A Decision Model for Production Inventory Management in a Manufacturing Firm

Nnadi, Daniel C  
Department of Mechanical Engineering, Micheal Okpara University Of Agriculture, Umudike, P.M.B 7267, Abia State, Nigeria

Aniekan Offiong  
Department of Mechanical Engineering, University of Uyo, Uyo, P.M.B. 1017, Akwa Ibom State, Nigeria

Abstract--This paper presents a working inventory model to incorporate the partial back ordering inventory with references to the back ordering inventory model. The re-order point models of maximum expected cost and service level approach were established. Data, covering the demand, set-up cost, and the holding cost were obtained from the Champion Breweries Plc, other related data were estimated. These enabled the application of the models to obtain, the review period, (T) the fill rate (F), the stock-out (S) and finally the order quantity (Q). It was found and could be recommended that quantity (Q) of 285022 tons of plan sorghum be placed bi-ennially with a lead time of four (4) weeks. At a time the stock level is expected to be at 58,746 ton.

Keywords - Inventory, Inventory Level, Fill rate (F), Stock-out (S), Holding Cost, Demand, Period, Re-order quantity, Lead Time, Base stock level.

I. INTRODUCTION

Various definitions have been given by different authors to inventory management. [1] described inventory as a buffer between supply and demand. [2] defined it as stock of goods or material awaiting delivery or dispatch. While [3] described it as a stock of goods or item held for future use. From the foregoing one can see that good inventory management in a firm would lead to greater profits, minimized losses, greater customer satisfaction, stabilized employment, improve product quality and other latent benefits of inventory. Failure to meet demand in any company usually compromises customer satisfaction and attracts high cost that characterizes emergency production. Efficient management of inventory system is therefore very critical in the operations of the firm. [1] outline the basic benefits of inventory management to the customer as off-the-shelf availability of products while to the management as reduced tied-up investment capital on inventory, reduced operating cost and carrying associated with warehousing and reduction in the obsolescence of product. In this paper our emphasis is on the backordering. As [4] observes, there may be some economic reasons for a company to decide not to satisfy all demand, but rather lose some sales in the interest of the company. We consider a situation where rather than accumulating a lot of buffers stocks and attracting spoilage, some stocks can be back-ordered, some lost and sufficient customer and company satisfaction achieved. [2] modeled the system using the length of stock-out period and the length of the inventory review period. They also tried to deal with the problem of “reorder point” by developing a closed form model parameters. In this paper a modification of the model by Jonah and Chukwu would be attempted, and an application of same to the Brewery industry would be explored.

Consider a firm that operates a random placement of orders on raw materials, upholds an infinite production process and delivers to customers when the customers are available. If supplies are made to a clone system of customers the firm may run into a problem of over-production and its attendant consequences. According to [5] the problem of most companies is that of inadequate planning and control of production activities. Many companies lack the technical know-how while others ignore the practice of inventory control entirely. Another version of problem associated with inventory is the non-placement of order expect there is requisition from customers. Akin to this are delays in supplies to customers and the likely losses of customers and sometimes permanently. The objective of this paper is to establish a good inventory management policy that would bridge the gap between overstocking and under-stocking and also enhance quick delivery of orders. Also, this paper will capture prompt delivery to customers, to reduce buffer stocks to a known quality such that even during uncertainties losses are minimized, and to smooth demand even it is erratic. This paper would be important and applicable to companies that operate on either of two classes of inventory, the raw material inventory and finished product inventory. It is therefore expected that good inventory control and in fact the application of the model would go a long way to harness the wealth of the company of application. In this paper a model which can be used to determine the total cost of inventory with backordering the quality and the re-order point will be developed. The model will be tested with a given company and recommendations would be made following the results obtained.

The terminologies of the inventory analysis are as follows:

1. **Demand:** This is the sum total of customers requirements for a given time, usually per year.
2. **Lead Time:** This is the time limit between the placement of an order and the subsequent arrival of same to fill the inventory.
3. **Base Stock Level:** This is the maximum level that the replenishment should bring the stock level to.
4. **Inventory Level:** This is instant level of on hand inventory.
5. **Backorder:** This is a given quantity of customer demand during stock-out of which he is prepared to wait and receive after replenishment.

