A Handy Analytical Approximate Solution for A Heat Transfer Problem by using A Version of Taylor Method with Boundary Conditions

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Abstract— **This work presents the modified Taylor series method (MTSM) with the purpose to find an approximate solution for the nonlinear problem that describes the steady state one dimensional conduction of heat in a slab with thermal conductivity linearly dependent on the temperature, After comparing MTSM approximation with the exact solutions, we will conclude that the proposed solution is besides of handy, accurate and therefore it follows that the proposed method is potentially an efficient tool for practical applications.**

Keywords— Nonlinear Differential Equations; Boundary Value Problems; Taylor series; Heat problems.

I. INTRODUCTION

The objective of this article is to find a handy analytical approximate solution for the boundary value problem of the steady state one dimensional conduction of heat in a slab with thermal conductivity linearly dependent on the temperature [1] (see Figure 1). Given the importance of the heat transfer phenomena, both in pure problems as well as in the design and operation of equipment in applications, it is primordial to research for analytical approximate solutions for the equations describing these phenomena [1,2,9,10].

The Taylor series method (TSM) is employed with the purpose to depict a function in a certain open interval [3]. This method is based on the successive obtaining of the derivatives of the interest function, in such a way that to obtain a better approximation more terms of the series are required. As a matter of fact, an inconvenient is that the above-mentioned method provides just a local convergence [4] which it means that for obtaining a good approximation for a function by using TSM, it is required to employ many terms of the Taylor series. For this same reason, the use of TSM for obtaining solutions for nonlinear differential equations has been rather limited. Taylor series method has been employed above all to find solutions to differential equations with initial conditions because TSM is directly related with them. Following the idea of [5] we will employ a version of TSM useful to solve nonlinear problems with boundary conditions, by calculating some shooting constants (in our case some initial conditions). Therefore, this work presents the modified Taylor series method (MTSM) in order to find an analytical approximate solution for the nonlinear ordinary differential equation that describes the steady state one dimensional conduction of heat in a slab with thermal conductivity linearly dependent on the temperature, which is defined on a finite interval with Dirichlet boundary conditions.

The importance of nonlinear problems is that many nature phenomena are nonlinear. For this reason, several methods have been proposed in order to find approximate solutions to nonlinear differential equations: variational approaches [6,7,29,34], tanh method [13], exp-function [14,15], Adomian's decomposition method [16-21], parameter expansion [22], homotopy perturbation method [23-34], homotopy analysis method [35], and perturbation method [10,36,37] among many others.

The rest of the paper is as follows. In Section 2, we introduce the idea of modified Taylor series method (MTSM). For Section 3 we briefly introduce the heat problem to solve. Additionally, Section 4 presents the application of MTSM in the search for an approximate solution for nonlinear ordinary differential equation that describes the steady state one dimensional conduction of heat in a slab with thermal conductivity linearly dependent on the temperature. Besides a discussion on the results is presented in Section 5. Finally, a brief conclusion is given in Section 6.

II. MTSM METHOD

To figure out how MTSM method works, consider a general nonlinear equation in the form [5]

$$u^{(n)} = N(u) - f(r), \qquad x \in \Omega, \tag{1}$$

with the following boundary condition

$$B(u, \partial u / \partial n) = 0, \qquad x \in \Gamma, \tag{2}$$

where n is the order of the differential equation, N a general differential operator, B is a boundary operator, $f(x)$ a known analytical function, Γ is the domain boundary for Ω

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and $\partial u / \partial \eta$ denotes differentiation along the normal drawn outwards from Ω.

First, we increase the order of the differential equation

$$u^{(n+k)} = \frac{d^k}{dx^k}\left[N(u) - f(x)\right], \tag{3}$$

where k is a constant related to the number of the desired shooting constants (SC)

It is possible to express the Taylor series solution for (3) in the following form

$$u_T(x) = u(x_0) + \frac{u'(x_0)}{1!}(x - x_0) + \frac{u''(x_0)}{2!}(x - x_0)^2 + \ldots, \tag{4}$$

where, x_0 is the expansion point and derivatives evaluated in the same point $u^{(i)}(x_0)$ $(i = 0, 1,2,3..)$ are expressed in terms of the parameters and boundary conditions of (3)

As it is required to solve boundary condition problems, boundary conditions not located at the chosen expansion point will be replaced by shooting constants giving as result traditional DC conditions. Next, with the purpose to get the coefficients of (4) $u^{(i)}(x_0)$ $(i = 0, 1, \ldots)$, MTSM requires (I) calculating the successive derivatives of (3) and (II) evaluating each derivative using the Dirichlet conditions. Finally, in order to satisfy the boundary conditions which were replaced by the shooting constants, it is required to evaluate (4) in such points and the resulting system of equations is solved with the end to get the value of the SC constants. It is important to note that the order of the Taylor expansion (4) is chosen in such a way that it includes all the shooting constants in the polynomial; as long as we satisfy such condition, the order of the Taylor expansion may be increased to improve accuracy.

