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A Hybrid Reduction Technique for Transformer Linear Section Model

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Abstract-The authors proposed a mixed method for model order reduction for transformer linear section. In this method reduced denominator polynomial is obtained by modified pole clustering method and reduced numerator polynomial is obtained by cauer second form. The obtained reduced model is stable, provided that if higher order system is stable. The proposed method is explained with the help of transformer linear section and the results are compared with the other methods.

Key words-Order reduction; Pole cluster; Dominant pole

INTRODUCTION

The mathematical model of a system can be formulated based on the theoretical approach of that particular system. Modeling of a mathematical system may lead to comprehensive description of a process in a higher order transfer function of which leads to difficulty either for analysis or for controlling. In order to minimize the complexity of solving, it is necessary to find the equivalent lower order transfer function without any change in the characteristics of higher order transfer function.

Several model order Reduction techniques for time domain are available in literature such as Hutton and Friedland [1], Shamash [2], Krishnamurthy and Seshadri [5], J.Pal[6]. Further some methods have also been included, by combining features of two different methods which can be termed as mixed methods [10-12]. Each method includes advantages as well as disadvantages too. Even though there are plenty of methods, no method always gives the best result.

In the clustering technique [13] the poles and zeroes are to be grouped separately to form clusters which are to be replaced by their cluster centers. In the literature, only poles are grouped together to generate cluster centers and then denominator polynomial of the reduced model is synthesized from these cluster centers. In this method, a modified pole clustering technique is adapted, which generates more effective cluster centers. If a cluster contains r number of poles, then IDM criterion is repeated r times with the most dominant pole available in that cluster.

One of the other popular methods for Model order reduction is cauer second form of continued fraction expansion [11]. All methods have their own advantages and disadvantages. In the proposed method denominator of reduced model is determined by using modified pole cluster while the numerator by cauer second form to combine advantages of both the methods and to minimize the disadvantages.

An Error index ISE (Integral Square Error) [16] between original order and reduced order systems is tabulated in this paper which is given by

$$ISE = \int_0^\infty [y_o(t) - y_r(t)]^2 \qquad (1)$$

Here $y_o(t)$, $y_r(t)$ are the unit step responses of original and reduced order systems respectively at t^{th} instant of time limits 0 and ∞.

PROBLEM STATEMENT

Let 'n' be order of higher order transfer function and it can be given by

$$G(s) = \frac{A_{21}+A_{22}s+\cdots\ldots\ldots A_{2n-1}s^{n-1}}{A_{11}+A_{12}s+\cdots\ldots.+A_{1n-1}s^{n-1}+A_{1n}s^n} \qquad (2)$$

Let the corresponding 'k^{th}' order reduced model be

$$G_k(s) = \frac{B_{21}+B_{22}s+\ldots.+B_{2,k-1}s^{k-1}}{B_{11}+B_{12}s+\ldots.+B_{1,k-1}s^{k-1}+B_{1,k}s^k} \qquad (3)$$

Procedure for reducing order:

1. Finding the denominator of 'k^{th}' order reduced model, using the modified pole clustering.

The cluster center can be formulated based on the concept of 'inverse distance measure', which is explained as follows:

Let there be ‘r’ poles in i^{th} cluster which can be given by (p_1, p_2 ...... p_r.) then the Inverse Distance Measure (IDM) criterion identifies the cluster center as

$$p_c = \left\{\left[\sum_{i=1}^{r} \frac{-1}{|p_i|}\right] \div r\right\}^{-1} \quad (4)$$

Where $|p_1| < |p_2| < \ldots < |p_r|$, and then modified cluster center p_{ei} can be obtained by using modified pole clustering method as followed.

STEP-1: let there be ‘r’ poles in a cluster

$|p_1|<|p_2|<\ldots<|p_r|$. (5)

STEP-2: Set a=1.

STEP-3: Find pole cluster center

$$p_a = \left\{\left[\sum_{i=1}^{r} \frac{-1}{|p_i|}\right] \div r\right\}^{-1}$$

STEP-4: Set a=a+1.