6. **Continuous Review:** This is an inventory replenishment policy whereby replenishment is done at any point.

7. **Economic Order Quantity (EOQ):** The optimal replenishment level that would best minimize the holding cost and order cost of on-hand inventory.

8. **Fill Rate:** This is the fraction of the demand that is filled from on-hand inventory.

9. **Holding Cost:** This is the cost per unit of holding stock inventory and comprises of rents, insurance and opportunity cost of tied up capital.

10. **Inventory Level:** This is the on-hand inventory less backorders.

11. **Lost Sales:** These are sales that are lost due to non-availability of stock. If the waiting time for delivery of an order is too long.

12. **Obsolescence:** Where stock is no more useful for its intended purpose, by way of expiration damage, contamination or shift of market.

13. **On-hand Inventory:** The instant available stock in inventory.

14. **Lower Bound:** This is the least value that partial backordering rate assumes for the partial backordering model to be applicable.

15. **Yield Variance:** This is the variance of the yield distribution. The quantity that makes the quantity ordered to differ from the quantity received.

16. **Re-order Quantity:** The number of items to be ordered during replenishment.

17. **Stock:** The items in the warehouse to support operations.

18. **Service Level:** This demonstrates the fraction or percentage of order cycle during the year in which there are no stock-outs.

19. **Stationary Demand:** Demand having a single probability distribution that does not change order time.

20. **Set-up Cost:** These are the costs of labour materials and marginal costs of machines or work station set-up.

**II. LITERATURE REVIEW**

Before reviewing literature directly related to our focal point, we will relate some fundamental literature so as to reveal some other works by other researchers, such as the theory of EOQ (economic order quantity), ELS (economic lot size), the re-order point and EOQ will price break. The foremost theory in inventory management is what is known as Economic Order Quantity (EOQ) where independent demand was used to established cost minimization. This theory was developed in 1913, [6]. As shown in figure 2.1, the dynamic minimum cost is at the point where the cost curve is minimum. The EOQ is given by:

$$Q = \frac{\sqrt{2KD}}{h}$$

Where,

- \(K\) = Ordering cost
- \(D\) = Average demand
- \(h\) = Holding cost (percent)
- \(Q\) = Quantity ordered

$$\text{EOQ} = \frac{\sqrt{2DS}}{H}$$

Where,

- \(D\) = Annual Demand
- \(S\) = Ordering Cost
- \(H\) = Holding Cost

EOQ = Economic order quantity [7].

The average order interval per year which also gives the optimal re-order point is given by average order interval:

$$R = \frac{\text{EOQ}}{D}$$

Where,

- \(R\) = Reorder point
- \(D\) = Quantity demand

[8] extended the EOQ model to what he called EOQ with price breaks which is like EOQ model except that the item may be purchased at a discount if the size of the order \(y\) exceeds a given limit, \(q\) that is the unit purchasing price \(C\) is given by:

$$Q = \begin{cases} c_1, & \text{if } y \leq q \\ c_2, & \text{if } y > q \end{cases}, \quad c_1 > c_2$$

