A Finite source Perishable Inventory system with Retrial demands and Multiple server vacation

C. Periyasamy
Department of Mathematics,
Madurai Kamaraj University Constituent College,
Sattur - 626 203, INDIA.

Abstract

In this article, we consider a continuous review perishable inventory system with a finite number of homogeneous sources of demands. The inventory is replenished according to a state dependent (s, S) ordering policy and the lead times are assumed to follow an exponential distribution. The life time of each item is assumed to be exponential. The server goes for a vacation of an exponentially distributed duration whenever the inventory level reaches zero. If the server finds empty stock when he returns to the system, he continues his vacation. The demands that occur during stock out period and/or during the server vacation period enter into the orbit. These orbiting demands send out signal to compete for their demand and the retrial times are distributed as exponential. The joint probability distribution of the inventory level and the number of demands in the orbit are obtained in the steady state case. Various system performance measures are derived and the results are illustrated numerically.

Keywords : Continuous review Inventory System, Random life time, State dependent (s, S) policy, Finite population, Retrial demands Multiple vacation

1 Introduction

In most of the inventory models considered in the literature, the demanded items are directly delivered from the stock (if available). The demands occurring during the stock-out period are either lost (lost sales) or satisfied only after the arrival of ordered items (backlogging). The often quoted review articles Nahmias [7] and Raafat [10] and Goyal and Giri [5] provide excellent summaries of many of these modelling efforts.

However, in Queueing systems with server vacations have been widely studied in different contexts in the literature. Continuous review inventory system with server vacation has been received little attention in the literature. Daniel and Ramanarayanan [3] have first introduced the concept of server vacation in inventory with two servers. In [4], they have studied an inventory system in which the server takes a rest when the level of the inventory is zero.

The concept of retrial demands in inventory was introduced by Artalejo et al. [2]. They have assumed Poisson demand, exponential lead time and exponential retrial time. In that work, the authors proceeded with an algorithmic analysis of the system. Ushakumari [14] considered a retrial inventory system with classical retrial policy. Krishnamoorthy and Jose [6] analysed three different retrial inventory with positive service time and positive lead-Time. Sivakumar [12] has considered a retrial inventory system with multiple server vacation and in[11] he has considered a perishable inventory system with retrial demands.

In this paper, we address a continuous review perishable inventory system with a finite number of homogeneous sources of demands. The operating policy is a state dependent ordering policy. According to this policy the placement of order occurs in the following situations:

1. When the on hand inventory level reaches the prefixed level s, he places an order for $Q(= S - s)$ items.
2. When the server returns to the system (the following situations may arise because of the nature of the item),
   - If the on hand inventory level is i which is less than or equal to s, he place an order for Q items and terminates his vacation.
   - If the inventory level is zero and the ordered items are not pending, he place an order for Q units and continues his vacation.

The server terminates his vacation only when he finds the positive inventory level. During the vacation period, any arriving primary demands enter the orbit. These orbiting demands compete for their demands after a random time. The inter-retrial times follows exponential distribution.

The rest of the paper is organized as follows. In Section 2, we describe the problem and in the next section analyse the mathematical model of the problem under study. The steady-state analysis of the model is presented in section 4 and some key system performance measures are derived in Section 5. In the last section, we perform sensitivity analyses on the total expected cost rate in terms of numerical illustrations.

2 Problem formulation

We consider an inventory system with a maximum stock of S units and the items are distributed by the server to the demands. The items are perishable in nature. The lifetime of each item is exponential with parameter $\gamma(> 0)$. The demands are originated from the population of finite size N. The demand time points form a Quasi-random distribution with parameter α, demand only single unit at a time. The operating policy is the state dependent (s, S) ordering policy. The replenishment of stock occur after some random time. The lead time is exponentially distributed with mean rate $\mu(>0)$. When the on-hand inventory level zero, the server goes for vacation. The duration of the server vacation is an exponential random variable with parameter $\beta(>0)$. Due to the perishable nature of the items, during the server vacation period the stock replenished and the items may perish. The situation makes to place an order. According to our ordering policy, when the server returns to the system, if the on hand inventory level is $i(>0)$ which is less than or equal to the prefixed level s, he place an order for Q items and he terminates his vacation or the inventory level zero and the ordered items are not pending, he place an order for Q units and continues his vacation. Server terminates his vacation only when he finds the positive stock. Demands that occur during stock-out period and/or during server vacation periods enters into the orbit. These orbiting demands compete for their demands according to an exponential distribution with parameter $\theta(>0)$. We consider the classical policy where each demands in the orbit conducts his own attempts to obtain service independently from the other demands present in the orbit. We can then assume that the probability of a retrial during the time interval $(t, t+dt)$, given that j demands were in orbit at time t, is $j\theta dt + o(dt)$. Each source is either free or in the orbit at any time. We also assume that the inter-demand times between the primary demands, lead times, lifetime of each items, retrial demand times and server vacation time are mutually independent random variables.

