

# A Dynamic Programming Approach for Preventive Maintenance of Tea Sieving Machine Components

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**Abstract** - Indonesia is rich in tea plantations. The management of tea leaves is carried out by state-owned company, that produces orthodox black tea in Indonesia. Currently, the company often experiences problems due to frequent breakdowns of the tea sieving machine, production machines used in the tea industry, especially for processing orthodox tea in the withering and grinding rooms. Machine, particularly in the lower arm component. Sudden damage to the lower arm component reduces production, resulting in production targets not being met. The company needs a maintenance schedule for the lower arm component along with the actions to be taken during periodic inspections. Dynamic programming with three decisions, that continuing to use the previous lower arm component, welding or replacing it, along with an inspection stage every two weeks with a total cost expectation minimization objective function. Based on the calculations performed, the total maintenance cost using dynamic programming is lower than the company's current total maintenance cost.

**Keywords** - dynamic programming, preventive maintenance schedule, minimization of total maintenance costs

## I. INTRODUCTION

The tea production process is divided into three production rooms, namely the withering and grinding room, the drying room, and the sorting and packing room. The production process involves operators and machines. Problems that often arise in the process of using machines are machine breakdowns, which can hinder production when the machines are not in use. The machines that experience the most breakdowns and cause downtime are located in the grinding room.

The machine with the highest damage rate in the milling room is the tea sieving machine. The lower arm component of this machine often breaks due to high and prolonged use, namely 23-30 tons with 15-24 hours per day depending on the daily tea supply. Company conducts routine inspections every two weeks, where the company waits for the lower arm component to become damaged to the point where it can no longer be used before welding the damage or replacing the lower arm component with a new one.

In view of this, to reduce downtime and prevent damage to the lower arm of the machine, the company needs to implement proper scheduling so that it can determine the optimal

preventive maintenance measures for vital components. The appropriate method for creating a maintenance schedule for the machine is dynamic programming, where the characteristic of dynamic programming is to divide the problem into several stages and at each stage only one optimal decision is taken. By using dynamic programming, it is hoped that damage to the machine can be prevented by determining a preventive maintenance schedule for critical components with the criterion of minimizing the total expected cost.

## II. THEORETICAL REVIEW

### A. Maintenance

Maintenance is an activity required to maintain or preserve the quality of a facility so that it can function properly in a ready-to-use condition. The main objectives of maintenance are to ensure equipment reliability and availability, improve safety, and minimize maintenance and operational costs [6]

### B. Preventive Maintenance

Preventive maintenance is a type of maintenance performed periodically to prevent damage to a system (equipment or machinery) during operation.[6] This periodic maintenance includes several maintenance actions, such as:

- 1) Inspection
- 2) Repair
- 3) Replacement
- 4) Cleaning, lubrication, and adjustment

### C. Corrective Maintenance

Corrective maintenance is maintenance activity performed after a machine or production facility experiences a malfunction or damage that prevents it from functioning properly. Corrective maintenance is not a scheduled maintenance activity because it is performed after a component has been damaged.

### D. Dynamic Programming

Dynamic programming is a mathematical technique used to make a series of interrelated decisions. Some keywords to remember in dynamic programming are stage, state, and

policy. Dynamic Programming is an optimization methodology used to solve complex problems by decomposing them into smaller, simpler subproblems. The technique is applicable when the problem exhibits overlapping subproblems and optimal substructure, meaning that an optimal solution to the overall problem can be constructed from optimal solutions of its subproblems.[2]

Dynamic programming was first introduced by **Richard Bellman** in the 1950s as a mathematical framework for multistage decision-making processes.[1]

A dynamic programming problem can be mathematically represented as [2]:

- State variable:  $u_t$
- Decision variable:  $x_t$
- Transition function:  

$$x_{t+1} = f(x_t, u_t) \quad (1)$$

- **Objective function:**

$$\min \sum_{t=1}^T g(x_t, u_t) \quad (2)$$

The optimal value function is defined recursively using the **Bellman equation**:

$$V_t(x_t) = \min_{u_t} [g(x_t, u_t) + V_{t+1}(f(x_t, u_t))] \quad (3)$$

### III. RESEARCH METHOD

The step undertaken in this research are as determining data collection and processing data.

#### A. This research requires several types of data, including:

- 1) Failure occurrence data,
- 2) Corrective replacement time,
- 3) Preventive replacement time,
- 4) Preventive maintenance cost,
- 5) Corrective maintenance cost.

