

A Confidence-Weighted Approach for Solving Grey Transportation Problems under Cost Uncertainty

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Abstract - Transportation issues are very important for planning logistics and optimizing the supply chain. Conventional transportation models presume that transportation costs are accurately understood; however, in several real-world scenarios, such prices remain unclear due to variable traffic conditions, environmental disturbances, and insufficient information. Grey system theory is a good way to simulate these kinds of uncertainty since it shows transportation costs as interval grey values. This paper suggests a Confidence-Weighted Grey Transportation Method (CWGT) to help with transportation challenges where the costs are not known. The suggested method uses the breadth of the uncertainty of gray intervals to create confidence weights for each transportation route. These weights change grey transportation costs into certain values that show both the predicted cost and how reliable the information is. Then, linear programming techniques are used to solve the deterministic transportation model that comes out of this. To show that the suggested method works, a numerical example and a number of computer experiments are done. The results show that the CWGT approach is a more reliable way to move things than standard midpoint and average transformation methods. Also, the suggested approach is easy to compute and may be simply used with optimization tools like MATLAB. The suggested framework is a useful decision-support model for planning logistics when things are unknown, and it may be used for more complicated transportation systems as well.

Keywords - Grey transportation problem, optimization of uncertainty, grey system theory, transportation cost uncertainty, and logistics optimization.

I. INTRODUCTION

Transportation issues are among the most essential optimization models in operations research and logistics management. The main goal of a transportation model is to find the best way to move items from many supply sources to many demand destinations while keeping the overall cost or time of transportation as low as possible. The transportation problem was formalized by the French Mathematician Monge [1]. Major advances were made in the field during World War II by the Russian Mathematician and Economist Leonid Vitaliyevich Kantorovich [2]. The standard form of the transportation problem was first presented by Frank Lauren Hitchcock [3].

Traditional transportation models assume that all the characteristics, such as costs, amounts of supply, and needs for demand, are known exactly. Based on this idea, several

algorithms have been made to find the best answers quickly. The Northwest Corner Method, the Least Cost Method, and Vogel's Approximation Method are all well-known ways to find initial workable solutions. After that, optimization processes like the Modified Distribution Method (MODI) or the stepping-stone method are used. These methods are used a lot since they are fast and easy to use, but they only work if shipping costs are known for sure. Besides this a large number of research works on distribution issues has been done by several researcher such as [4-15] solved a transportation problem with market choice.

In decision-making theory, uncertain decision-making is a vital branch. There are various approaches to dealing with uncertainty problems [16]. To deal with uncertainties, a number of methods were developed, including interval, fuzzy, Rough and stochastic numbers [17,18,19]. Many researchers [20-30] have proposed different techniques for solving transportation problem in uncertain environment.

So, it is clear that in many real-world logistics systems, it's not always possible to figure out exactly how much transportation will cost. Things like changing fuel costs, unpredictable traffic, infrastructural problems, changing weather, and missing information can all make transportation cost parameters less definite. In these kinds of situations, using fixed numbers to show transportation costs might lead to solutions that aren't realistic or dependable. To deal with such situations, Deng [31] first introduced the concept of grey systems theory. Grey system theory has been successfully applied in various fields including decision-making, forecasting, optimization, and system analysis [32]. Based on the framework of grey system theory, Bai et al.[33] introduced the Grey Transportation Problem (GTP), in which transportation costs are modeled as grey numbers rather than deterministic values. Grey system theory is a good way to represent issues with just some information available since it works around these problems. Grey system theory was first used to examine systems with partial or unclear data. It uses interval grey numbers to show unknown parameters. The grey transportation problem builds on this idea by using grey intervals instead of fixed numbers to show transportation costs. This picture helps people who make decisions better understand uncertainty while planning transportation. Palanci et al. [34] studied on uncertainty under grey goals in a cooperative game. Nasser et al. [35] proposed a direct

approach for solving the grey assignment problem based on grey arithmetic. Pourofoghi et al. [36] proposed a new method to find an optimal solution for grey transportation problems where transportation cost, supply and demand are interval grey numbers. Since the introduction of grey transportation models, several approaches have been proposed to convert grey costs into deterministic values for optimization purposes. One common technique involves the midpoint approximation, where the grey interval is replaced by its average value. Another widely used approach is the risk coefficient method, which introduces a parameter representing the decision-maker's risk preference and transforms the grey interval accordingly. These methods simplify the solution process by converting the grey transportation problem into a deterministic transportation problem.