Notation :

A_{ij} : element/sub-matrix at ith row, jth column of the matrix A.
$\mathbf{e}$: a column vector of appropriate dimension containing all ones.
$\mathbf{I}$: an identity matrix of appropriate dimension.

3 Analysis

Let $X(t), Y(t), Z(t)$ and $Z'(t)$, respectively, denote the inventory level, number of demands in the orbit, server status(0-is on vacation & 1-is available for provide item) and the status of the ordered item(0-received & 1-not received) at time t. From the assumption made on the input and output processes, it may be verified that the stochastic process $\{X(t), Y(t), Z(t), Z'(t) : t \geq 0\}$ is a Markov process with state space E, which is defined as, Here

$$\begin{aligned}
E &= E_1 \cup E_2 \cup E_3 \cup E_4, \\
E_1 &= \{(i,j,k,l) \mid i = s+1, s+2, \ldots, S,\ j = 0,1,2,\ldots,N,\ k = 0,1,\ l = 0\} \\
E_2 &= \{(i,j,k,l) \mid i = 1,2,\ldots,s,\ j = 0,1,2,\ldots,N,\ k = 0,\ l = 0\} \\
E_3 &= \{(i,j,k,l) \mid i = 1,2,\ldots,s,\ j = 0,1,2,\ldots,N,\ k = 1,\ l = 1\} \\
E_4 &= \{(i,j,k,l) \mid i = 0,\ j = 0,1,2,\ldots,N,\ k = 0,\ l = 0,1\}
\end{aligned}$$

The values taken by these random variables are listed in the following table.

<table>
<tr><th>i</th><th>j</th><th>k</th><th>l</th></tr>
<tr><td>0</td><td>0
1
2
⋮
N</td><td>0</td><td>0
1</td></tr>
<tr><td rowspan="2">1
2
⋮
s</td><td rowspan="2">0
1
⋮
N</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td rowspan="2">$s+1$
$s+2$
⋮
S</td><td rowspan="2">0
1
⋮
N</td><td>0</td><td rowspan="2">0</td></tr>
<tr><td>1</td></tr>
</table>

The state space of the stochastic process $\{X(t), Y(t), Z(t), Z'(t) | t \geq 0\}$ is the collection of all quadruples $(i) = (i, j, k, l)$ where each entry is selected from each column as we move from left to right; we may cross vertical lines but not horizontal ones. These quadruples can be ordered in the lexicographic order in each box separated by the horizontal lines. Define the following sets:

$$\begin{aligned}
(i) &= (<< i, 0 >>, << i, 1 >>, \ldots, << i, N >>), \\
<< i, k >> &= (< i, k, 0 >, < i, k, 1 >), \\
< i, k, 0 > &= ((i, k, 0, 0)), \\
< i, k, 1 > &= ((i, k, 1, 1)) \text{ for } i = 1, 2, \ldots, s, \\
< i, k, 1 > &= ((i, k, 1, 0)) \text{ for } i = s+1, s+2, \ldots, S.
\end{aligned}$$

Then the state space of the process can be ordered as $\{(0), (1), (2), \ldots, (S)\}$, where $(0) = ((0, j, 0, 0), (0, j, 0, l))$ for $j = 0, 1, \ldots, N$,