STEP-5: Find a modified cluster center

$$c_a = \left[\left(\frac{-1}{|p_1|} + \frac{-1}{|c_{a-1}|}\right) \div 2\right]^{-1} \quad (6)$$

STEP-6: Check for is r=j , if No, and then go to step-4 else go to step-7.

STEP-7: Now the modified cluster center of k^{th} cluster is $p_{ek} = c_a$.

Then the denominator polynomial of the k^{th} order reduced model can be formulated as

$$D_k(s) = (s - p_{e1})(s - p_{e2}) \ldots (s - p_{ek}) \quad (7)$$

Where $p_{e1}, p_{e2}, \ldots, p_{ek}$ are modified cluster centers of $1^{st}, 2^{nd}, \ldots, k^{th}$ pole cluster centers respectively. Therefore the denominator polynomial can be obtained as

$$D_k(s) = B_{11} + B_{12}s + \ldots + B_{I,k}s^k \quad (8)$$

2. Finding the numerator of the reduced model by cauer second form

Evaluate cauer second form coefficients by forming routh algorithm

Formulation of ‘A’ array

$$\begin{array}{cccccc}
A_{11} & A_{12} & A_{13} & \ldots\ldots & A_{1,n-1} & A_{1,n} \\
A_{21} & A_{22} & A_{23} & \ldots\ldots & A_{2,n-1} & \\
A_{31} & A_{32} & \ldots\ldots\ldots\ldots & & & \\
A_{41} & \ldots\ldots & & & &
\end{array} \quad (9)$$

1^{st} and 2^{nd} rows of A array are formed by denominator and numerator coefficients of higher order system. Remaining elements are obtained by using routh algorithm and are given as follows.

$$A_{i,j} = A_{i-2,j+1} - h_{i-2}A_{i-1,j+1} \quad (10)$$

Where j=1, 2, 3……,

$$h_p = \frac{A_{i,1}}{A_{i+1,1}} \quad (11)$$

Formation of ‘B’ array

$$\begin{array}{ccccc}
B_{11} & B_{12} & \ldots\ldots\ldots & B_{1,k-1} & B_{I,k} \\
B_{21} & B_{22} & \ldots\ldots\ldots & B_{2,k-1} & \\
B_{31} & B_{32} & \ldots\ldots\ldots & & \\
B_{41} & \ldots\ldots & & &
\end{array} \quad (12)$$

1^{st} row of B array is obtained from modified pole cluster form and rests of the elements are obtained by formulation of inverse routh algorithm, which can be obtained as follow

$$B_{i+1,1} = \frac{B_{i,1}}{h_i} \quad (13)$$

Where i=1,2,…k

$$B_{i+1,j+1} = \frac{(B_{i,j+1} - B_{i+2,j})}{h_i} \quad (14)$$

Where i=1,2……(k-j)

j=1,2……....(k-1)

Now the numerator polynomial can be obtained as

$$N_k(s) = B_{21} + B_{22}s + \ldots + B_{2,k-1}s^{k-1} \quad (15)$$

NUMERICAL EXAMPLE

In order to obtain a transformer model, a lumped linear coil is used as the test system having10 section [19-23]

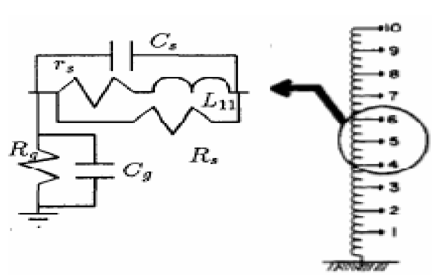

Figure 1: Air core transformer section

Fig.1 shows the typical section of transformer contains series resistance r_s, a shunt resistance R_s, a self inductance L_{11} and a series capacitance C_s, and a parallel combination of a resistance R_g and a capacitance C_g with respect to ground node. It includes concept of mutual inductances between the sections.

$$i = i_l + i_e + i_r \quad (16)$$

$$i = \frac{1}{L}\int e\,dt + Ge + C\frac{de}{dt} \quad (17)$$

Where i is the total currents, i_l is the current through inductor, e is voltage at node, i_c is the current through capacitor and i_r is the current through resistor. Taking the derivatives of (17) with respect to t, it can be formulated as

$$\frac{di}{dt} = \frac{1}{L}e + G\frac{de}{dt} + C\frac{d}{dt}\left(\frac{de}{dt}\right) \quad (18)$$

