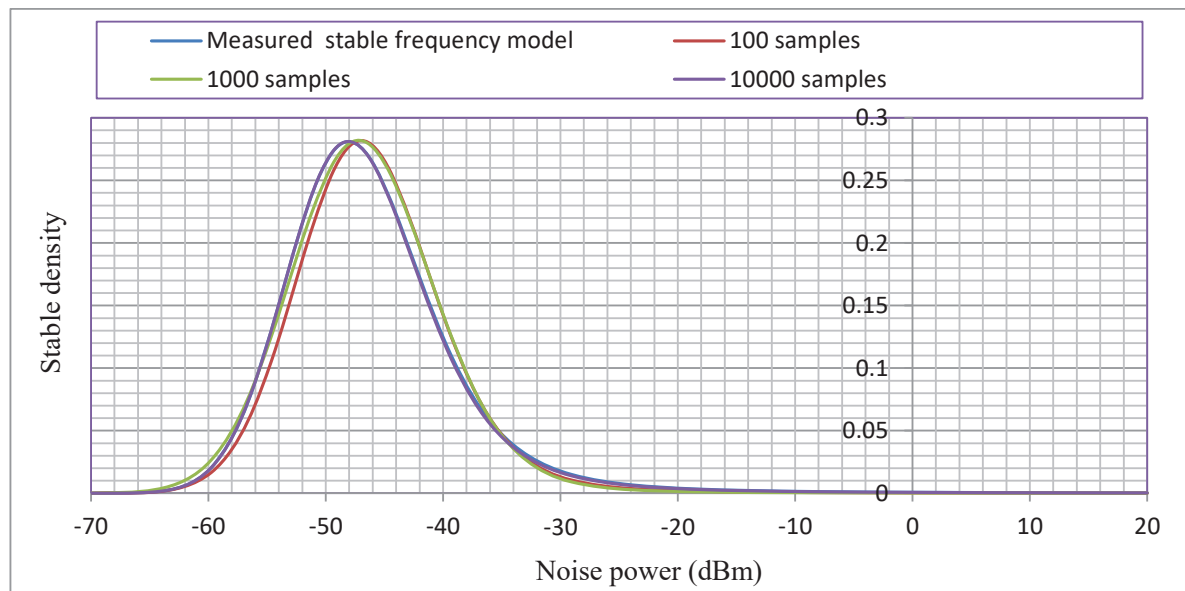


Figure 8: Frequency domain alpha stable power line noise models

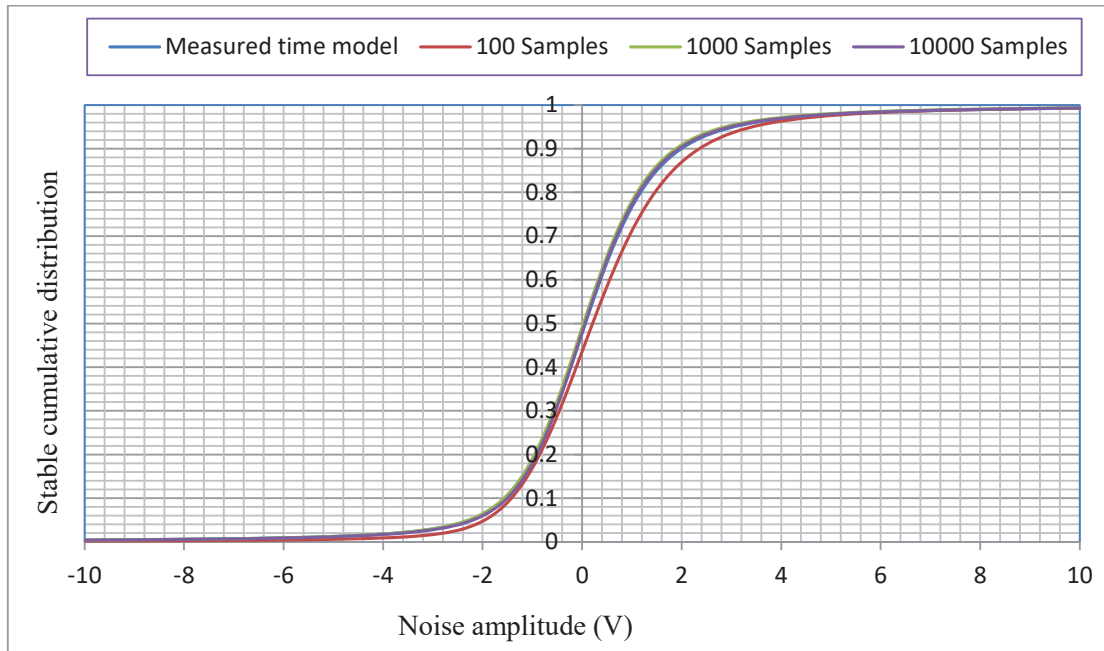


Figure 9: Time domain alpha stable power line noise cdf plots

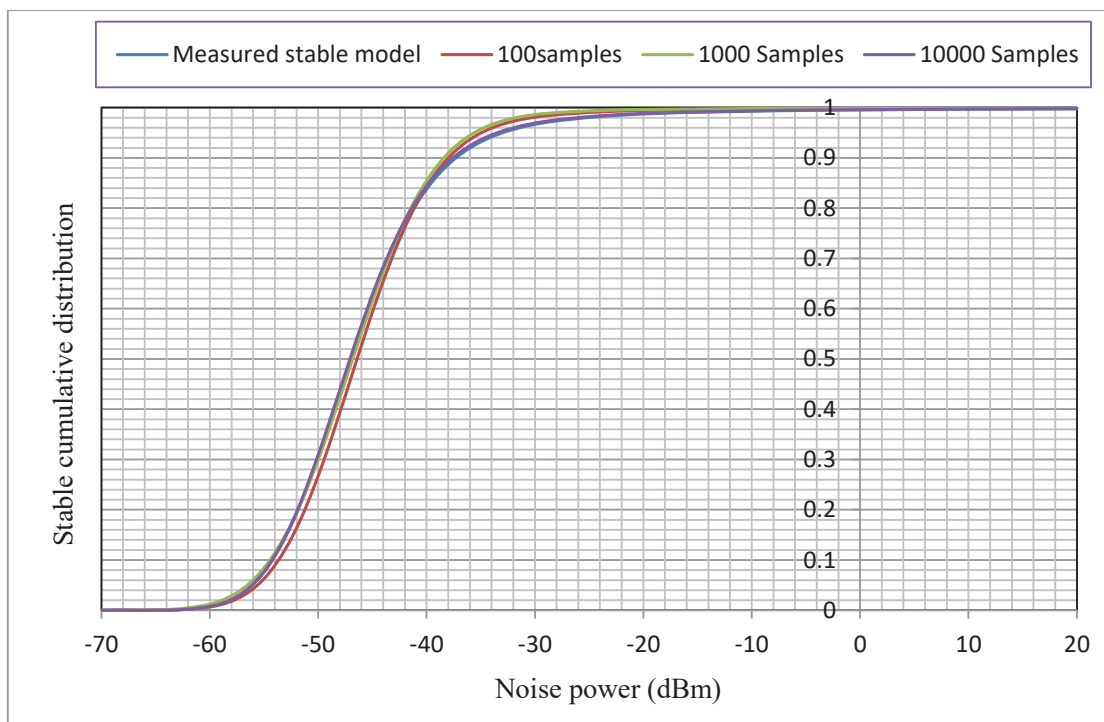


Figure 10: Frequency domain alpha stable power line noise cdf plots

Table 3: Time domain noise goodness of fit results

Number of noise samples	Time domain			
	RMSE	χ^2	DF	CV
100	0.0104	0.34	450	500.456
1000	0.0026	0.00003	450	500.456
10000	0.0022	0.003	450	500.456

Table 4: Frequency domain noise goodness of fit results

Number of noise samples	Frequency domain			
	RMSE	χ^2	DF	CV
100	0.0109	1.254	450	500.456
1000	0.0079	0.270	450	500.456
10000	0.001	0.00006	450	500.456

From the results in Tables 3 and 4, we see that the least RMSE is obtained for 10000 random noise samples, which means that a higher number of noise samples improves the algorithm prediction accuracy. Also, the highest χ^2 values are obtained for 100 noise samples which means that the algorithm reliability is slightly reduced when the noise samples are very small. The smallest χ^2 values are obtained for 1000 noise samples for the time domain while the same is obtained for 10000 noise samples for the frequency domain. However, at the 0.05 significance level used for this test, we see that the critical values (CVs) are much much higher than the computed χ^2 values, for the same degrees of freedom (DF). Therefore, the null hypothesis is accepted for all cases. Thus for all random noise samples, the PLC noise synthesis framework developed proves to be very reliable. Overall, we also see that all RMSEs are very small, which means the noise distribution prediction accuracy of the algorithm proposed is very high.

Conclusion

