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A Cost-Effective Mission to Mercury Using Multiple Gravity Assist

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***Abstract*:- Interplanetary missions are required to gather information necessary to ensure the long terms survival of the human race. However, such missions are costly and it is required to reduce the cost. For this purpose, trajectories of an interplanetary mission to the planet Mercury are considered. In this paper, cost reductions of such missions by reducing the change in velocity requirements are discussed. When the change in velocity requirement is small, fuel consumption is reduced. This reduces the overall cost of the mission. The technique involves using multiple gravity-assisted maneuvers to achieve the same. The amount by which the change in velocity is reduced depends not only on the type of planet used for the flyby but also on the sequence in which the planets are used. In this paper, different types of multiple gravity-assisted missions are designed. Their performances with respect to single gravity-assisted missions and direct missions are compared. The different planets that were used for flyby are Mars, Earth, Venus and Mercury. Along with this, launch opportunities and mass savings of each mission are also found.**

NOMENCLATURE

r Radial distance of the secondary body from the primary body (km)

V Velocity of the secondary body with respect to the primary body (km/s)

Φ Phase angle of the secondary body with respect to horizontal (rad)

γ Flight path angle of the secondary body from the primary body (rad)

μ The product of universal gravitational constant and the mass of the primary body ($km^3 s^{-2}$)

r_{sat} Distance of satellite with respect to Sun (km)

Φ_{sat} Phase angle of satellite with respect to Sun (rad)

V_{sat} Velocity of satellite with respect to Sun (km/s)

γ_{sat} Flight path angle of satellite with respect to Sun (rad)

μ_s The product of universal gravitational constant and the mass of the Sun ($km^3 s^{-2}$)

μ_{pi} The product of universal gravitational constant and the mass of the i^{th} planet ($km^3 s^{-2}$)

Ψ_{satpi} Inclination of gravity direction of i^{th} planet with the local horizontal (rad)

x_{sat} Horizontal distance of satellite from Sun (km)

y_{sat} Vertical distance of satellite from Sun (km)

x_{pi} Horizontal distance of i^{th} planet from Sun (km)

y_{pi} Vertical distance of i^{th} planet from Sun (km)

k_{satpi} Sign of inclination vector of i^{th} planet

V_a Apogee velocity of the satellite with respect to the Sun (km/s)

a Semi-major axis of the transfer ellipse (km)

ω_t Angular velocity of the target planet (rad/s)

t_p Time period required for the transfer (s)

e eccentricity of the transfer ellipse

r_p Perigee distance of the transfer ellipse (km)

r_{pt} Initial targeted distance of the spacecraft during the initial mission design (km)

r_{pi} Radial distance of the i^{th} planet from the primary body (km)

θ_{ps} True anomaly that the source planet make in the transfer ellipse (rad)

θ_{pt} True anomaly that the target planet makes in the transfer ellipse (rad)

E_{ps} Eccentric anomaly of source planet (rad)

E_{pt} Eccentric anomaly of target planet (rad)

t_{ps} Time since periapsis passage of source planet (s)

t_{pt} Time since periapsis passage of target planet (s)

V_{sa} Spacecraft velocity with respect to Sun at the arrival of the SOI of the target planet (km/s)

V_{sd} Spacecraft velocity with respect to Sun at departure from the SOI of the target planet (km/s)

V_{ps} Velocity of target planet with respect to Sun (km/s)

$V_{\infty a}$ Velocity at infinity on the arrival asymptote (km/s)

$V_{\infty d}$ Velocity at infinity on the departure asymptote (km/s)

δ Angle through which the spacecraft velocity vector is to be turned (rad)

β Hyperbolic asymptote angle (rad)

α_a Angle between velocity of the planet and the velocity of the spacecraft (rad)

τ Angle between V_{sa} and $V_{\infty a}$ (rad)

ϕ Angle between V_{ps} and $V_{\infty a}$ (rad)

G Universal gravitational constant
M_{sun} Mass of sun in kg

I. INTRODUCTION

The cost of an interplanetary mission is dependent on fuel consumption. Now, the amount of fuel consumed is a function change in velocity or the delta-V (δV) requirement. The time taken for a particular mission is inversely proportional to its velocity. Hence, a low-cost mission cannot guarantee a minimum time mission. However, in a low-cost mission, when the fuel requirement is reduced, the payload capacity increases. This in turn increases the scope of the mission. Hence low-cost missions are necessary.

