A Fullly Backlogged Deteriorating Inventory Model with Price Dependent Demand using Preservation Technology Investment and Trade Credit Policy

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Abstract: **Here we developed an EOQ inventory model using price dependent demand taking deterioration as a factor. Also we use preservation technology cost to control deterioration and apply trade credit policy to attraact customers to buy more products.In this work shortages are allowed and fullly backlogged during the specific time period.Our main objective in this paper is to find optimal cycle length and preservation technology strategies while maximized the total profit. We have presented a numerical example to validate the work and analysed the sensitivity of different parameters used in this work using LINGO software and also shown graphically the profit function is concave using MATLAB software.**

Keywords: Deterioration, Preservation technology, Trade credit policy, Shortage,Backlogged.

1. INTRODUCTION

Generally the word inventory defined as stock of goods and it has three different stages i.e raw materials,work-in-process products and fininished goods that are considered to be the part of a business assets that are ready or will be ready for sale.So, inventory is a most important part in a business, considering the vital role of inventory in a business , business organisation always put emphasis on proper management of inventory to run their business smoothly.So proper inventory management gives profit to a business organisation. Here we have taken some factors related to management of inventory properly which helps the business organisation to take better decisions. Deterioration of inventory is a key factor in almost all business organisation which affects the decision related to inventory management.The word deterioration defined as decay or damage or worst or out dated etc. according to the different products. There are some products deteriorate or decay during their storage period such as fruits , vegetables,eggs, fishes, rice,wheat and seasonal products etc. and some products are out-dated due to arrival of new products in the market with new technology such as electronic items, automobiles and radioactive substances etc. Many researchers developed their work taking deterioration in their model such as Darwiash & Odah(2010) has developed vendor managed inventory for single-vendor multi retailer supply chains and Chang et el (2010) has developed a non-instantaneous deteriorating inventory models with stock-dependent demand . In this way researcher like Huang et al(2011) has introduced preservation technology and developed an inventory model. In this connection we may refer several related research work was discussed by Bhunia and Shaikh (2011 a,b) , Lee & Dye(2012) ,Bhunia et al (2013) , Hseih & Dye (2013) , Bhunia et al (2015), Bhunia & Shaikh(2016), Shaikh (2016 a,b) and Bhunia et al (2017) and others.

Demand plays an important role in a business.So researcher gives importance to demand and developed their model taking different types of demand according to the market needs. In earlier inventory models , generally demand rate assumed to be either constant, time dependent and stock dependent etc. How ever it is observed that selling price of a product is also most important factor in customer point of view because selling price of a products always present in the mind of a customer before buying a product. So business organisation always changing the price of their products according to the market demand to attracts the customers. So sometimes demand is dependent upon price of a products. Several researchers developed inventory model taking priced dependent demand like Sana(2011) has developed an inventory model taking price sensitive demand with perishable items. Similarly researcher like Maihami et al (2012) ,Avinadav et al (2013) and Shaikh et al (2017) and others have discussed on price dependent demand in their inventory models.

Now-a-days, trade credit policy plays an important role in a business scenario. Trade credit is the credit extended by the supplier to the customers for the puchase of goods and services. Trade credit helps the retailer to purchase the supplies of goods by the supplier without immediate payment. Trade credit is commonly used by the business organisation as a source of short-term financing. There are many forms of trade credit in many forms , different

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business organisation use various specialized forms of trade to attracts the customers to bye more products from their organisation. Many researchers discussed trade credit in their research work. Researcher like Min et al (2010) has discussed an inventory model with stock dependent demand and two level trade credit. In this connection we may refer several related research work discussed by Liang & Zhou(2011) , Mahata (2012),Teng et al (2013) , Shaikh (2017 a, b) and others.

In this work , we developed an EOQ inventory model using price dependent demand taking deterioration as a factor.Also we use preservation technology cost to control deterioration and apply trade credit policy to attraact customers to buy more products.In this work shortages are allowed and fullly backlogged during the specific time period.Our main objective in this paper is to find optimal cycle length and preservation technology strategies while maximized the total profit. The We have presented a numerical example to validate the work and analysed the sensitivity of different parameters used in this work. Lastly we have given some concluding remarks and future research .