Then the infinitesimal generator P can be conveniently expressed in block partitioned matrix with entries,

$$P = \begin{array}{c} \\ (0) \\ (1) \\ \vdots \\ (s) \\ \vdots \\ (Q) \\ (Q+1) \\ \vdots \\ (S) \end{array}
\begin{array}{c}
\begin{array}{ccccccccccc} (0) & (1) & (2) & \cdots & (s) & \cdots & (Q) & (Q+1) & \cdots & (S) \end{array} \\
\left(\begin{array}{cccccccccc}
A_0 & & & & & & C_0 & & & \\
B_1 & A_1 & & & & & & C_1 & & \\
 & & \ddots & \ddots & & & & & \ddots & \\
 & & & B_s & A_s & & & & & C_1 \\
 & & & & \ddots & \ddots & & & & \\
 & & & & & B_Q & A_Q & & & \\
 & & & & & & B_{Q+1} & A_{Q+1} & & \\
 & & & & & & & \ddots & \ddots & \\
 & & & & & & & & B_S & A_S
\end{array} \right)
\end{array}$$

For $i = 0, 1, 2, \ldots, S$,

$$A_i = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \vdots \\ N-1 \\ N \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & \cdots & N-1 & N \end{array} \\ \begin{pmatrix} D_{i0} & E_{i0} & & & & \\ & D_{i1} & E_{i1} & & & \\ & & D_{i2} & E_{i2} & & \\ & & & \ddots & \ddots & \\ & & & & D_{iN-1} & E_{iN-1} \\ & & & & & D_{iN} \end{pmatrix} \end{array}$$

For $i = 1, 2, \ldots, S$,

$$B_i = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \vdots \\ N-1 \\ N \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & \cdots & N-1 & N \end{array} \\ \begin{pmatrix} F_{i0} & & & & & \\ G_{i1} & F_{i1} & & & & \\ & G_{i2} & F_{i2} & & & \\ & & \ddots & \ddots & & \\ & & & G_{iN-1} & F_{iN-1} & \\ & & & & G_{iN} & F_{iN} \end{pmatrix} \end{array}$$

For $i = 0, 1, \ldots, s$,

$$C_i = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \vdots \\ N \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & \ldots & N \end{array} \\ \begin{pmatrix} H_{i0} & & & & \\ & H_{i1} & & & \\ & & H_{i2} & & \\ & & & \ddots & \\ & & & & H_{iN} \end{pmatrix} \end{array}$$

The dimension of the main matrices are defined in Table 1 and the dimension of the sub-matrices are explicitly from the structure of the matrices. The sub-matrices are defined as, For $j = 0, 1, 2, \ldots, N$,

$$D_{ij} = \begin{cases} \begin{array}{c} \\ 0 \end{array} \begin{array}{c} 0 \\ \begin{pmatrix} D_{ij}^{00} \end{pmatrix} \end{array} & i = 0 \\ \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{pmatrix} D_{ij}^{00} & D_{ij}^{01} \\ \mathbf{0} & D_{ij}^{11} \end{pmatrix} \end{array} & i = 1, 2, \ldots, Q \\ \begin{array}{c} \\ 1 \end{array} \begin{array}{c} 1 \\ \begin{pmatrix} D_{ij}^{11} \end{pmatrix} \end{array} & i = Q+1, Q+2, \ldots, S \end{cases}$$

$$
E_{ij} = \begin{cases}
\begin{array}{cc} & \begin{array}{c} 0 \end{array} \\ \begin{array}{c} 0 \end{array} & \begin{pmatrix} E^{00}_{0j} \end{pmatrix} \end{array} & i = 0 \\[2ex]
\begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} E^{00}_{ij} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array} & i = 1, 2, \ldots, Q
\end{cases}
$$

$$
F_{ij} = \begin{cases}
\begin{array}{cc} & \begin{array}{c} 0 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} F^{00}_{1j} \\ F^{10}_{1j} \end{pmatrix} \end{array} & i = 1 \\[2ex]
\begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} F^{00}_{ij} & \mathbf{0} \\ \mathbf{0} & F^{11}_{ij} \end{pmatrix} \end{array} & i = 2, 3, \ldots, Q \\[2ex]
\begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 1 \end{array} & \begin{pmatrix} \mathbf{0} & F^{11}_{ij} \end{pmatrix} \end{array} & i = Q + 1 \\[2ex]
\begin{array}{cc} & \begin{array}{c} 1 \end{array} \\ \begin{array}{c} 1 \end{array} & \begin{pmatrix} F^{11}_{ij} \end{pmatrix} \end{array} & i = Q + 2, Q + 3, \ldots, S.
\end{cases}
$$