#### B. Determination of Failure Rate Distribution (Goodness-of-Fit Test)

This step aims to identify the most appropriate probability distribution that represents the failure behavior of the system based on historical failure data.

#### C. Estimation of Parameters of the Selected Failure Distribution

The parameters of the selected failure distribution are estimated using appropriate statistical estimation methods.

#### D. Evaluation of the Failure Distribution Function

The probability density function (PDF), cumulative distribution function (CDF), and reliability function are calculated based on the selected distribution.

#### E. Dynamic Programming Formulation Modeling

A dynamic programming model is formulated by defining system states, decision alternatives, transition probabilities, and cost functions.

#### F. Dynamic Programming Model Computation

The optimal preventive maintenance policy is obtained by solving the dynamic programming formulation to minimize the expected total maintenance cost.

## IV. DISCUSSION

This stage involves calculations performed to obtain the optimal solution based on the available data.

#### A. Distribution Testing and Determination of Distribution Parameters

The results of the distribution test for the right and left lower arm components can be seen in Table I and Table II.

TABLE I. DISTRIBUTION OF RIGHT LOWER ARM COMPONENTS

Right Lower Arm		
Distribution	Index of Fit	Selected Distribution
Normal	0.9385	Weibull
Lognormal	0.9787	
Exponential	0.7868	
Weibull	0.9818	

Table I shows that the damage data for the right *lower arm* component is distributed according to the Weibull distribution. Table II shows that the left *lower arm* component is also distributed according to the Weibull distribution:

TABLE II. DISTRIBUTION OF LEFT LOWER ARM COMPONENTS

Left Lower Arm		
Distribution	Index of Fit	Selected Distribution
Normal	0.8399	Weibull
Lognormal	0.9378	
Exponential	0.8944	
Weibull	0.9521	

After testing the distribution of the right lower arm component and the left lower arm component, the parameters of the selected distribution were determined. The parameters of the right lower arm and left lower arm distributions are shown in Table III.

TABLE III. DISTRIBUTION PARAMETERS OF THE RIGHT LOWER ARM

Notation	Description	
	Right Lower Arm	Left Lower Arm
$b$	0.9104965	1.1256586
$a$	3.4191321	3.2864777
$\alpha$	30.542895	26.748482
$\beta$	1.0983019	0.8883688

#### B. Dynamic Programming Formulation Modeling

This research was conducted using dynamic programming to determine the optimal maintenance schedule with minimum cost. [7]

##### 1) Notation Research

The following are the notations used in this study.

$T$  : inspection cycle length

$N$  : number of inspection points per cycle

$s$  : time interval between inspections

$j$  : starting point of the inspection interval  $j = 0, 1, 2, \dots, N$

$t$  : age of the lower arm component at a given  $j$

$x_j$  : action selected at a given  $j$ ;

$x_j$  : Do Nothing, Welding, Replace

$\varepsilon$  : improvement factor

Do nothing : No action is taken on the lower arm component at point  $j$

Welding : Perform welding on the damaged lower arm at point  $j$

Replace : Replace the damaged lower arm at point  $j$

$c_1$  : Cost of the lower arm component is no action at  $j$

$c_2$  : Cost of welding the damaged lower arm at  $j$

$c_3$  : Cost of replacing the damaged lower arm at  $j$

$c_4$  : inspection cost

$c_5$  : downtime cost

##### 2) Objective Function, Stages, State, and Decision Variables Used

The following are the references used in this research using dynamic programming:

###### a) Objective Function

The objective function is to determine the optimal preventive maintenance schedule by minimizing the total maintenance cost of the lower arm components on the machine.

###### b) Stage ( $j$ )

The stage or decision point is taken during routine inspections by the company, which are conducted every 2 weeks or 14 days with a planning period of  $T = 70$  days, where the length of the planning period is taken from the average age of the components when new. Each decision made at a stage will affect the decisions at the next stage. In this study, there are  $N=5$  stages. At each inspection point ( $j$ ), there are decision options to consider  $x_j$  namely replacing the component with a new one, welding the damaged lower arm component, or not replacing or welding the component.

##### c) State

In this case, the decision selection ( $x_j$ ) is determined based on the total costs incurred, and these costs are also influenced by the service life of the components. At stage  $j+1$ , the decision  $x_j$  taken determines the machine's lifespan at that stage. The service life of the component and the actual condition of the component in the field will be the basis for selecting a decision at stage  $j$ . The decision is made when  $j \geq 1$  because at  $j=1$  the component has not yet been used, so  $t=0$ . The component to be replaced or welded is the lower arm component of the machine.