The constants due to the extra *k*-derivatives (see (3)) are used in order to minimize the mean square residual error defined (MSR) as

$$\int_{x_i}^{x_f} \left(u_T^{(n)} - N(u_T) + f(x)\right)^2 dx, \tag{5}$$

where u_T is the approximated TSM solution (4) and $\left[x_i, x_f\right]$ is the finite interval delimited by the boundary conditions.

For the case where only is required adjust one parameter, a good alternative for (5) condition is by substituting the MTSM approximation into one of the boundary conditions (see Section 4).

III. MATHEMATICAL DESCRIPTION OF THE PHENOMENON

The purpose of this article is the search for an analytical approximate solution for the problem, which describes the steady-state one-dimension conduction of heat in a slab with thermal conductivity linearly dependent on the temperature (Figure 1).

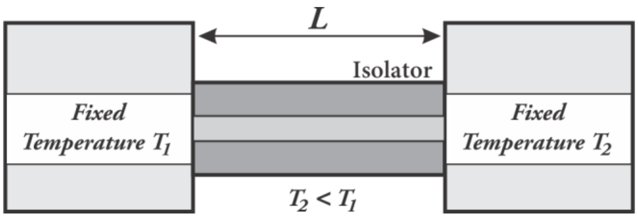

Figure 1. The 1-D conduction of heat through an insulated slab

Let $T(x, y, z, t)$ be the temperature of the abovementioned slab at a point (x, y, z) at time t, where K, σ, and μ are the thermal conductivity, specific heat, and density respectively.

It is possible to verify, following [7] that T satisfies.

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) = \sigma\mu\frac{\partial T}{\partial t} \tag{6}$$

For steady conditions $\partial T / \partial t = 0$, and

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) = 0 \tag{7}$$

For the case of steady conditions for the one-dimensional conduction of heat in a slab of thickness L, we will assume that the uniform temperatures of the faces obey the relation $T_2 < T_1$ (See Figure 1) in such a way that (7) is reduced to

$$\frac{d}{dx}\left(K\frac{dT}{dx}\right) = 0, \tag{8}$$

with boundary conditions

$$T(0) = T_1, \quad T(L) = T_2. \tag{9}$$

Assuming for simplicity that thermal conductivity varies linearly with the temperature [9] then

$$K = K_2(1 + \beta(T - T_2)) \tag{10}$$

Using the dimensionless quantities [8]

$$y = \frac{T - T_2}{T_1 - T_2}, z = \frac{x}{L}, \varepsilon = \beta(T_1 - T_2) = \frac{K_1 - K_2}{K_2}, \tag{11}$$

into (8), and after some algebraic steps it is obtained

$$\frac{d^2y}{dz^2}+\varepsilon y\frac{d^2y}{dz^2}+\varepsilon\left(\frac{dy}{dz}\right)^2=0, \qquad (12)$$

where (9) adopts the form:

$$y(0)=1, \qquad y(1)=0. \qquad (13)$$

IV. Case Study.

This section presents the application of MTSM in the search for an approximate solution for nonlinear ordinary differential equation that describes the steady state one dimensional conduction of heat in a slab with thermal conductivity linearly dependent on the temperature (see (12) and (13)). This work will follow the general idea of MTSM but will employ just one shooting parameter correspondent to initial derivative. From this view to calculate it will be sufficient ensure that the MTSM solution satisfies the right boundary condition $y(1)=0$, besides to exemplify we will work the case of $\varepsilon=1.$

We will get an approximate solution for the system (12) and (13) with good precision performing very few effort.

From (12) we obtain the following successive derivatives

$$y''=\frac{y'^2}{1+y},\ \ y'''=\frac{-3y'y''}{1+y},\ \ y^{(4)}=\frac{-4y'y'''-3y''^2}{1+y},\ldots \qquad (14)$$

After defining the hitherto unknown quantity $y'(0)=\alpha$, and assuming that a fourth-degree polynomial is enough we obtain from (4)

$$y_T(z)=1+\alpha z-\frac{\alpha^2}{4}z^2+\frac{\alpha^3}{8}z^3-\frac{15\alpha^4}{192}z^4+\ldots, \qquad (15)$$

after we defined the expansion point $x_0=0.$

Instead of (5), the only unknown parameter of (15) is easier evaluated applying the boundary condition

$$y_T(1)=0, \qquad (16)$$

so that we get the following fourth degree algebraic equation

$$15\alpha^4-24\alpha^3+48\alpha^2-192\alpha-192=0. \qquad (17)$$

The real solution of (17) is given by

$$\alpha=-0.768425. \qquad (18)$$

By substituting (17) into (15) we get the following approximate solution for (15)

$$\begin{aligned}y_T(z)=1&-0.768425z-0.147619245z^2\\&-0.056717159z^3-0.027239301z^4\end{aligned}. \qquad (19)$$

Next we will discuss the accuracy of (19).