To move a spacecraft from one orbit to another, a change in velocity is required which is usually achieved by thrusters. In a low-cost mission, the use of thruster is reduced by employing other means to supply the change in velocity. This can be achieved by using different types of propulsion techniques, such as solar sails [1], beam-powered propulsion, laser propulsion [2], solar electric propulsion [3,4], etc. However, this paper focuses on the use of multiple-gravity assist maneuvers for achieving a low-cost mission. Hohmann transfer to Mercury requires a net velocity change of about 17 km/s. The high value of δV requirement, as well as technological challenges, calls for a reduction in the overall cost of the mission.

Missions that used multiple gravity assists are NASA's Voyager 1 and 2, launched in 1977; MESSENGER [5] launched on 3rd August 2004; etc. The Bepi-Colombo mission of the European Space Agency is supposed to have eight flybys around Earth, Venus and Mercury, [6]. The gravity assist techniques are often accompanied by the use of solar sails or solar electric propulsion techniques in their mission.

Low thrust and multiple gravity assist trajectory have also been designed for Mercury in 2011 [7-10]. The aero-gravity assist (AGA) technique was designed in 2006 Ref. [11]. Packages for the design and optimization of low thrust gravity assist trajectories were developed in 2002. [12, 13], mentions patching different low thrust trajectories together based on the shape of the trajectory.

Packages to develop trajectories for different planetary sequences were developed [14, 15]. This paper is based on our previous work regarding single gravity-assisted missions to Mercury [16] in which only one planet is used for the flyby purpose. Based on this, reduced velocity requirement and mass savings are analyzed. Whereas, the current paper proposes the use of more than one planet for gravity-assisted interplanetary travel. Same planets in different sequences are also evaluated and the corresponding velocity requirements are tabulated. The objective of this paper is to design a low-cost mission to Mercury using multiple gravity-assisted maneuvers. Different types of flyby sequences are simulated and their performances are compared. Launch opportunities are also calculated for the mission.

This paper discusses various concepts in the following order. The mathematical model is described in section II. Multiple gravity-assisted missions and their performance are discussed in sections III and IV respectively, along with their simulation results. Launch opportunities are covered in section V. The paper ends with a conclusion in section VI.

II. MATHEMATICAL MODEL

The spacecraft is to be transferred from the low earth orbit to Mercury by applying a decremental velocity tangential to the spacecraft's motion. Once the spacecraft reaches Mercury, another velocity is provided to the spacecraft tangentially so that it follows the orbit of Mercury. The same technique is done for a direct or a gravity assist transfer. The mathematical model is similar to the one mentioned in [17] and [18]. The following assumptions are made for the mathematical model of the system.

- Sun is located at the center of the coordinate system and it is stationary.
- The planet is only under the influence of the Sun.
- The eccentricity of the Earth's orbit is zero. Hence the orbit is circular.
- The orbits are coplanar.

A. *Planet around Sun*

The mathematical model of a planet orbiting the sun is discussed in this section. Figure 1 shows a planet (P) orbiting the Sun in a restricted circular orbit.

The coordinate system is Sun Centric Coordinate System (SCS). The motion of the planet P around the Sun is governed by the equations. (1)-(4).