2.NOTATIONS AND ASSUMPTIONS

In order to develop the inventory models we have been used the following notations and assumptions:
Notation:

Notations	Units	Description
c	\$/order	Purchasing cost per order.
h	\$/unit	Holding cost per unit
b	\$/unit	Shortage cost per unit
A	\$/unit	Replenishment cost per order.
δ	\$/unit	Backlogging parameter
p	\$/unit	Selling price per unit
θ	Constant	Deterioration rate
M	Month	Period of permissible delay in payments offered by the supplier.
a	Constant	
b	Constant	
ξ	\$/unit	Preservation technology cost
I_e	\$/unit	Rate of interest earned by the retailer.
I_c	\$/unit	Rate of interest payable by the supplier
m_1	\$/unit	Mark up rate
R	Units	Maximum shortage quantity per cycle.
S	Units	Initial inventory level.
Q	Units	Order size per cycle
t_1	Month	Time point at which the inventory level reaches zero
T	Month	The total length of the inventory cycle.
$Z^1(t_1,T,\xi)$	\$/month	The total profit per unit time for the interval $0<M<t_1$
$Z^2(t_1,T,\xi)$	\$/month	The total profit per unit time for the interval $t_1<M<T$
		Decision variable
t_1	Month	Time at which the stock reaches zero
T	Month	The total length of the inventory cycle.
ξ	\$/unit	Preservation technology cost

Assumptions:

The model is developed for a single deteriorating item for linearly price a dependent demand pattern $D(p)=a-bp$ i.e., demand function depends on price ,where $a>0$ and $b>0$.

The deterioration rate $\theta(0<\theta<<1)$ is constant and depends on the stock amount.

There is a no replacement or repair for deteriorated products during the period under consideration.

Replenishment rate is infinite and Lead-time is negligible or zero.

The total planning horizon of the inventory system is infinite.

The relationship of deterioration rate and the preservation technology investment parameter satisfies the following $\frac{\partial m(\xi)}{\partial \xi} < 0$, $\frac{\partial^2 m(\xi)}{\partial \xi^2} > 0$. Therefore, this research work considers that $m(\xi) = \theta - e^{-a_1 \xi}$; where, $m(\xi)$ is the deterioration rate when there is investing preservation technology, θ is the deterioration rate without preservation technology investment , and a_1 is the sensitive parameter of investment to the deterioration rate.

3. MATHEMATICAL FORMULATION

During the time period $[0, t_1]$, the inventory level decreases due to both demand and deerioration and drop to zero.Thus the inventory level can be represented in the form of the following differential equation.

$$\frac{dI_1(t)}{dt} + (\theta - m(\xi)) I_1(t) = -(a - bp) \qquad 0 < t < t_1 \qquad (1)$$

With the boundary condition $I_1(t_1) = 0$.Solving equation (1) , we have

$$I_1(t) = \frac{(a - bp)}{(\theta - m(\xi))} \left(e^{(\theta - m(\xi))(t_1 - t)} - 1 \right) \qquad 0 < t < t_1 \qquad (2)$$

The inventory level reaches zero at $t = t_1$, then shortage occured during the time period $[t_1, T]$ and the unsatisfied demand is completely backlogged. The level of inventory during the time period $[t_1, T]$ can be represented in the form of following differential equation:

$$\frac{dI_2(t)}{dt} = -\delta(a - bp) \qquad t_1 < t < T \qquad (3)$$

With the boundary conditions $I_2(T) = -R$. Solving equation (3), we have

$$I_2(t) = \delta(a - bp)(T - t) - R \qquad t_1 < t < T \qquad (4)$$

Now using the continuity property at $t = t_1$, we have $I_1(t_1) = I_2(t_1)$ which gives the maximum amout of shortages which is backogged per cycle

$$R = \delta(a - bp)(T - t_1) \qquad (5)$$

The maaximum invetory level is $I_1(t) = S$ at $t = t_1$ is

$$S = \frac{(a - bp)}{(\theta - m(\xi))} \left(e^{(\theta - m(\xi)) t_1} - 1 \right) \qquad (6)$$