Fig. 1. shows the graphical representation of EOQ

![Graphical representation of EOQ](image-url)
[9] presented a system dynamic modeling of Nigerian automotive battery production organization, where policy runs of the system simulation were done to evaluate the impact of the size and pattern of demand. [10] developed a model for progressive inventory management for job shops where the developed model incorporates salient characteristics like uncertainty of demand, limitations of space and funds and multiple materials. [11] presented a paper which considers an economic lot production quantity problem for an unreliable manufacturing system which the machine is subjected to random failure. They established a model whereby shortages can be managed by accepting the existence of an on-hand inventory. [12] established an EQQ model for deteriorating items where supplier offers a permissible delay in payment. Their model allows not only the partial backlogging rate to be related to the waiting time but also the unit selling price to be larger than the unit purchase cost. [13] defined a time dependent partial backlogging rate and introduced an opportunity cost due to lost sales. He established that the larger the waiting time for the next replenishment the smaller the backlogging rate would be. [14] addressed the economic lot scheduling problem where a manufacturer makes a variety of products types on a single facility or assembly line. The model accounts for the time to set up the facility and charges a penalty on each unit short regardless of whether shortages actually result in lost sales or not. [15] presented the willingness of customers to backorder as a function of customer incentive that accompany the backorder as a function of customer incentive that accompany the backorder. They concluded that under some technical assumption other competitor’s optimal inventory policies are monotone in the amount of incentive offered. [8] divided inventory into two parts, the deterministic and the probabilistic inventories, showing that the deterministic is a case where the demand is known and certain, while the probabilistic is a case of unknown previous demand. [16] illustrated a proposition which goes beyond production based on his experience at Toyota Company. The main concept deals with linking quality cost and delivery research activities of all departments concerned with development production and sales. [17] established the optimal quality that should be ordered in a situation of combined effect of deterioration and inflation. [18] studied a model having a constant rate of deterioration and a constant rate of demand over a finite planning state. More work on this area was done by [19] who added a rate of deterioration and [19] who considered the model allowing complete backlogging of the unsatisfied demand. [21] studied models allowing partial backlogging of unsatisfied demand and deterioration of inventory. [22] who considered where unit selling price is affected by demand, as low selling prices generated demand, while high selling prices decline demands. [23] established that the demand rate is “described by any convex decreasing function of selling price and instead of constant rate of partial backlogging considered a variable backlogging rate which is found in the work of [24].

III. Methodology

This paper would be carried out through the following approach:

i. The model by [2] will be slightly modified. The research of Jonah and Chukwu is basically a theoretical analysis of a general situation, bringing both total backordering, total lost sales and partial backordering cases. In their analysis they made use of figures not obtained from real situation and also assumed some basic factors and variance which they used in their theoretical analysis. However this situation will endeavor to break down the case of partial back ordering by analyzing the development of the model and then employing same in the estimation of the quantity and reorder point make use of a real situation.

ii. Data were collected from champion brewery plc and they are:
   a) Demand rate
   b) Set-up cost
   c) Variable Costs
   d) Carrying or Holding cost
   e) Shortage costs
   f) Backordering costs
   g) Profits

iii. The lower bound for the partial backordering rate will be determined

iv. The length of inventory period (T), the fill rate (F) and consequently the order quantity (Q) and shortages (S) will be determined

v. The re-order point will be established by applying the result above and point equation

A. Model Development

This model is developed on the assumption that in a company that intends to operate on a zero buffer stock, the expectation should be that not all the customers that arrive during stock-out period would be willing to wait for the arrival of new stock. Other assumptions made includes: the arrival of orders should be able to meet all backorders and bring the on hand inventory above the re-order points, the carrying cost of the inventory is applicable to only the units of acceptable quantity, the paper is made on a single item inventory, demand is treated as deterministic, customer demand is considered linear, and lead time is constant. Fig. 2. shows graph of partial back-ordering model showing the random yield curve.
Fig. 2. Graph of partial back-ordering model showing the random yield curve

The shortage period is further divided into two parts, BS and (1 - B)S. \( \mu Q_1 - BS \) is the function for the quantity while stock lasts which when divided by the function for total quantity \( \mu Q_1 + (1 - B) S \) we obtain the fill rate. \( \frac{\mu Q_1 - BS}{\mu Q_1 + (1 - B) S} \).