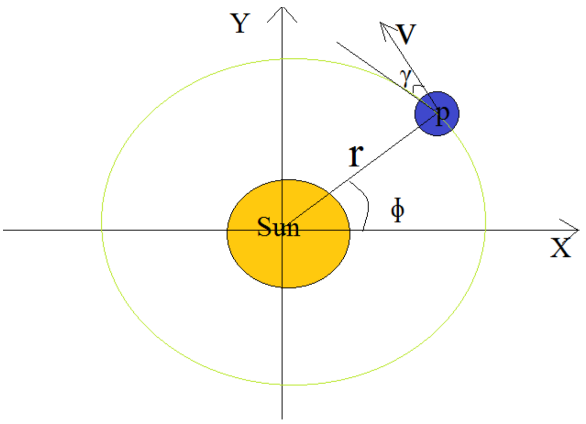

Figure 1. Illustration of planet around the Sun

$$\frac{dr_p}{dt} = V_p \sin\gamma_p \quad (1)$$

$$\frac{d\emptyset_p}{dt} = \frac{V_p}{r_p}\cos\gamma_p \quad (2)$$

$$\frac{dV_p}{dt} = -\frac{\mu_s}{r_p{}^2}\sin\gamma_p \quad (3)$$

$$\frac{d\gamma_p}{dt} = \left(\frac{V_p}{r_p} - \frac{\mu_s}{V_p r_p{}^2}\right)\cos\gamma_p \quad (4)$$

Where, μ_s=G× M_{sun}.
Value of G is taken as $6.678 \times 10^{20} km^3 kg^{-1} s^{-2}$

B. Spacecraft around Sun

In this system, three planets viz., P1, P2 and P3 influence the motion of a satellite around the Sun. If the satellite is orbiting around P1 initially, then the motion of the spacecraft is defined as shown in eqns. (6)-(9).

$$\frac{dr_{sat}}{dt} = V_{sat}\sin(\gamma_{sat}) \quad (6)$$

$$\frac{d\emptyset_{sat}}{dt} = \frac{V_{sat}}{r_{sat}}\cos(\gamma_{sat}) \quad (7)$$

$$\frac{dV_{sat}}{dt} = \sum_{i=1}^{n}\frac{\mu_{pi}}{r_{satpi}{}^2}\cos(\varphi_{satpi} - \gamma_{sat}) - \frac{\mu_{sat}}{r_{sat}{}^2}\sin(\gamma_{sat}) \quad (8)$$

$$\frac{d\gamma_{sat}}{dt} = \left(\frac{V_{sat}}{r_{sat}} - \frac{\mu_{sat}}{V_{sat}\, r_{sat}{}^2}\right)\cos\gamma_{sat} + \sum_{i=1}^{n}\frac{\mu_{satpi}}{V_s r_{satpi}{}^2}\sin(\varphi_{satpi} - \gamma_s) \quad (9)$$

Where,

$$r_{sati} = \sqrt{\left[(x_{sat}-x_{pi})^2 + (y_{sat}-y_{pi})^2\right]} \quad (10)$$

$$X_{sat} = r_{sat}\cos\emptyset_{sat} \quad (11)$$

$$y_{sat} = r_{sat}\cos\emptyset_{sat} \quad (12)$$

$$x_{pi} = r_{pi}\cos\emptyset_{pi} \quad (13))$$

$$y_{pi} = r_{pi}\cos\emptyset_{pi} \quad (14)$$

$$\varphi_{satpi} = -\frac{\pi}{2} - k_{satpi} \cos^{-1}\left\{\frac{[x_{sat}(x_{sat}-x_{pi}) + y_{sat}(y_{sat}-y_{pi})]}{r_{sat}r_{satpi}}\right\} \quad (15)$$

$$k_{satpi} = \sin[x_{sat}(y_{sat}-y_{pi}) - y_{sat}(x_{sat}-x_{pi})] \quad (16)$$

III. MULTIPLE GRAVITY ASSISTED MISSION

Multiple gravity-assisted mission involves the use of more than one planet for gravity-assisted maneuver. Trajectories can be designed initially to reach either Mars or Venus since these planets lie next to Earth. The first gravity assist can take place as soon as the spacecraft reaches the first planet. If the spacecraft passes through the inner circular position, then the velocity of the spacecraft decreases. However, the velocity of the spacecraft increases if it passes through the outer circular position of the planet. The different planets used in this paper for multiple gravity-assisted designs are Mars, Earth, Venus and Mercury. The trajectory must be designed so that the flyby planet is in the right position when the spacecraft passes by it.