$$
G_{ij} = \begin{cases}
\begin{array}{cc} & \begin{array}{c} 0 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} \mathbf{0} \\ G^{10}_{1j} \end{pmatrix} \end{array} & i = 1 \\[2ex]
\begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & G^{11}_{ij} \end{pmatrix} \end{array} & i = 2, 3, \ldots, Q \\[2ex]
\begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 1 \end{array} & \begin{pmatrix} \mathbf{0} & G^{11}_{ij} \end{pmatrix} \end{array} & i = Q + 1 \\[2ex]
\begin{array}{cc} & \begin{array}{c} 1 \end{array} \\ \begin{array}{c} 1 \end{array} & \begin{pmatrix} G^{11}_{ij} \end{pmatrix} \end{array} & i = Q + 2, Q + 3, \ldots, S.
\end{cases}
$$

$$H_{ij} = \begin{cases} \begin{array}{c} \\ 0 \end{array} \begin{array}{cc} 0 & 1 \\ \left(H_{ij}^{00} \right. & \left. \mathbf{0} \right) \end{array} & i = 0 \\ \\ \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{cc} 0 & 1 \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & H_{ij}^{11} \end{array} & i = 1 \end{cases}$$

Define $\eta_j = (N-j)\alpha$, $\psi_{ij} = (N-j)\alpha + i\gamma$

$$D_{0j}^{00} = \begin{array}{c|cc} & 0 & 1 \\ \hline 1 & & -\eta_j - \mu \end{array}$$

For $i = 1, 2, \ldots, s$

$$D_{ij}^{00} = \begin{array}{c|c} & 0 \\ \hline 0 & -\beta - \psi_{ij} \end{array}$$

$$D_{ij}^{01} = \begin{array}{c|c} & 0 \\ \hline 0 & \beta \end{array}$$

$$D_{ij}^{11} = \begin{array}{c|c} & 0 \\ \hline 0 & -\psi_{ij} - j\theta \end{array}$$

$$D_{ij}^{00} = \begin{array}{c|c} & 0 \\ \hline 0 & -\beta - \psi_{ij} \end{array}$$

For $i = s+1, s+2, \ldots, S$

$$D_{ij}^{01} = \begin{array}{c|c} & 1 \\ \hline 0 & \beta \end{array}$$

$$D_{ij}^{11} = \begin{array}{c|c} & 1 \\ \hline 1 & -\psi_{ij} - \mu \end{array}$$

$$E_{0j}^{00} = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & \eta_j & \\ 1 & & \eta_j \end{array}$$

For $i = 1, 2, \ldots, S$

$$E_{ij}^{00} = \begin{array}{c|c} & 0 \\ \hline 0 & \eta_j \end{array}$$

$$F_{1j}^{00} = \begin{array}{c|c} & 0 \\ \hline 0 & \gamma \end{array}$$

$$F_{1j}^{10} = \begin{array}{c|cc} & 0 & 1 \\ \hline 1 & & \psi_{1j} \end{array}$$

For $i = 2, 3, \ldots, s+1$,

$$F_{ij}^{00} = \begin{array}{c|c} & 0 \\ \hline 0 & i\gamma \end{array}$$

$$F_{ij}^{11} = \begin{array}{c|c} & 1 \\ \hline 1 & \psi_{ij} \end{array}$$

For $i = s+2, s+3, \ldots, S$,

$$F_{ij}^{11} = \begin{array}{c|c} & 0 \\ \hline 0 & \psi_{ij} \end{array}$$

$$F_{ij}^{00} = \begin{array}{c|c} & 0 \\ \hline 0 & i\gamma \end{array}$$

$$G_{1j}^{10} = \begin{array}{c|cc} & 0 & 1 \\ \hline 1 & & j\theta \end{array}$$

For $i = 2, 3, \ldots, s+1$,

$$G_{ij}^{11} = \begin{array}{c|c} & 1 \\ \hline 1 & j\theta \end{array}$$

For $i = s+2, s+3, \ldots, S$,

$$G_{ij}^{11} = \begin{array}{c|c} & 0 \\ \hline 0 & j\theta \end{array}$$

$$H_{0j}^{00} = \begin{array}{c|c} & 0 \\ \hline 0 & 0 \\ 1 & \mu \end{array}$$

$$H_{0j}^{11} = \begin{array}{c|c} & 0 \\ \hline 1 & \mu \end{array}$$

Matrix	Dimension	Matrix	Dimension
A_0	$(2(N+1), 2(N+1))$	C_1	$(2(N+1), N+1)$
$A_i,\ i = 1, 2, \ldots, Q$	$(2(N+1), 2(N+1))$	$B_i,\ i = 1, 2, \ldots, Q$	$(2(N+1), 2(N+1))$
$A_i,\ i = Q+1, \ldots, S$	$(N+1, N+1)$	B_{Q+1}	$(N+1, 2(N+1))$
C_0	$(2(N+1), 2(N+1))$	$B_i,\ i = Q+2, \ldots, S$	$(N+1, N+1)$