However, existing grey transportation methods exhibit several limitations. Most of these approaches treat all grey intervals equally during the transformation process, regardless of the level of uncertainty associated with each transportation route. In practical situations, some transportation routes may have relatively narrow cost intervals and therefore more reliable cost estimates, while others may have wide intervals indicating significant uncertainty. Ignoring this variation in uncertainty may lead to transportation plans that do not adequately reflect the reliability of the available information.

To overcome this limitation, this study proposes a Confidence-Weighted Grey Transportation Method (CWGT) for solving grey transportation problems. The proposed approach incorporates the uncertainty width of each grey cost interval into the cost transformation process by introducing a confidence weight. Transportation routes with smaller interval widths receive higher confidence weights, while routes with larger uncertainty intervals receive lower influence in the optimization model. By integrating uncertainty reliability into the transportation cost transformation, the proposed method provides a more realistic representation of transportation decision-making under uncertainty.

The resulting deterministic transportation problem can then be solved using classical optimization techniques such as linear programming. The proposed CWGT method remains computationally simple while improving the reliability of transportation cost estimation in uncertain environments.

The primary contributions of this work are encapsulated as follows. First, a new approach for transforming confidence weights is suggested to include uncertainty breadth in estimating the cost of gray transit. Second, a computational algorithm is created to put the suggested strategy into action inside a traditional framework for optimizing transportation. Third, numerical tests and comparisons are done to show that the suggested method works better than other gray transportation solution approaches.

II. PRELIMINARIES

2.1 CLASSICAL TRANSPORTATION PROBLEM

The classical transportation problem in operations research that involves finding the optimal way to move goods from one place to another. It involves allocating resources in the most efficient way while minimizing the cost of transportation. It is based on objective function. An objective function is a

function whose value we try to maximize (profit) or minimize (travel length, costs, time, ...) in the process of optimization.

Then the linear programming model representing the transportation problem is generally given as

$$\begin{aligned} \text{Minimize } z &= \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \\ \text{subject to } \sum_{j=1}^n x_{ij} &\leq s_i && ; \\ i &= 1, 2, \dots, m \\ \sum_{i=1}^m x_{ij} &\geq d_j && ; \\ j &= 1, 2, \dots, n \\ x_{ij} &\geq 0. && \text{ for all } i \text{ and } j. \end{aligned}$$

In mathematical terms the above problem can be expressed as finding a set of x_{ij} 's,

$$\begin{aligned} i &= 1, 2, \dots, m; j = 1, 2, \dots, n \text{ to} \\ \text{Minimize } z &= \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \\ \text{subject to } \sum_{j=1}^n x_{ij} &= s_i ; \quad i = 1, 2, \dots, m \\ \sum_{i=1}^m x_{ij} &= d_j && ; \\ j &= 1, 2, \dots, n, \quad x_{ij} \geq 0. && \text{ for all } i \text{ and } j. \end{aligned}$$

2.2 GREY SYSTEM THEORY

Mathematically, systems containing both known and unknown parts can be better understood by applying grey system theory. Deng Julong first suggested it in 1982. Grey is a middle ground between knowing everything and knowing nothing. Most real-world systems (economics, transportation, engineering, decision making) are grey systems because data are incomplete, uncertain, or imprecise. Grey theory aims to model uncertain systems, work with small sample data, handle interval uncertainty, support decision making. The major components of grey system theory are grey numbers, grey relational analysis, grey prediction model, grey decision making, grey optimization models. Thus, grey systems lie between certainty and uncertainty.

Grey systems frequently exhibit the following characteristics:

- ❖ Small Sample Size: Grey models might only work with a small number of observations (4–10 data points).
- ❖ Not Enough Information: It's not always easy to find exact information on parameters.
- ❖ Unclear Data: Data are sometimes shown as ranges instead than exact amounts.
- ❖ Dynamic Systems: Grey systems can simulate real-world events that change over time.

2.3 GREY NUMBERS

A Grey Number is a number whose exact value is unknown but lies within a known interval.

Mathematically it is written as, $G = [G_{ij}, G^{ij}]$

Where,

\mathcal{G} = Grey Number

\mathcal{G}_{ij} = Lower bound

\mathcal{G}^{ij} = Upper Bound, and it is most common type grey number which is called interval grey number. Besides this discrete grey number and continuous grey number are another type of grey numbers.