A known fraction B of the demand during the lead time is backlogged while the remaining (1 - B) is lost. Since S is the maximum stock-out BS is denoting the total amount backlogged while (1 - B)S is totally lost. Considering first the length of the inventory review period can be presented mathematically as:

\[
T = \frac{Q_2 - BS}{D} + \frac{S}{D} = \frac{(Q_2 + (1 - B)S)}{D} \tag{5}
\]

Considering that due to shortages which generate the random yield problem, not all \( Q_1 \) will result to \( Q_2 \) therefore consider the explanation of the review period. The expected length of the inventory is given as:

\[
E \left( \frac{T}{Q} \right) = \int_0^\infty TDQ_2 = \int_0^\infty \frac{Q_2 + (1 - B)S}{D} = \frac{\mu Q_1 + (1 - B)S}{D} \tag{6}
\]

This is the expected length of the inventory review period. Looking at the total cost \( TC \) as a function of the individual cost:

\[
= \int_0^\infty \left( \frac{C_2 + C_1v[(Q_2 - BS)^2]}{2D} + C_3S + P(1 - B)S + \frac{C_0BS^2}{2D} \right) \tag{7}
\]

Resolving this (integrating the total cost function) gives:

\[
TC = C_2 + \frac{C_1}{2D}(\delta^2 + \mu^2Q_1^2) - 2BS(\mu Q_1) + (BS)^2 + C_3S + P(1 - B)S + \frac{C_0BS^2}{2D} \tag{8}
\]

To determine the expected cost per unit time:

\[
\frac{1}{T} = \frac{\mu Q_1 + (1 - B)S}{D}
\]

we obtain the expected cost per unit time:

\[
TC(Q_1S) = \frac{C_2D}{\mu Q_1 + (1 - B)S} + \frac{C_1V\sigma^2}{2[\mu Q_1 + (1 - B)S]C_0SD} + \frac{\mu Q_1 + (1 - B)S}{P(1 - B)SD} + \frac{C_0BS^2}{2[\mu Q_1 + (1 - B)S]} \tag{9}
\]

To transform the above equation into the expected length of the review period and fill rate, \( F \) is

\[
Represent = \frac{\mu Q_1 - BS}{\mu Q_1 + (1 - B)S} \quad \text{and} \quad T = \frac{\mu Q_1 + (1 - B)S}{D}
\]

We obtain the expected total cost

\[
TC(T, F) = \frac{C_2C_1V}{T} \left( \frac{\sigma^2}{2DT} + \frac{D^2TF^2}{2} \right) + \frac{C_3BDT(1 - F)^2}{2} + \frac{(P + C_3D)(1 - B)(1 - F)}{2} \tag{10}
\]

Differentiating equation 3.6 partially with respect to \( T \) and \( F \) and equating each to Zero obtain the minimum cost and fill rate

\[
\frac{\partial TC}{\partial T} = -\frac{C_2}{T^2} - \frac{C_1V\sigma^2}{2DT^2} + \frac{C_1VD^2}{2} + \frac{C_3BD(1 - F)^2}{2} = 0
\]

and finally to obtain the optimal values of \( T \) and \( F \), the two equations are solve simultaneously

\[
T = \frac{C_1V + C_0BS}{C_0BDT} + \frac{2C_2D}{C_1V + C_0BS} \left( C_1V - D^2C_0^2 + \frac{[C_0]^2}{P} \right) \tag{12}
\]

The solution to (11) and (12) exist if and only if

\[
\beta \geq 1 + \frac{C_3}{P} \left( \frac{2C_2D + C_1V\sigma^2}{P^2D^2} \right) C_1V - D^2C_0^2 + \frac{[C_0]^2}{P} \tag{13}
\]

These theorems are presented in our notion.

To obtain the quantity to be taken backordered we multiply the review period \( T \) with demand \( D \), subtract the fraction that is totally lost \( (1 - \beta)S \) and divide through by the bias factor \( \mu \).

\[
Q = \frac{TD - (1 - \beta)S}{\mu} \tag{14}
\]

And to obtain the maximum stock out we multiply the review period \( T \) with the demand \( D \) and the conserve of the fill rate \( (1 - F) \)

\[
S = TD(1 - F) \tag{15}
\]

B. Determination of the Maximum Expected Cost

The customer service level is described as the percentage of orders filled from stock on hand which is also called the fill rate. This together with its counterpart: the stock-out rate therefore the service level is that given by [25].
\[
SS = Z \sqrt{LT \cdot (SD)^2} + SS
\]

R.O.P = D (LT) + SS

(16)

Where,
- SS = Safety stock
- Z = Value from normal distribution table
- LT = Lead time
- SD = Standard deviation of demand
- R.O.P = Re-order point