Table 1: Dimension of the matrices

4 Steady state analysis

It can be seen from the structure of P that the homogeneous Markov process $\{(X(t), Y(t), Z(t), Z'(t)), t \geq 0\}$ on the finite state space E is irreducible. Define

$$\Phi((i,j,k,l;i_1,j_1,k_1,l_1),t) = Pr\left[(X(t),Y(t),Z(t),Z'(t)) = (i,j,k,l)|(X(0),Y(0),Z(0),Z'(0)) = (i_1,j_1,k_1,l_1)\right]$$

Hence the limiting distribution
$\pi^{(i,j,k,l)} = \lim_{t\to\infty} \Phi((i,j,k,l;i_1,j_1,k_1,l_1),t)$ exists.
Let $\Pi = (\pi^{<<<0>>>}, \pi^{<<<1>>>}, \ldots, \pi^{<<<S>>>})$ where,

$$\begin{aligned}
\pi^{<<<i>>>} &= (\pi^{<<i,0>>}, \pi^{<<i,1>>}, \ldots, \pi^{<<i,N>>}),\ i = 0,1,\ldots,S \\
\pi^{<<i,j>>} &= (\pi^{<i,j,0>}),\ i = 0,\ j = 0,1,\ldots,N, \\
\pi^{<i,j,0>} &= (\pi^{(i,j,0,0)}, \pi^{(i,j,0,1)}),\ i = 0,\ j = 0,1,\ldots,N, \\
\pi^{<<i,j>>} &= (\pi^{<i,j,0>}, \pi^{<i,j,1>}),\ i = 1,2,\ldots,Q,\ j = 0,1,\ldots,N, \\
\pi^{<i,j,0>} &= (\pi^{(i,j,0,0)}),\ i = 1,2,\ldots,Q,\ j = 0,1,\ldots,N, \\
\pi^{<i,j,1>} &= \begin{cases} (\pi^{(i,j,1,1)}),\ i = 1,2,\ldots,s,\ j = 0,1,\ldots,N, \\ (\pi^{(i,j,1,0)}),\ i = s+1,s+2,\ldots,Q,\ j = 0,1,\ldots,N \end{cases} \\
\pi^{<<i,j>>} &= (\pi^{<i,j,1>}),\ i = Q+1,Q+2,\ldots,S,\ j = 0,1,\ldots,N, \\
\pi^{<i,j,1>} &= (\pi^{(i,j,1,0)}),\ i = Q+1,Q+2,\ldots,S,\ j = 0,1,\ldots,N.
\end{aligned}$$

The limiting distribution Π can be computed by using

$$\Pi P = 0 \quad \text{and} \quad \Pi \mathbf{e} = 1. \tag{4.1}$$

The first equation of the above yields the following set of equations :

$$\begin{aligned}
\pi^{<<<i+1>>>}B_{i+1} + \pi^{<<<i>>>}A_i &= \mathbf{0}, && i = 0,1,\ldots,Q-1, \\
\pi^{<<<i+1>>>}B_{i+1} + \pi^{<<<i>>>}A_i + \pi^{<<<i-Q>>>}C_0 &= \mathbf{0}, && i = Q, \\
\pi^{<<<i+1>>>}B_{i+1} + \pi^{<<<i>>>}A_i + \pi^{<<<i-Q>>>}C_1 &= \mathbf{0}, && i = Q+1,\ldots,S-1, \\
\pi^{<<<i>>>}A_i + \pi^{<<<i-Q>>>}C_1 &= \mathbf{0}, && i = S.
\end{aligned} \tag{4.2}$$