Let $\quad w = \frac{de}{dt}$, $u = \frac{di}{dt} \quad (19)$

There fore

$$u = L^{-1}e + Gw + C\frac{dw}{dt} \quad (20)$$

$$\frac{dw}{dt} = C^{-1}u - C^{-1}L^{-1}e - C^{-1}Gw \quad (21)$$

Equation (7) is in form of state space equation i.e.

$$\frac{d}{dt}x = Ax + Bu \quad (22)$$

Where

$$A = \begin{bmatrix} 0 & I \\ -C^{-1}L^{-1} & -C^{-1} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ C \end{bmatrix} \quad (23)$$

The assumed transformer parameters for transformer section are

$C_g = 8.5*10^{-10}$, $C_S = 3.4*10^{-12}$, $R_g = 2.1*10^{11}$, $R_S = 1.65*10^5$, $r_s = 22.6$, $L_{1-1} = 28.998$, $M_{1-2} = 13.537$, $M1_{-3} = 6.231$, $M_{1-4} = 3.379$, $M_{1-5} = 1.987$, $M_{1-6} = 1.242$, $M_{1-7} = 0.817$, $M_{1-8} = 0.560$, $M_{1-9} = 0.398$ and $M_{1-10} = 0.292$.

The state variables are used in identification process and the full order model poles and zeros are obtained as

Poles: -3.06,-10.37, -19.84, -31.33, -44.51,-58.69, -72.94, -86.39, -97.59, -105.09

Zeros: -7.39,-16.8,-28.45, -41.92, -56.54, -71.44, -85.45, -97.14, -104.97

The transfer function of air core transformer linear section having 10 sections is

$$G(s) = \frac{s^9 + 510.1s^8 + 1.106\,e5\,s^7 + 1.33\,e7\,s^6 + 9.691\,e8\,s^5 + 4.393\,e10\,s^4 + 1.223\,e12\,s^3 + 1.981\,e13\,s^2 + 1.652\,e13\,s + 5.211\,e14}{s^{10} + 529.8s^9 + 120200\,s^8 + 1.527\,e7\,s^7 + 1.191\,e9\,s^6 + 5.892\,e10\,s^5 + 1.842\,e12\,s^4 + 3.513\,e13\,s^3 + 3.784\,e14\,s^2 + 1.965\,e15\,s + 3.32\,e15} \quad (24)$$

The poles of the given transfer function are [19]

-3.06, -10.37, -19.84, -31.33, -44.51, -58.69, -72.94, -86.39, -97.59, -105.09

Let the reduced order to be realized is 2nd order.

Therefore no. of clusters centers formed is 2

1st cluster is (-3.06, -10.37, -19.84, -31.33, -44.51) and 2nd cluster is (-58.69, -72.94, -86.39, -97.59, -105.09)

The cluster centers obtained as $p_{e1} = -3.1725$ and $p_{e2} = -59.1156$

Then the denominator polynomial of the 2nd order reduced model by using modified pole cluster is

$$D_k(s) = (s + 3.1725)(s + 59.1156)$$

$$D_2(s) = s^2 + 62.29s + 187.5 \quad (25)$$

For numerator

From the higher order transfer function **A table** is formulated as

$3.3*10^{15}$	$1.965*10^{15}$	$3.784*10^{14}$	……
$5.211*10^{14}$	$1.652*10^{14}$	$1.981*10^{13}$	……
$9.125*10^{14}$	$0.2522*10^{15}$	………	

From A table the values of $h_1=6.4$, $h_2=0.57$

From lower order reduced model denominator **B table** is formulated.