C. Determination of the Service Level

To adopt the service level approach of [26]. If a service level approach of \(x\%\) is desired in inventory decision making then:

\[
(1 - \delta) = \frac{\delta_l}{Q_1}
\]

\[
NL - \frac{r - E(L)}{Q_1} = [\mu Q_1 + (1 - B)S] \left(1 - \frac{x}{\delta_l}\right)
\]

(17)

Where,
- \(X\) = Desired service level
- NL = Normal loss function
- \(R\) = re-order point
- \(E(L)\) = Expectation of lead time demand

IV. ANALYSIS AND RESULTS

Having been able to determine the length of the inventory period (\(T\)), the fill rate (\(F\)), partial back-ordering inventory rate (\(\beta\)), and the re-order point (\(R\)). A numerical application of these theories so far obtained will be tested using data collected from the Champion Brewery Nig. Plc, Uyo, Akwa Ibom State.

A. Estimation of Review Period, Fill Rate, Quantity and Stock-out

As mentioned in the methodology we obtained data of the following on Plain Sorghum.

- Demand (D) - 300,000 tons
- Set up Cost (\(C_2\)) - N\(100,000\)/replenishment
- Variable cost (\(V\)) - N\(1000\)/ton
- Holding cost (\(C_1\)) - N\(900\)/ton
- Shortage cost (\(C_3\)) - N\(500\)/ton
- Backordering cost (\(C_B\)) - N\(300\)/ton
- Profit (\(P\)) - N\(7,000\)/ton

Seeing that these figures as large we may reduce all by a scale of \(X\ 10^2\) for easy calculation therefore we use the following figure: \(D = 3000, C_2 = 100, V = 10, C_3 = 5, P = 70, C_B = 3, C_1 = 9\).

We also assume a variance of “0” and a bias factor “\(\mu\)” of “1”

Recall (13)

\[
\beta \geq 1 + \frac{C_3}{P} \left( \frac{2C_2 D + C_1 V \sigma^2}{P \sigma D^2} \right) \left( C_1 V - D^2 C_3 + \frac{[C_B^2]}{P} \right)
\]

\[
= \frac{5 \left( 2 \times 1000 \times 300 \right) 9 \times 100 - 3000^2 \times 5^2}{70^2} + \frac{5^2}{70^2} \]

\[
\beta \geq 1 + \frac{0.9489}{0.9489}
\]

This is the lower bound for the partial back-ordering rate which we wish to establish. Then proceed to solve for the review and the fill rate using (11) and (12).

\[
T = \frac{C_1 V + C_B B}{C_B B D^2} - \frac{P(1 - B)^2}{C_1 V + C_B B C_1 V} + \delta^2
\]

\[
= \frac{9 \times 1000 + 3 \times 0.9489 \times 3000}{3 \times 0.9489 \times 3000^2} - \frac{2 \times 1000 \times 300}{9 \times 100}
\]

\[
= 0.5
\]

And \(F\) is given as

\[
F = \frac{C_B B T + (P - C_3)(1 - B)}{(C_1 V + C_B B) T}
\]

\[
= \frac{3 \times 0.9489 + (70 + 5)(1 - 0.9489)}{9 \times 100 + 3 \times 0.9489}
\]

\[
F = 0.02296
\]

\[
F = 2.3\%
\]

Also,

\[
S = TD (1 - F)
\]

\[
S = 0.5 \times 3000 (1 - 0.02296)
\]

\[
S = 1,465.56
\]

Also, \(Q\) is given as:

\[
Q = \frac{TD - (1 - \beta)S}{\mu}
\]

\[
= \frac{0.5 \times 3000 - (1 - 0.9489)1465.56}{0.5}
\]

\[
Q = 2850.22
\]

Therefore, \(Q = 2850.22 \times 10^2 = 285022\) tons

\(Q\) is the quantity of plain sorghum that should be ordered.

B. Estimation of Re-order Point

To find the re-order point we assume one month (\(L\)) (4 weeks) lead time

\[
\text{Demand} E (D) = 3000
\]

Demand variances \(\delta\) of 20.