The equations (except (4.2)) can be recursively solved to get

$$\pi^{<<<i>>>} = \pi^{<<<Q>>>}\Omega_i, \quad i = 0,1,\ldots,S,$$

where

$$\Omega_i = \begin{cases}
(-1)^{Q-i} B_Q A_{Q-1}^{-1} B_{Q-1} \cdots B_{i+1} A_i^{-1}, & i = 0,1,\ldots,Q-1, \\
I, & i = Q, \\
(-1)^{2Q-i+1} \sum_{j=0}^{S-i} \left[\left(B_Q A_{Q-1}^{-1} B_{Q-1} \cdots B_{s+1-j} A_{s-j}^{-1} \right) C_1 A_{S-j}^{-1} \right. & \\
\quad \left. \times \left(B_{S-j} A_{S-j-1}^{-1} B_{S-j-1} \cdots B_{i+1} A_i^{-1} \right) \right], & i = Q+1,\ldots,S,
\end{cases}$$

and $\pi^{<<<Q>>>}$ can be obtained by solving

$$\pi^{<<<Q>>>}\Bigg[(-1)^{Q}\sum_{j=0}^{s-1}\Big[\Big(B_QA_{Q-1}^{-1}B_{Q-1}\cdots B_{s+1-j}A_{s-j}^{-1}\Big)C_1A_{S-j}^{-1} \times\Big(B_{S-j}A_{S-j-1}^{-1}B_{S-j-1}\cdots B_{Q+2}A_{Q+1}^{-1}\Big)\Big]B_{Q+1}+A_Q +(-1)^{Q}B_QA_{Q-1}^{-1}B_{Q-1}\cdots B_1A_0^{-1}C_0\Bigg]=\mathbf{0},$$

and

$$\pi^{<<<Q>>>}\Bigg[\sum_{i=0}^{Q-1}\Big((-1)^{Q-i}B_QA_{Q-1}^{-1}B_{Q-1}\cdots B_{i+1}A_i^{-1}\Big)+I + \sum_{i=Q+1}^{S}\Bigg((-1)^{2Q-i+1}\sum_{j=0}^{S-i}\Big[\Big(B_QA_{Q-1}^{-1}B_{Q-1}\cdots B_{s+1-j}A_{s-j}^{-1}\Big)C_1A_{S-j}^{-1} \times\Big(B_{S-j}A_{S-j-1}^{-1}B_{S-j-1}\cdots B_{i+1}A_i^{-1}\Big)\Big]\Bigg)\Bigg]\mathbf{e}=1.$$

5 System performance measures

In this section , we derive some system performance measures in the steady-state case.

5.1 Expected inventory level

Let ζ_i denote the expected inventory level in the steady-state. Then ζ_i is given by

$$\zeta_i=\sum_{i=1}^{S} i\pi^{<<<i>>>}\mathbf{e}$$

5.2 Expected reorder rate

Let ζ_r denote the expected reorder level in the steady-state. Then ζ_r is given by

$$\zeta_r=\sum_{k=0}^{N}\left[[(s+1)\gamma+(N-k)\alpha+k\theta]\,\pi^{<s+1,k,1>}\mathbf{e}+\sum_{i=0}^{s}\beta\pi^{<i,k,0>}\mathbf{e}\right]$$

5.3 Expected perishable rate

Let ζ_p denote the expected perishable rate in the steady-state. Then ζ_p is given by

$$\zeta_i = \sum_{i=1}^{S} i\gamma\pi^{<<<i>>>}\mathbf{e}$$

5.4 Expected number of demands in the orbit

Let ζ_o denote the expected number of demands in the orbit in the steady-state. Then ζ_o is given by

$$\zeta_o = \sum_{k=1}^{N}\sum_{i=0}^{S} k\pi^{<<i,k>>}\mathbf{e}$$

5.5 The blocking probability

Let ζ_B denote the probability that the demands is blocked and ζ_B is given by

$$\zeta_B = \sum_{i=1}^{Q}\sum_{k=0}^{N-1} \pi^{<i,k,0>}\mathbf{e} + \pi^{<<<0>>>}\mathbf{e}$$

5.6 The overall rate of retrial

The overall rate of trials at which the orbiting demands request his demand is given by

$$\zeta_{SR} = \sum_{i=1}^{S}\sum_{k=1}^{N} k\theta\pi^{<<i,k>>}\mathbf{e}$$

5.7 The successful rate of retrial

The rate at which the orbiting demands successfully receive his demands is given by

$$\zeta_{SR} = \sum_{i=1}^{S}\sum_{k=1}^{N} k\theta\pi^{<i,k,1>}\mathbf{e}$$

5.8 The fraction of time the server is on vacation

The fraction of time the server is on vacation is given by

$$\zeta_{SV} = \sum_{k=0}^{N}\left[\sum_{i=1}^{Q} \pi^{<i,k,0>}\mathbf{e} + \pi^{(0,k,0,0)}\right]$$