2.4 SOME ALGEBRAIC OPERATIONS OF GREY NUMBERS

Let \mathcal{G}_1 and \mathcal{G}_2 be two grey numbers define as $\mathcal{G}_1 = [a_1, a_2]$ and $\mathcal{G}_2 = [b_1, b_2]$ where a_1, b_1 are lower bound and a_2, b_2 are upper bound.

Addition of two grey number:

$$\mathcal{G}_1 + \mathcal{G}_2 = [a_1 + b_1, a_2 + b_2]$$

Subtraction of two grey number

$$\mathcal{G}_1 - \mathcal{G}_2 = [a_1 - b_1, a_2 - b_2]$$

Multiplication of two grey numbers:

$$\mathcal{G}_1 \times \mathcal{G}_2 = [\min S, \max S],$$

where $S = \{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}$

Division of two grey numbers: $\mathcal{G}_1/\mathcal{G}_2 = [\min T, \max T]$

where $T = \{a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2\}$.

2.5 GREY TRANSPORTATION PROBLEM

A Grey Transportation Problem (GTP) is a form of transportation model that uses grey numbers instead of exact numbers to show costs, supplies, or needs. The goal is still the same as it was in the old transportation problem: The whole cost of transportation should be lower. But gray intervals are utilized to show what is in dispute. The mathematical model of grey transportation problem is stated below:

Objective Function,

Objective Function,

$$\text{Minimize } \check{Z} = \sum_{i=1}^m \sum_{j=1}^n \check{c}x_{ij}$$

Subject to,

$$\text{Supply constraints: } \sum_{j=1}^n x_{ij} = \check{a}$$

$$\text{Demand constraints: } \sum_{i=1}^m x_{ij} = \check{b} \quad \text{and } \forall x_{ij} \geq 0$$

Where notations represent,

$$\check{c} = [\check{c}_{ij}, \check{c}^{ij}] = \text{Grey Transportation cost,}$$

$$\check{a} = [a_i, a^i] = \text{Grey Supply}$$

$$\check{b} = [b_j, b^j] = \text{Grey Demand}$$

x_{ij} = Total transported unit.

Tbale-1: Matrix of Grey Transportation Problem with mixed constraints

D/S	1	2	n	Supply
1	$[\check{c}_{11}, \check{c}^{11}]$	$[\check{c}_{12}, \check{c}^{12}]$	$[\check{c}_{13}, \check{c}^{13}]$	$[\check{c}_{1n}, \check{c}^{1n}]$	$[\check{a}_1, \check{a}^1]$
2	$[\check{c}_{21}, \check{c}^{21}]$	$[\check{c}_{22}, \check{c}^{22}]$	$[\check{c}_{23}, \check{c}^{23}]$	$[\check{c}_{2n}, \check{c}^{2n}]$	$[\check{a}_2, \check{a}^2]$
⋮	⋮	⋮	⋮	⋮	⋮
m	$[\check{c}_{m1}, \check{c}^{m1}]$	$[\check{c}_{m2}, \check{c}^{m2}]$	⋮	$[\check{c}_{mj}, \check{c}^{mj}]$	$[\check{c}_{mn}, \check{c}^{mn}]$	$[\check{a}_m, \check{a}^m]$
Demand	$[\check{b}_1, \check{b}^1]$	$[\check{b}_2, \check{b}^2]$	$[\check{b}_3, \check{b}^3]$	$[\check{b}_n, \check{b}^n]$	

2.6 SOLUTION PROCEDURE

To solve a grey transportation problem, researchers usually convert grey numbers into crisp values. By reviewing various research papers, we find four notable methods. There are also some other methods that are not as widely accepted.

1. Lower Limit Method (LLM) (Bai et al., 2004).
2. Upper Limit Method (ULM) (Bai et al., 2004).
3. Midpoint Method (MM) (Deng, 1988; Liu & Lin, 2010).
4. Risk Coefficient Method (RCM) (Xu, G. et al. 1999).

However, existing grey transportation methods exhibit several limitations we mentioned above. To overcome these limitations, this study proposes a Confidence-Weighted Grey Transportation Method (CWGT) for solving grey transportation problems.