V. DISCUSSION.

In this work MTSM was employed in order to find an approximate solution, for the nonlinear ordinary differential equation with Dirichlet boundary conditions, which describes the steady state one dimensional conduction of heat in a slab with thermal conductivity linearly dependent on the temperature. Since MTSM is expressed in terms of initial conditions for a differential equation, the procedure consisted of expressing the approximate solution in terms of $\alpha=y'(0)$. We determined its value requiring that approximate solution satisfies the boundary condition $y_T(1)=0.$ This condition defines an algebraic equation for α, whose solution provides the sought analytical approximation solution.

Figure 2 shows the comparison between numerical solutions and approximate solution (19) for $\varepsilon=1$. It can be noticed that curves are in good agreement, whereby it is clear that MTSM method is potentially useful in the search for approximate solutions of nonlinear problems definite with boundary conditions.

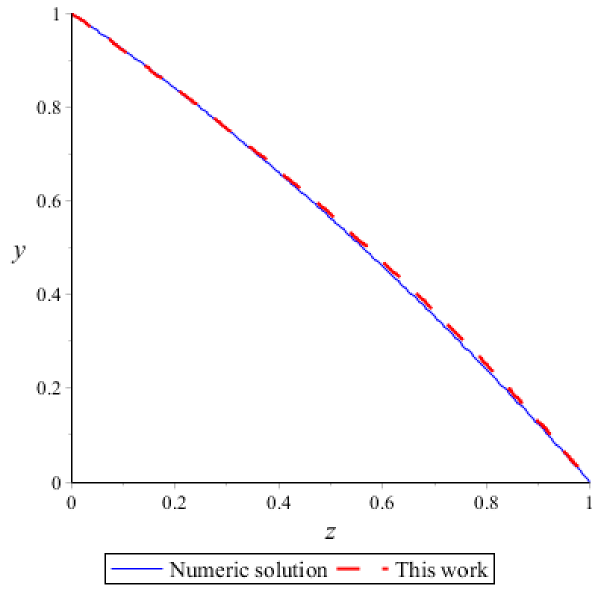

Figure 2. Comparison between numerical solution and approximate solution (19) for ε=1

As a matter of fact, Table 1 shows a more detailed comparison between numerical solution for (12) and approximations: (19) of this work, (47) of MHPMLT [9], and (19) of PM [10] for $\varepsilon=1$. It is clear that (19), is competitive with the second best accuracy; it's Average Absolute Relative Error (A.A.R.E) is $1.66348x10^{-2}$, behind of PM fifth order approximation PM method with accuracy $1.57033x10^{-2}$, but better than MHPMLT second order approximation ($2.04270x10^{-2}$) despite of the fact that MHPMLT method is considered more difficult to use, it requires to command MHPMLT algorithm that implies employing Laplace transform. On the other hand, PM method required solving five recurrence differential equations. From Table 1, we note that the difference of accuracy between MTSM and Perturbation Method correspond scarcely a difference of A.A.R.E $9.315x10^{-4}$. This small difference shows the convenience of employing MTSM given the difference of effort employed for both methods. While PM solved five recurrence differential equations, the proposed method required to solve essentially three derivatives and an algebraic equation. From above it is very important to emphasize that, it is possible to improve the accuracy of our MTSM approximation keeping more terms in Taylor series expansion (19).

x	Exact	MTSM (19)	MHPMLT (47) [9]	P.M (19) [10]
0.1	0.9270953420	0.9216218664	0.899999999	0.9345155625
0.2	0.8489990321	0.8399129100	0.843983936	0.8514560000
0.3	0.7654913465	0.7544347663	0.762539659	0.7625813125
0.4	0.6763157367	0.6646836965	0.676971888	0.6719040000
0.5	0.5811698795	0.5700905875	0.585906125	0.5791015625
0.6	0.4796933928	0.4700209520	0.487967872	0.4819840000
0.7	0.3714504952	0.3637749282	0.381782631	0.3780163125
0.8	0.2559046829	0.2505872801	0.265975904	0.2648960000
0.9	0.1323801709	0.1296273972	0.139173193	0.1401855625
	A.A.R.E	$1.66348x10^{-2}$	$2.04270x10^{-2}$	$1.57033x10^{-2}$

Table 1: Comparison between (19), exact solution and other reported approximate solutions, using $\varepsilon = 1$

VI. CONCLUSIONS.

This work presented MTSM method in order to calculate an analytical approximate solution for the problem that describes the steady state one dimensional conduction of heat in a slab with thermal conductivity linearly dependent on the temperature. The method basically works calculating derivatives of several orders and expresses the solution of a differential equation in terms of the solution of one or more algebraic equations. The comparison with other methods of the literature show the convenience of employ MTSM as a practical tool with the purpose to obtain accuracy solutions for boundary value problems instead of using other more sophisticated and cumbersome procedures.

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