5.9 Fraction of successful rate of retrials

The fraction of successful rate of retrials is given by

$$\zeta_{FR} = \frac{\zeta_{SR}}{\zeta_{OR}}$$

6 Cost analysis

The long-run expected cost rate for this model is defined to be

$$TC(S, s) = c_h\zeta_i + c_s\zeta_r + c_p\zeta_p + c_w\zeta_o.$$

where,

c_s : Setup cost per order
c_h : The inventory carrying cost per unit item per unit time
c_w : Waiting cost of a demand in the orbit per unit time
c_p : The cost per unit failure

Due to the complex form of the limiting distribution, it is difficult to discuss the properties of the cost function analytically. Hence, a detailed computational study of the cost function is carried out.

6.1 Numerical Examples

We have studied the effect of varying the cost and the other system parameters on the optimal values and the results agreed with what one would expect.

Example 1. *In this example, we study the impact of the setup cost c_s, holding cost c_h, perishable cost c_p and the waiting cost c_w on the the total expected cost rate $TC(s, S)$. Towards this end, we first fix the parameter values as $\alpha = 7,\ \beta = 0.2,\ \gamma = 0.5,\ \theta = 4$ and $\mu = 5$. We observe the following from Table 1.*

- *The optimal cost increases when c_h, c_s, c_w and c_p increase.*

Example 2. *Here, we study the impact of the primary demand rate α, the lead time rate μ, the retrial demand rate θ and the vacation time β on the total expected cost rate $TC(s, S)$. We first fix the cost values as $c_h = 0.02,\ c_s = 25,\ c_p = 2$ and $c_w = 8$ We observe the following from Figure 1 to 4.*

- *The optimal expected cost rate increases when α increases.*
- *As is to be expected, μ increases the total expected cost rate decreases.*
- *The total expected cost rate increases when γ increases and decreases when β and θ increases.*

c_h	c_s	c_p	c_w			
			3	6	9	12
0.1	5	2	78.570140	150.559809	222.549478	294.539147
		4	83.999053	155.988721	227.978390	299.968059
		6	89.427965	161.417634	233.407303	305.396972
	10	2	79.333920	151.323589	223.313258	295.302926
		4	84.762832	156.752501	228.742170	300.731839
		6	90.191745	162.181413	234.171082	306.160751
	15	2	80.097699	152.087368	224.077037	296.066706
		4	85.526612	157.516281	229.505950	301.495618
		6	90.955524	162.945193	234.934862	306.924531
0.2	5	2	78.957920	150.947589	222.937257	294.926926
		4	84.386832	156.376501	228.366170	300.355839
		6	89.815744	161.805413	233.795082	305.784751
	10	2	79.721699	151.711368	223.701037	295.690706
		4	85.150612	157.140281	229.129949	301.119618
		6	90.579524	162.569193	234.558862	306.548531
	15	2	80.485479	152.475148	224.464817	296.454486
		4	85.914391	157.904060	229.893729	301.883398
		6	91.343304	163.332972	235.322641	307.312310
0.3	5	2	79.345699	151.335368	223.325037	295.314706
		4	84.774611	156.764280	228.753949	300.743618
		6	90.203524	162.193193	234.182862	306.172530
	10	2	80.109479	152.099148	224.088817	296.078485
		4	85.538391	157.528060	229.517729	301.507398
		6	90.967303	162.956972	234.946641	306.936310
	15	2	80.873258	152.862927	224.852596	296.842265
		4	86.302171	158.291840	230.281508	302.271177
		6	91.731083	163.720752	235.710421	307.700090

Table 2: Effect of cost parameters on total expected cost rate

References

[1] Artalejo, J. R., (1998), *Retrial queues with a finite number of sources*, Journal of the Korean Mathematical Society, **35**, 503 - 525.

[2] Artalejo, J. R., Krishnamoorthy, A. and Lopez-Herrero, M. J., (2006), *Numerical analysis of* (s, S) *inventory systems with repeated attempts*, Annals of Operations Research, **141,** 67 - 83.