1) Design

The design of a gravity-assisted mission can be done by designing a Fixed Time (FT) transfer or a Hohmann transfer to a nearby planet, as discussed in [16]. Figure 2 shows the velocities of the flyby planet and the spacecraft. The velocity of the spacecraft as it approaches the sphere of influence of the flyby is considered as the approach velocity, while the spacecraft velocity as it departs the sphere of influence is called the departure velocity. The velocity of the spacecraft changes near the arrival and departure points according to equations (17)-(21).

$$\delta = 180 - 2\beta \quad (17)$$

$$\boldsymbol{\Delta v = 2v_{\infty a}\cos\beta} \quad (18)$$

$$\sin\tau = \frac{v_{ps}\sin\alpha_a}{v_{\infty a}} \quad (19)$$

$$\emptyset = 180 - \tau - \alpha_a \quad (20)$$

$$v_{sd}{}^2 = v_{sa}{}^2 + \Delta v^2 - 2v_{sa}\Delta v\cos(\beta+\tau) \quad (21)$$

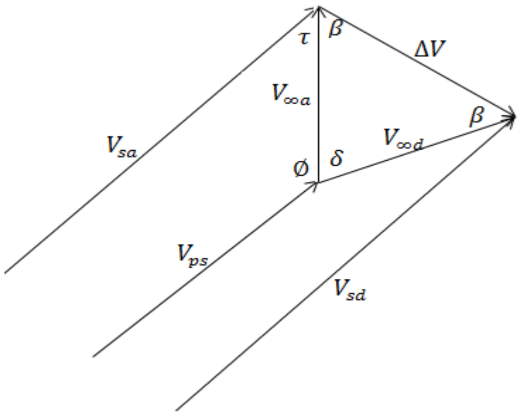

Figure 2 Gravity assist

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Where, $\cos(\beta) = \frac{1}{e}$.

The spacecraft is passed through the outer or inner positions of the planet so that required velocity change is achieved to reach the next planet for a gravity assist. The process is repeated until the spacecraft reaches Mercury. Trajectory designs of four different types of multiple gravity-assisted missions to Mercury are discussed in this paper. The nomenclature of the mission is such that the first and the last planet name initials stand for the starting and the target planet. The intermediate ones are the flyby planets. The different mission names discussed here are Earth-Venus-Mars-Mercury (E-V-Ma-M), Earth-Mars-Venus-Mercury (E-Ma-V-M), Earth-Earth-Venus-Mercury (E-E-V-M), Earth-Earth-Venus-Mercury Bi-elliptical (E-E-V-M Bi) and Earth-Mars-Venus-Venus-Mercury (E-Ma-V-V-M). The trajectory of the missions to the first flyby planet was designed either by using Hohmann transfer or by using Fixed Timed (FT) transfer, discussed in [16]. The parameters used to design fixed time transfers are enclosed in table 2.

IV. SIMULATION RESULTS

A. E-V-Ma-M

In Earth-Venus-Mars-Mercury, the satellite is initially designed to reach Venus using fixed time transfer. The spacecraft, on reaching Venus does a flyby so that its velocity is minimally increased so as to reach Mars. This is done by adjusting the flyby distance. The Mars flyby decreases the spacecraft velocity, due to which it reaches Mercury. The result of this mission is shown in figure 3.

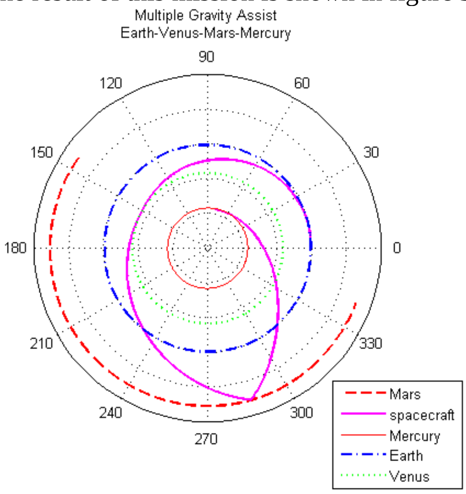

Figure 3 Multiple gravity assist E-V-Ma-M

The velocity changes in the spacecraft after each flyby are mentioned in Table 1.