To calculate the expectation of lead time
\[ E(L) = L \times E(D) \]
\[ = \frac{4}{52 \times 3000} \]
\[ = 230.76 \text{ tons} \]

C. Variance of Lead Time
\[ \delta_L = \sqrt{L} \times \delta_D \]
\[ = \frac{4}{52} \times 20 \]
\[ = 5.5470 \]

D. To Check the Re-order Point

From the forgoing, when the stock reduces to 588 tons, 285022 tons of plain sorghum should be ordered. This is expected to fail twice in one production year. The bases of the study on inventory are to reduce cost which includes reduction of losses. Thus, study on back ordering is targeted towards reducing losses that arise from deterioration, spoilage and damages in stocking. Therefore, ordering for shorter period of time eliminates or reduces this problem. By placing orders biennially holding cost of the stock for half a year has been eliminated, the haulage cost and ordering cost remain. However, the last two are minimal compared to the holding cost. Furthermore looking at Papachristos et al (2003), the holding cost was put at 2.3%. If we estimate the same in our case it would be found that the holding cost on 9m worth of plain sorghum would be about Two Hundred Thousand Naira (₦200,000), whereas information obtained from the production manager of the company shows that cost on bringing down the product of each order fall within the range of Fifty Thousand Naira (₦50,000) eliminating a lot of costs.

V. CONCLUSIONS

In this paper the problems of shortages, yield uncertainty or yield randomness were considered as major inventory problems. It was found that in attempt to solve this problem and obtain acceptable lot size models other researcher have employed different models, including, lost sale backordering and partial backorder. The partial backordering model of Jonah and Chukwu was adopted and review. This enable the obtaining of models for review period (T), the fill rate (F), the stock (S) and consequently the order quantity (Q). Initially the company used to order six hundred tones (600 tons) of plain sorghum once per year, but due to spoilages and general depreciation, not all are used. Another problem posed was that of holding cost. To circumvent these problems a working parameter of three hundred thousand tons of plain sorghum was chosen; this gave an inventory review period (T) of 0.5 which meant order could be placed every six months. The order quantity (Q) for this period was found to 285022 tons. Choosing a four weeks (4wk) lead time it was found that orders should be placed when stock dropped to 58,746 tons. By this shortages are minimized, and keeping extremely large stock is avoided.

VI. RECOMMENDATIONS

Many companies are still operating today without any inventory control at all not even the early generation first-in-first-out (FIFO), hence inconsistencies of production are experienced. We are therefore by this encouraging all recognized companies to:

- Inculcate inventory control systems in their operations for a more successful operation
- Companies that have existing inventory system should attempt to implement the partial backordering system enable the minimization of shortages and losses
- That seminar should be held in companies and Government parastatals to give orientation to storekeepers and administrative officers on the importance of inventory control.
- That inventory course be introduced to most faculties of the university

For further research, other inventory problems should be review in order to improve productivity of any firm.

ACKNOWLEDGMENT

Nnadi Daniel C., and Aniekan Offiong thanks the entire staffs of Champion Brewery Plc, Uyo, Akwa Ibom State, for their numerous assistance during the course of this study.
REFERENCES


AUTHORS BIOGRAPHY

Nhadi, Daniel C is a lecturer in Mechanical Engineering at the Department of Mechanical Engineering, Micheal Okpara University of Agriculture, Umudike, Abia State, Nigeria. He holds a Bachelors Degree in Mechanical Engineering obtained from the University of Uyo, Nigeria in 2003 and Master Degree in Mechanical Engineering obtained from the University of Nigeria, Nsukka in 2012. His area of research interest is Industrial Engineering and Management.

Aniekan Offiong is a Senior Lecturer in the Department of Mechanical Engineering, University of Uyo, Akwa Ibom State, Nigeria. He holds a Bachelor, Master and Doctorate Degree in Mechanical Engineering obtained from the University of Nigeria, Nsukka in 1989, 1994 and 1997 respectively. His area of research interests is Industrial Engineering and Management.