[3] Daniel, J. K. and Ramanarayanan, R., (1987), *An inventory system with two servers and rest periods*, Cahiers du C.E.R.O,Universite Libre De Bruxelles, **29,** 95 - 100.

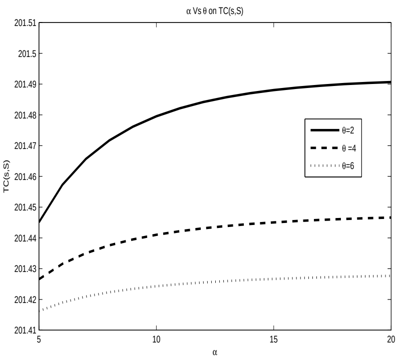

Figure 1: α vs θ variation on $TC(s, S)$

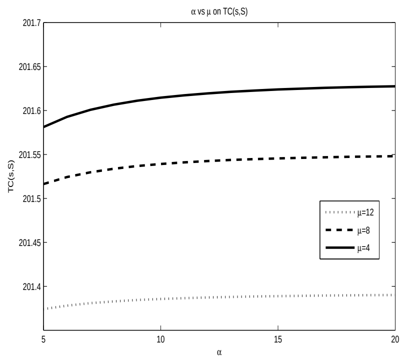

Figure 2: α vs μ variation on $TC(s, S)$

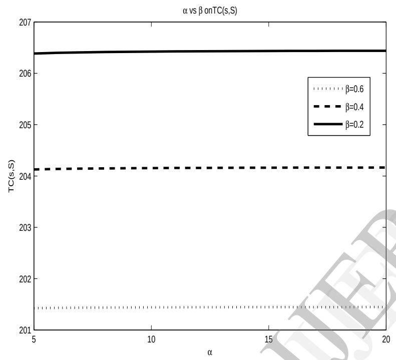

Figure 3: α vs β variation on $TC(s, S)$

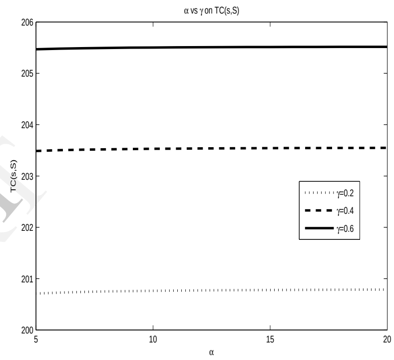

Figure 4: α vs γ variation on $TC(s, S)$

[4] Daniel, J. K. and Ramanarayanan, R., (1988), *An (s, S) inventory system with rest periods to the server*, Naval Research Logistics, John Wiley & Sons **35**, 119 - 123.

[5] Goyal, S. K. and Giri, B. C., (2001), *Recent trends in modeling of deteriorating inventory*, European Journal of Operational Research, **34(1),** 1-16.

[6] Krishnamoorthy, A. and Jose, K. P., (2007), *Comparison of inventory systems with service, positive lead-lime, loss, and retrial of customers*, Journal of Applied Mathematics and Stochastic Analysis, **Article ID 37848,** 1-23.

[7] Nahmias, S.,(1982), *Perishable inventory theory : A review,* Operations Research, **30,** 680 - 708.

[8] Narayanan, V.C., Deepak, T. G., Krishnamoorthy, A., and Krishnakumar, B., (2008), *On an (s, S) inventory policy with service time, vacation to server and correlated lead time*, Qualitative Technology & Quantitative Management, **5(2),** 129 - 143.

[9] Neuts, M. F., (1994), *Matrix-geometric solutions in stochastic models : an algorithmic approach,* Dover Publication Inc. New York.

[10] Raafat, F., (1991), *A Survey of Literature on Continuously Deteriorating Inventory Models*, Journal of Operational Research Society, **42,** 27- 37.

[11] Sivakumar, B., (2009), *A perishable inventory system with retrial demands and a finite population*, Journal of Computational and Applied Mathematics, **224**, 29 - 38.

[12] Sivakumar, B., (2009), *An inventory system with retrial demands and Multiple server vacation*, Quality Technology & Quantitative Management, **224**, 29 - 38.

[13] Tian, N. and Zhang, Z. G., (2006) *Vacation Queueing Models - Theory and Applications*, Springer Science.

[14] Ushakumari, P. V., (2006) *On (s, S) inventory system with random lead time and repeated demands*, Journal of Applied Mathematics and Stochastic Analysis, **2006**, 1 - 22.