Hence the total ordering quantity per cycles is given as follows

$$Q = S + R$$

$$= \frac{(a - bp)}{(\theta - m(\xi))} \left(e^{(\theta - m(\xi)) t_1} - 1 \right) + \delta(a - bp)(T - t_1) \qquad (7)$$

Now the different cost assoccieated in this model is

Sales revenue

$$SR = p \int_0^{t_1} D dt + PR$$

$$= PDt_1 + PR$$

Purchasing cost

$$PC = cQ$$

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$$=c\left[\frac{(a-bp)}{(\theta-m(\xi))}\left(e^{(\theta-m(\xi))t_1}-1\right)+\delta(a-bp)(T-t_1)\right]$$

Holding cost

$$HC=h\int_0^{t_1} I_1(t)\,dt$$

$$=h\left[\frac{e^{(\theta-m(\xi))t_1}}{(\theta-m(\xi))}-\frac{1}{(\theta-m(\xi))}-t_1\right]$$

Backlogging cost

$$BC=b\int_{t_1}^{T}\left[-I_2(t)\right]dt$$

$$=b\left[R(T-t_1)-\delta(a-bp)\left(\frac{T^2}{2}-Tt_1+\frac{t_1^{\,2}}{2}\right)\right]$$

Preservation technology cost

$$PTC=\xi T$$

Trade credit is described in two different interval.

Case 1:- $0<M<t_1$

Case 2:- $t_1<M<t_2$

$$0<M<t_1$$

$$IE_1=pI_e\int_0^{t_1}\int_0^{t} Ddudt+pI_eRt_1$$

$$=\frac{pI_eDt_1^{\,2}}{2}+pI_eRt_1$$

$$IC_1=cI_c\int_M^{t_1} I_1(t)\,dt$$

$$=cI_c\left[\frac{(a-bp)}{(\theta-m(\xi))}\left(\frac{e^{(\theta-m(\xi))(t_1-M)}-1}{(\theta-m(\xi))}\right)-(t_1-M)\right]$$

Hence the total profit function is for two case is

$$Z^1(t_1,T,\xi)=\frac{X}{T}$$

Where $X=(SR-PC-OC-HC-BC+IE_1-IC_1)$

$$X=PDt_1+PR-c\left[\frac{(a-bp)}{(\theta-m(\xi))}\left(e^{(\theta-m(\xi))t_1}-1\right)+\delta(a-bp)(T-t_1)\right]$$

$$-A-h\left[\frac{e^{(\theta-m(\xi))t_1}}{(\theta-m(\xi))}-\frac{1}{(\theta-m(\xi))}-t_1\right]$$

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$$-b\left[R\left(T-t_1\right)-\delta\left(a-bp\right)\left(\frac{T^2}{2}-Tt_1+\frac{t_1^{\,2}}{2}\right)\right]-\xi T$$

$$+\frac{pI_eDt_1^{\,2}}{2}+pI_eRt_1-cI_c\left[\frac{\left(a-bp\right)}{\left(\theta-m\left(\xi\right)\right)}\left(\frac{e^{\left(\theta-m\left(\xi\right)\right)\left(t_1-M\right)}-1}{\left(\theta-m\left(\xi\right)\right)}\right)-\left(t_1-M\right)\right]$$

$t_1 < M < t_2$

In this case the interest earned is

$$IE_2 = pI_e\int_0^{t_1}\int_0^{t} Ddudt+\left(M-t_1\right)pI_eD+pI_eRM$$

There is no interest charged for this case .