In this paper, a framework for synthesizing noise in PLC systems has been developed. The mathematical background for the algorithm proposed has been presented as well. Results from the synthesis of the power line noise as an alpha stable stochastic process show that the random nature of the power line noise is well captured. High variability is observed in the synthesised noise series for both time and frequency domains, for a random number of alpha stable noise samples. Also, an ever present low power background noise component is observed in the synthesised noise, coupled with a lesser number of high power impulses. The

Levy stable noise parameters obtained from the synthesised power line noise are very close to those in alpha stable models, for all random number of samples considered, an indication that the algorithm prediction accuracy is very good. Also, the synthesised noise has the same distribution as the measured noise. All goodness of fit and reliability test results obtained show that the algorithm accuracy is very satisfactory. Also, the algorithm is applicable for all admissible values of alpha stable parameters. Thus, the proposed algorithm is able to synthesise any level of impulsiveness ranging from very impulsive noise processes to background noise cases, by considering the PLC noise process as a Levy stable stochastic process. Overall, the long-tailed behaviour as well as overall shape of the pdfs and cdfs is retained in all the synthesized noise models. Therefore, the PLC noise synthesis framework developed here is very crucial in the study and development of high performing and reliable PLC systems, as well as stimulating further research in PLC noise in general (for LV, MV and HV networks) in line with the outcomes presented in this paper.

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