##### d) Decision Variables ( $x_j$ )

Decision variables describe the decisions that can be made for preventive maintenance of components. There are three alternative decisions that can be made, namely:

- Do Nothing

If the decision at  $j$  is  $x_j = \text{Do Nothing}$ , then the component life at  $j+1$  becomes  $(t+s)$ . The decision to Do Nothing the lower arm component unchanged does not alter the performance of the component used, or no action is taken on the lower arm component. If the decision chosen at stage  $j$  is to Do Nothing, the component life at stage  $j+1$  increases by 14.

- Welding

If the decision at  $j$  is  $x_j = \text{Welding}$ , then the component life at  $j+1$  becomes  $[t + (s, \varepsilon)]$ . The welding decision aims to reduce the probability of damage to the lower arm, in this case by welding the lower arm. If the decision selected at  $j$  is welding, then the life of the lower arm component at stage  $j+1$  increases by 7, because there is an improvement factor (lower arm life renewal) of 0.5.

- Replace

If the decision at  $j$  is  $x_j = \text{Replace}$  ( $t=0$ ), then the component life at  $j+1$  becomes  $(t=s)$ . The replacement decision is the action of replacing the damaged lower arm component. If the decision selected at  $j$  is replace, then the chisel life at stage  $j+1$  becomes 0 again.

##### C. Dynamic Programming Formulation Calculation

The calculation of the dynamic programming formulation is performed by input data into the model that has been created. After performing the calculation, the result that appears is the machine maintenance action schedule with minimum cost. Using the decision cost and the relationship between age  $t$ , decision  $x_j$  and the contribution of  $x_j$  to the total cost at stage  $j$  can be shown in Table IV.

TABLE IV. THE DECISION COST AND THE RELATIONSHIP BETWEEN AGE  $t$ , DECISION  $x_j$

Age $t$	Decision $x_j, 1 \leq j < N$			Minimize cost with decision $x_j$	Optimal Decision $x_j$
	Do Nothing	Welding	Replace		
	Cost	Cost	Cost		
$0 < t \leq j_s$	$c_1$	$c_2$	$c_3$	$C_j^*(t)$	$x_j$

Total cost is the decision cost from stage  $j$  to  $N$ .

Using Table IV, the value of the optimal cost function can be expressed in the following recursive equation:

$$C_j^*(t) = \min \begin{cases} C_{x_j} = \sum_{j+1}^{j=1,2..N} \text{Do Nothing} \\ C_{x_j} = \sum_{j+1}^{j=1,2..N} \text{Welding} \\ C_{x_j} = \sum_{j+1}^{j=1,2..N} \text{Replace} \end{cases} \quad (4)$$

The following maintenance action schedule can be seen in Table V.

TABLE V. RESULTS OF THE DYNAMIC PROGRAMMING MODEL

Right Lower Arm			Right Lower Arm		
Stage (j)	Decision	Age (t)	Stage (j)	Decision	Age (t)
1	Do Nothing	14	1	Do Nothing	14
2	Do Nothing	28	2	Do Nothing	28
3	Welding	35	3	Welding	35
4	Welding	42	4	Welding	42
5	Welding	49	5	Welding	49
Total Cost	IDR 4.555.720	Total Cost			IDR 4.711.391

## V. CONCLUSION

Based on the results of the analysis and data processing, the following conclusions can be drawn:

1. Based on the results of calculations using a dynamic programming model for the left and right lower arm components, there are two possible conditions: if at the inspection point the component is still in good condition, the decision is to Do Nothing it; if during the inspection interval the component is damaged, there are two possible decisions: weld or replace.
2. Action decisions are made at each inspection point, which occurs every 14 working days during a 70-day cycle,

resulting in 5 inspection stages. Action decisions include Do Nothing, Replace, and Weld.

3. During the 70-day cycle, the total expected maintenance cost is IDR 4,555,720 for the right lower arm and IDR 4,711,391 for the left lower arm. The cost for the right lower arm is lower than that for the left lower arm because there is a component replacement action that extends the component's lifespan, so no further action is required at the next inspection stage.
4. Based on the total cost data from this study and the company's current condition, it can be seen that the total maintenance cost in this study is lower than the company's current total maintenance cost.

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