Table 1 Change in velocity E-V-Ma-M

Planet	Altitude (km)	V_{sa} (km/s)	V_{sd} (km/s)	days
Venus	8	37.4436	41.6136	132
Mars	10	21.3949	20.8235	270

Total change in velocity ΔV=16.8 km/s
Time taken to reach Mercury = 375 days.

The total change in velocity required to reach Mercury is less than Earth-Mercury Hohmann transfer velocity change.

Table 2 Design parameters for fixed time transfer

Mission	$r_{pt} \times 10^8$ (km)	A× 10^8 (km)	E	θ_1 (deg.)	γ (deg.)
E-V-Ma-M	1.08	1.28	0.17	195	-3.05
E-E-V-M	1.498	1.426	0.04	300	-0.017
E-E-V-M bi-elliptical	1.496	1.4867	0.12	160	2.752

B. E-Ma-V-M

Another mission is designed so that the spacecraft initially possesses Earth-Mars Hohmann velocity, which is the minimum velocity to reach Mars. On reaching Mars, the spacecraft does a flyby around it. As a result, the spacecraft moves towards Venus where it performs another flyby and heads towards Mercury. The simulation result is shown in figure 4. The velocity reductions of Mars and Venus are mentioned in table 3.

Total change in velocity ΔV = 13.2147 km/s
Total time taken to reach Mercury = 290 days.

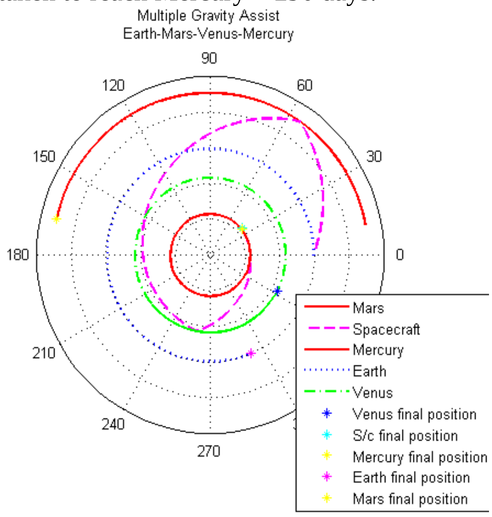

Figure 4: Multiple gravity assist E-Ma-V-M

Table 3 Change in velocity E-Ma-V-M

Planet	Altitude (km)	V_{sa} (km/s)	V_{sd} (km/s)	days
Earth	10	21.476	20.152	85
Venus	8	40.989	35.125	253

C. E-E-V-M

The initial trajectory of this mission is designed so as to meet the Earth again using fixed time transfer. The spacecraft intercepts Earth once again for its first flyby, after departing from Earth. It does its second flyby around Venus and heads towards Mercury. The simulation result for this mission is shown in Figure 5. The change in spacecraft velocity after each flyby is enclosed in table 4.

Total change in velocity ΔV = 7.216 km/s
Total time taken to reach Mercury = 220 days.

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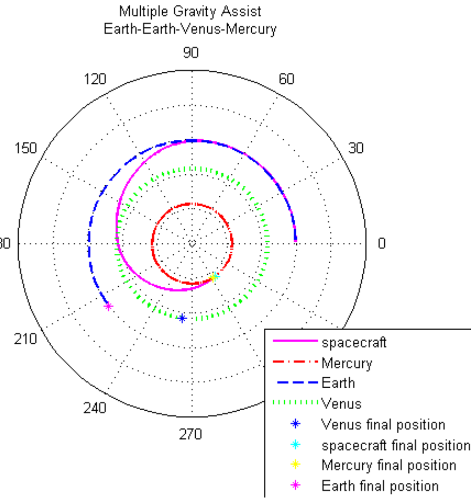

Figure 5: Multiple gravity assist E-E-V-M

Table 4 Change in velocity E-E-V-M

Planet	Altitude (km)	V_{sa} (km/s)	V_{sd} (km/s)	days
Earth	10	28.745	26.466	89
Venus	8	36.751	29.535	166

D. E-E-V-M Bi

A bi-elliptical transfer is a mission in which the spacecraft traverses only of the half parts of two different elliptical orbits. This mission is designed in such a way that the spacecraft initially does a half Hohmann transfer to an orbit with an apoapsis of 1.6875×10^8km. The second elliptical orbit through which the spacecraft moves is designed based on the fixed time transfer to the next target planet. Once it reaches the apoapsis of Hohmann transfer, a second impulse is provided so that the spacecraft is transferred to the target point. Hence with the help of two transfer ellipses, the spacecraft is made to reach its destination.