Now the total profit function is

$$Z^2\left(t_1,T,\xi\right)=\frac{X}{T}$$

Where $X=\left(SR-PC-OC-HC-BC+IE_2\right)$

$$X=PDt_1+PR-c\left[\frac{\left(a-bp\right)}{\left(\theta-m\left(\xi\right)\right)}\left(e^{\left(\theta-m\left(\xi\right)\right)t_1}-1\right)+\delta\left(a-bp\right)\left(T-t_1\right)\right]$$

$$-A-h\left[\frac{e^{\left(\theta-m\left(\xi\right)\right)t_1}}{\left(\theta-m\left(\xi\right)\right)}-\frac{1}{\left(\theta-m\left(\xi\right)\right)}-t_1\right]$$

$$-b\left[R\left(T-t_1\right)-\delta\left(a-bp\right)\left(\frac{T^2}{2}-Tt_1+\frac{t_1^{\,2}}{2}\right)\right]-\xi T$$

$$+\frac{pI_eDt_1^{\,2}}{2}+pI_eRM+PI_eD\left(M-t_1\right)$$

4. NUMERICAL EXAMPLE

To illustrate and validate of our proposed inventory model, we have considered two numerical examples with the following values of different parameters as given below:

Example 1:-

$A=\$200/odrer, p=\$60/unit, h=\$2/unit/year, b=\$6/unit,$

$I_c=\$0.12/\$/year, I_e=\$0.06/\$/year, M=90/365/year, \delta=.5, a_1=0.09, \theta=.5, a=220,$

$c=40, b=.4$.From the above numerical example, we have obtained case one gives better optimal solution which are described below:-

$Z^{1*}\left(t_1,T,\xi\right)=\$3223.413$,$t_1^*=0.2617378, T^*=0.2648429$ and $\xi^*=7.703062.$

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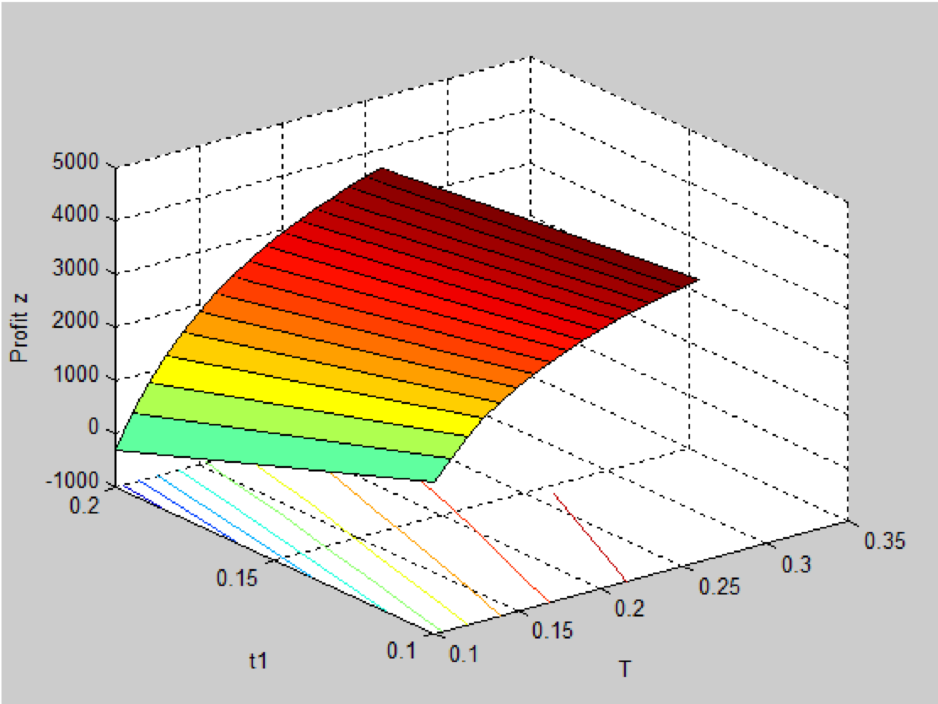

Figure-1 The above figure represent the concavity of the profit function.

5.SENSITIVITY ANALYSIS

The above described numerical, we have performed sensitivity analysis for example-1 to study the effect of under or over estimation of the inventory system parameters on the optimal values of the initial time period, cycle length, preservation cost, initial stock level , maximum shortage rate along with the maximum profit of the system. The percentage changes in the above mentioned optimal values are taken as measures of sensitivity. The analysis is carried out by changing (increasing and decreasing) the parameters by -20% to +20%. The results are obtained by changing one parameter at a time and keeping the other parameters at their original values. The results of these analyses are given in Tables 1.