III. PROPOSED METHOD: CONFIDENCE-WEIGHTED GREY TRANSPORTATION METHOD (CWGT)

Most grey transportation methods convert the grey number $\mathcal{G} = [\mathcal{G}_{ij}, \mathcal{G}^{ij}]$ into a single value by using average, risk coefficient, forecasting. But this method ignores uncertainty intensity. But our new proposed idea uses uncertainty width of grey number to determine the cost weight. We define

$$\check{w}_{ij} = \frac{1}{1 + (\mathcal{G}^{ij} - \mathcal{G}_{ij})}$$

Interpretation:
 Large interval gives more uncertainty which shows smaller weight
 From small interval get reliable cost and the larger weight

Then define the Confidence Weighted Cost:

$$\check{c}_{ij}^w = \check{w}_{ij} \left(\frac{\mathcal{G}_{ij} + \mathcal{G}^{ij}}{2} \right) + (1 - \check{w}_{ij}) \mathcal{G}^{ij}$$

Meaning:
 This reliable data close to midpoint also uncertain data move towards pessimistic value. This creates a risk aware grey cost transformation.

3.1 NOW WE CONSTRUCT THE ALGORITHM OF PROPOSED METHOD (CWGT)

Step-1: The primary transportation problem of the gray transportation problem that has been provided should be constructed.

Step-2: Compute uncertainty width: $(G^{ij} - G_{ij})$

Step-3: Using uncertainty width using subtraction from upper bound to lower bound, that is $(G^{ij} - G_{ij})$. Then determine weighted value as $\tilde{w}_{ij} = \frac{1}{1+(G^{ij}-G_{ij})}$

Step-4: Then compute weighted cost using the formula $\check{c}_{ij}^w = \tilde{w}_{ij} \left(\frac{G_{ij} + G^{ij}}{2} \right) + (1 - \tilde{w}_{ij}) G^{ij}$

Which we define in previous section.

Step-5: Solve transportation problem (least-cost method) to get optimal solution.

Step-6: Finally, compute total cost: $\check{Z} = \sum_{i=1}^m \sum_{j=1}^n \check{c}_{ij}^w x_{ij}$.

3.2 FLOW CHART OF PROPOSED METHOD (CWGT)

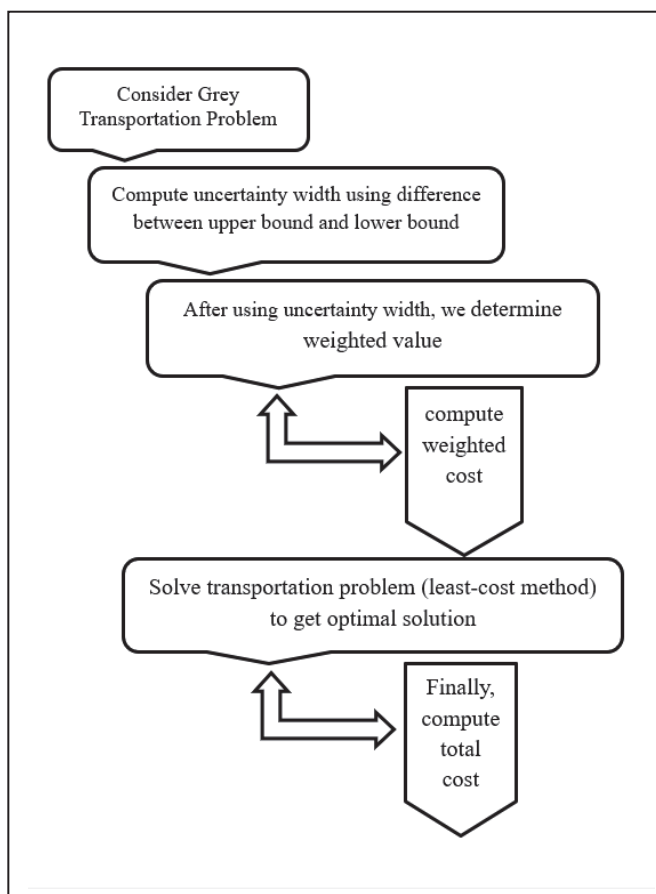


Fig. 1. Flow Chart of proposed method

IV. NUMERICAL EXAMPLE:

EXAMPLE 4.1: CONSIDER 2×2 BALANCED PROBLEM

Grey Cost Matrix:

	d_1	d_2	Supply
s_1	[4, 6]	[5, 7]	20
s_2	[3, 5]	[6, 8]	30
Demand	25	25	50

Compute uncertainty width using difference between upper bound and lower bound. All intervals have width 2, so weighted value is $\tilde{w}_{ij} = \frac{1}{1+(G^{ij}-G_{ij})} = \frac{1}{1+2} = \frac{1}{3}$