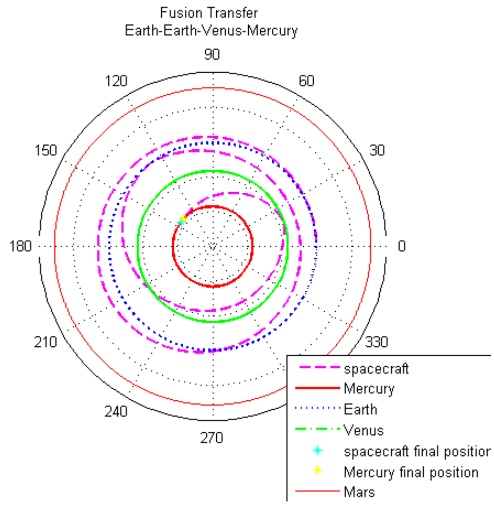

Figure 6 : Multiple gravity assist E-E-V-M bi-elliptical

The result of the Bi-elliptical Earth-Earth-Venus-Mercury transfer is shown in Figure 6. The change in velocity after each flyby is enclosed in table 5.

Table 5 change in velocity E-E-V-M bi-elliptical

Planet	Altitude (km)	V_{sa} (km/s)	V_{sd} (km/s)	days
Earth	1500	29.471	26.746	488
Venus	8	36.480	31.064	650

The first change in velocity given for Hohmann transfer is ΔV_1=0.8831 km/s

The second change in velocity given for FT transfer is ΔV_2=1.2044 km/s

The third change in velocity given on reaching Mercury is ΔV_3=5.78 km/s

Total change in velocity ΔV =7.8714 km/s

Time taken to reach Mercury = 700 days.

E. E-Ma-V-V-M

The initial trajectory of the mission is the same as the one discussed in E-Ma-V-M. Mars and Venus provide the spacecraft with first and second gravity assist. After the second gravity assist, the spacecraft takes a path that is similar to the orbit of Venus. The concept of mean motion resonance orbit is used to calculate the instant at which spacecraft and Venus meet again at the same point. Generally, resonant orbit gives an idea about when a spacecraft and a planet will meet again at the same position because their orbits are resonant according to [19]. Suppose the orbital period of the spacecraft has a time period 'q' and that of a planet is 'b'. Then the spacecraft and the planet will encounter at the same place when the spacecraft has performed 'b' revolutions while the planet performs 'q' revolutions around the Sun.

The initial change in spacecraft velocity, ΔV_1=2.9438 km/s.

The velocity change in the spacecraft due to the gravity assist of Mars and Venus are given in Table 6.

Table 6 change in velocity E-Ma-V-V-M

Planet	Altitude (km)	V_{sa} (km/s)	V_{sd} (km/s)	days
Mars	10	19.440	18.233	129
Venus	8	40.053	33.85	300
Venus	8	33.856	28.09	18.8yrs
Mercury	3400	54.206	47.88	19yrs

Once the spacecraft perform Venus gravity assists, its trajectory changes. The period of the new trajectory is found to be 207 days.

$$\frac{period\ of\ spacecraft}{period\ of\ venus} = \frac{207}{225} = \frac{23}{25}$$

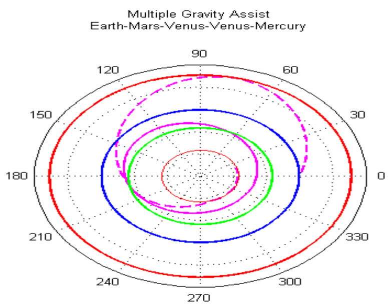

Figure 7: Multiple gravity assist E-Ma-V-V-M

The spacecraft should meet the planet, Venus, again at the same position by the end of its 23rd revolution around the Sun. However, results show that the spacecraft completes 24 revolutions around the Sun to meet Venus again at the position where the first Venus flyby occurred. This is due to the error in the measurement of the orbital period and integration errors. The simulation result for multiple gravity assist using resonant orbit concept is shown in figure 7.