187.5460	62.2882	1
29.4368	1.686	
51.5462		

From 2nd row of B table, reduced order numerator is given by

$$N_2(s) = 1.686s + 29.436 \quad (26)$$

Therefore reduced 2nd order transfer function is

$$G_2(s) = \frac{1.686s + 29.436}{s^2 + 62.29s + 187.5} \quad (27)$$

SIMULATION RESULTS

Step response:

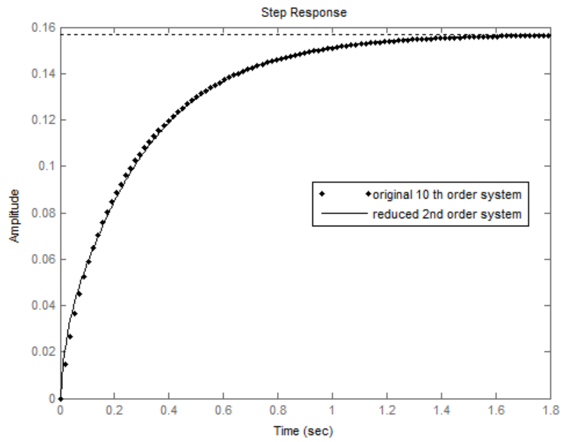

Figure 2: Step response comparison

The step responses for the original higher order system and the reduced model order are shown in Fig.1. It can be seen that step response of the reduced model is exactly matching with that of the original system.

Bode plot:

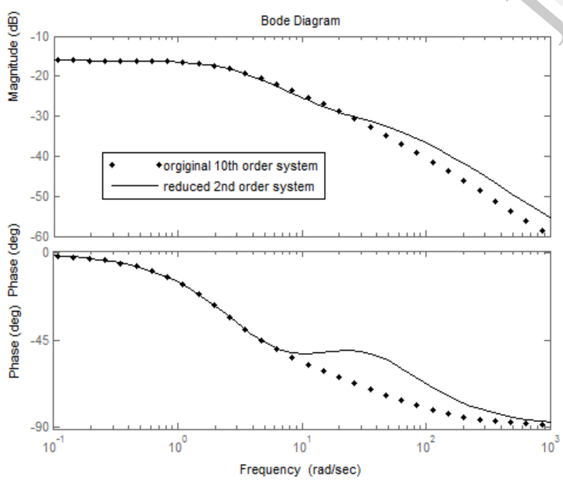

Figure 3: Bode plots comparison

The Bode plots for the original higher order system and the reduced model order are shown in Fig.2. It can be seen that bode plot of the reduced model is exactly matching with that of the original system.

TABLE – 1

COMPARISON OF PROPOSED METHOD USING ISE

Methods	**Reduced models**	**ISE**
Proposed method	$G_2(S)=\frac{1.686S+29.436}{S^2+62.29S+187.5}$	2.1009×10^{-9}
Hutton and Friedland [1]	$G_2(S)=\frac{0.5178S+1.633}{S^2+6.159S+10.41}$	2.2015×10^{-7}
Shamash[3]	$G_2(S)=\frac{0.5178S+1.633}{S^2+6.159S+10.41}$	2.2015×10^{-7}
Krishnamurthy and Seshadri [5]	$G_2(S)=\frac{0.518S+2.284}{S^2+6.725S+14.555}$	2.0742×10^{-7}
Jayanthapal [6]	$G_2(S)=\frac{0.427S+2.284}{S^2+6.725S+14.555}$	2.0742×10^{-7}
Vishwakarma [13]	$G_2(S)=\frac{-18.5S+119.3}{S^2+90S+762.6}$	7.3594×10^{-8}

The proposed method is compared with other methods available in literature and shown in table1, for which it is considered that this method has quality

TABLE – 2
QUALITATIVE COMPARISON WITH THE ORIGINAL SYSTEM

System	**Rise time t_r (sec.)**	**Settling Time t_s (sec.)**
10^{th} order	0.662	1.21
2^{nd} order	0.668	1.19

It is concluded that proposed method provides good approximation both in transient and study state regions.

CONCLUSION

Studying the effect of higher order model power transformer involves large amount of computations, difficult to simulate and includes complexity. To overcome these, the authors presented a mixed method for reducing the size of the detailed lumped parameter model normally used for transformer design to a size acceptable to a utility engineer performing systems studies.

In this the reduced denominator is obtained by modified pole clustering method while numerator is by cauer second form. This method is compared with different methods. The ISE (integral Square Error) of proposed method is compared with existing methods in the literature and is shown in Tab.1. From all these comparisons it can be concluded that the proposed method gives the best results and it is simple.

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