Table 1: Sensitivity analysis with respect to different parameters

Parameter	% Change In Parameters	% Change in	% Change in				
		Z^{1*}	R^*	S^*	t_1^*	T^*	ξ^*
c	-20	4389.374	0	50.61842	0.2464362	0.2464362	23.31297
	-10	4100.322	0.1487365	60.83170	0.3103647	0.3108706	7.702175
	10	2769.370	0.2468636E-03	110.3731	0.5631272	0.5631280	7.701725
	20	1972.910	8.274615	70.38822	0.3591233	0.3872683	7.701733
A	-20	3333.390	0.7834220	48.40948	0.2469867	0.2496514	7.701943
	-10	....	....	....	....	....	....
	10	3060.025	0.2170516E-01	48.34907	0.2466788	0.2467526	7.701699
	20	....	....	....	....	....	....
p	-20	....	....	....	....	....	....
	-10	2193.700	0.1235149	63.84792	0.3218141	0.3222291	7.701639
	10	....	....	....	....	....	....
	20	....	....	....	....	....	....
h	-20	3642.240	0.000000	167.3874	0.8540155	0.8540155	7.701739
	-10	4898.210	205.8461	48.32878	0.2465753	0.9467322	7.701673
	10	3154.611	0.8343884	48.39348	0.2464680	0.2493061	8.026030
	20	....	....	....	....	....	....
b	-20	3938.709	26.81266	52.08820	0.2657551	0.3569546	7.702283
	-10	....	....	....	....	....	....
	10	3119.092	0.000000	0.2464483	0.2464483	0.2464483	8.165308
	20	....	....	....	....	....	....

a_1	-20	4895.013	206.0336	48.32881	0.2465753	0.9473700	9.627235
	-10	....	....	....	....	....	....
	10	2916.870	0.000000	49.65414	0.2465221	0.2465221	12.86717
	20	4830.743	132.5408	48.32877	0.2465753	0.6973945	6.418031
θ	-20	3628.266	1.946967	178.7438	0.9119577	0.9185801	10.18103
	-10	3607.586	0.1082727E-01	163.2144	0.8327254	0.8327622	8.872389
	10	3619.749	0.7871395	164.2208	0.8378601	0.8405375	6.642698
	20	2885.661	0.000000	49.84394	0.2464831	0.2464831	11.73518
δ	-20	....	....	....	....	....	....
	-10	....	....	....	....	....	....
	10	3143.858	0.7204667E-01	48.37213	0.2467964	0.2470192	7.701728
	20	3168.504	0.8781295E-04	50.01328	0.2551697	0.2551700	7.701675
a	-20	2748.050	0.1613506E-01	145.0886	0.9545303	0.9545892	7.701645
	-10	2997.844	0.6726495E-03	68.75979	0.3951692	0.3951713	7.702207
	10	3470.987	0.000000	54.42541	0.2465443	0.2465443	10.22527
	20	4044.856	0.6072942E-04	60.32534	0.2513555	0.2513556	7.701704
b	-20	5329.602	97.23111	49.65601	0.2472908	0.5163012	7.701703
	-10						
	10	3566.794	0.8474837E-02	153.8597	0.7947300	0.7947543	7.701638
	20	3579.261	7.410067	230.2746	1.204365	1.225896	7.701637
M	-20	....	....	....	....	....	....
	-10	....	....	....	....	....	....
	10	3217.480	0.000000	53.16452	0.2712473	0.2712473	7.701748
	20	3296.985	0.000000	59.15894	0.3018312	0.3018312	7.701692

6. CONCLUSION

The purpose of this study , is to present a deteriorating inventory model with price dependent demand with shortage and fully backlogged. Here we have introduced preservation technology investment to control the deterioration rate for highly deteriorated products. Also we have apply trade credit policy in the perspective of retailers. We also provide a useful solution procedure to find the optimal cycle length and preservation technology investment strategies while maximizing the total profit per unit time. Here we have solved numerical examples which explained the importance of preservation technology investment and trade credit policy. Finally we have shown graphically the profit function is concave by using matlab software and analysed sensitivity of different parameters of the model by Lingo software which helps the business organisation for better managerial decisions.

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