V. COMPARISON OF PERFORMANCE OF MULTIPLE GRAVITY ASSISTED MISSION AND LAUNCH OPPORTUNITIES

Missions using multiple gravity-assisted techniques are feasible only if the planets are present at the required position and time. Accordingly, the mission duration also varies. Due to this, the comparison with respect to the direct missions to the same planet, based on mission duration is not significant. The values of direct mission, as well as single gravity-assisted mission to Mercury, were obtained from [16]. Using these results, a comparative study of the performance with a different sequence of planets considered for multiple gravity-assisted missions is done. In this paper, performance is considered as a measure of change in velocity. Table 7 gives the performance of multiple gravity-assisted missions for different planetary sequences when compared to the direct mission. A particular planetary sequence is of better performance if the total change in velocity is less. The maximum δV obtained for multiple gravity-assisted techniques is 16.8 km/s which is less than that required for a direct mission. Hence compared to direct missions, multiple gravity-assisted missions are economical.

Table 7 Mass savings

Planetary Sequence	ΔV (km/s)	Mass savings-1 (kg)	Mass savings-2 (kg)
E-V-Ma-M	16.8	66	-
E-Ma-V-M	13.21	1266.66	-
E-E-V-M bi-elliptical	7.871	3041.99	*928.67*
E-E-V-M	7.216	3261.33	*1148.00*
E-Ma-V-V-M	2.943	4685.66	*2572.3*

Mass savings-1 in Table 7 represents the savings in the propellant mass of a gravity-assisted mission compared to a direct Hohmann transfer to Mercury. This evaluation is based on the total change in velocity obtained for the missions. The total change in velocity for direct missions was obtained from [16]. Mass savings-2 of Table 7 gives the savings in the propellant mass of a multiple gravity-assisted mission when compared to the minimum change in velocity obtained for a single gravity-assisted mission to Mercury. This velocity was mentioned as 10.6 km/s in [16]. The performance of E-Ma-V-M is similar to that of E-V-Ma-M. It gives better performance than the single gravity-assisted transfer for 59 days mentioned in [16].

When compared to minimum δV for a single gravity-assisted mission, the performance of E-V-Ma-M and E-Ma-V-M are similar. Hence there are no mass savings for these missions. Mass savings can be observed for E-E-V-M, E-E-V-M (bi-elliptical) and E-Ma-V-V-M. However, E-Ma-V-V-M takes 19 years to complete. The launch opportunities for the various multiple gravity-assisted missions are enclosed in table 8. It is assumed that $\emptyset_{earth} = 0$ in all the cases while calculating the launch opportunities.

Table 8 Launch opportunities

Planet Sequence	$\emptyset_{venus}$	$\emptyset_{merc.}$	$\emptyset_{mars}$	Launch Opportunity
E-V-Ma-M	300	330	143	12-05-2038, 22-06-2068
E-Ma-V-M	220	260	11	15-01-2032
E-E-V-M bi-elliptical	66	145	-	13-10-2023
E-E-V-M	90	270	-	30-09-2020
E-Ma-V-V-M	75	85	7	26-07-2054

VI. CONCLUSION

The cost-effectiveness of multiple gravity-assisted interplanetary missions has been studied in this paper and the results are compared with the direct mission as well as single gravity-assisted mission. In general, the cost of the interplanetary mission is proportional to the change in velocity requirement of the spacecraft. This requirement is reduced in this paper with the help of multiple gravity-assisted flyby techniques. From the analysis carried out, it can be concluded that the change in velocity produced by each planet depends upon the planet and the flyby altitude. It also depends upon the entry flight path angle of the spacecraft. However, the entry flight angle is dependent on the initial mission design. Hence the performance of multiple gravity assists depends on the design of the initial phase of the mission. The E-E-V-M planetary sequence performs better when compared to the rest of the missions analyzed in this paper. Launch opportunities for multiple gravity-assisted maneuvers have also been found out.

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