Convert CWGT matrix:

	d_1	d_2	Supply
s_1	[5.67]	[6.67]	20
s_2	[4.67]	[7.67]	30
Demand	25	25	50

Using least cost method solve this Transportation problem we get:

	d_1	d_2	Supply
$X = s_1$	0	25	20
s_2	25	5	30
Demand	25	25	50

Finally, compute total cost:

$$\check{Z} = \sum_{i=1}^m \sum_{j=1}^n \check{c}_{ij}^w x_{ij} = 25(4.67) + 20(6.67) + 5(7.67) = 288.50$$

EXAMPLE 4.2: CONSIDER 2×3 BALANCED PROBLEM

Grey Cost Matrix:

	d_1	d_2	d_3	Supply
s_1	[4, 6]	[5, 8]	[6, 9]	20
s_2	[3, 5]	[4, 7]	[7, 10]	30
Demand	15	20	15	50

Using same procedure we get total cost:

$$\check{Z} = \sum_{i=1}^m \sum_{j=1}^n \check{c}_{ij}^w x_{ij} = 319.60$$

EXAMPLE 4.3: CONSIDER 3×3 BALANCED PROBLEM

Grey Cost Matrix:

	d_1	d_2	d_3	Supply
s_1	[3, 5]	[6, 8]	[7, 9]	15
s_2	[5, 7]	[4, 6]	[3, 5]	25
s_3	[6, 8]	[5, 7]	[4, 6]	20
Demand	10	20	30	60

Using same procedure we get total cost:

$$\check{Z} = \sum_{i=1}^m \sum_{j=1}^n \check{c}_{ij}^w x_{ij} = 330.20$$

EXAMPLE 4.4: CONSIDER 3×4 BALANCED PROBLEM

Grey Cost Matrix:

	d_1	d_2	d_3	d_4	Supply
s_1	[4, 6]	[7, 9]	[5, 7]	[8, 10]	20
s_2	[6, 8]	[5, 7]	[7, 9]	[6, 8]	30
s_3	[5, 7]	[6, 8]	[4, 6]	[7, 9]	25
Demand	15	20	25	15	75

Using same procedure we get total cost:

$$\check{Z} = \sum_{i=1}^m \sum_{j=1}^n \check{c}_{ij}^w x_{ij} = 485.25$$

EXAMPLE 4.5: CONSIDER 3×3 UNBALANCED PROBLEM

Grey Cost Matrix:

	d_1	d_2	d_3	Supply
s_1	[4, 6]	[5, 7]	[6, 8]	20
s_2	[3, 5]	[6, 8]	[5, 7]	25
s_3	[7, 9]	[4, 6]	[3, 5]	15
Demand	10	20	15	

(Unbalanced TP)

Grey Cost Matrix:

	d_1	d_2	d_3	d_4	Supply
s_1	[4, 6]	[5, 7]	[6, 8]	0	20
s_2	[3, 5]	[6, 8]	[5, 7]	0	25
s_3	[7, 9]	[4, 6]	[3, 5]	0	15
Demand	10	20	15	15	60

(Balanced TP)

Using same procedure we get total cost:

$$\check{Z} = \sum_{i=1}^m \sum_{j=1}^n \check{c}_{ij}^w x_{ij} = 250.15$$

Example 4.6: Consider 3×4 balanced problem

[Pourofoghi et al. [36]]

[supply demand also grey number]

	d_1	d_2	d_3	d_4	Supply
s_1	[1, 3]	[2, 4]	[4, 14]	[4, 12]	[2, 16]
s_2	[1, 3]	[3, 17]	[2, 8]	[2, 8]	[4, 38]
s_3	[2, 16]	[4, 18]	[2, 12]	[2, 12]	[2, 34]
Demand	[2, 22]	[2, 6]	[2, 28]	[2, 32]	[8, 88]

Using same procedure we get total cost:

$$\check{Z} = \sum_{i=1}^m \sum_{j=1}^n \check{c}_{ij}^w x_{ij} = 656.49$$

V. RESULT ANALYSIS AND COMPARISON OF METHODS

The effectiveness of the proposed Confidence-Weighted Grey Transportation Method (CWGT) was evaluated through five numerical examples and compared with four existing grey transportation approaches: the Lower Limit Method (LLM), Upper Limit Method (ULM), Midpoint Method (MM), and Risk Coefficient Method (RCM). The computational results obtained for these methods are summarized in Table 1 and represent it by using bar diagram in Fig 1.

Table 1. Summarized computational results

Example	LLM	ULM	MM	RCM	CWGT
4.1	205	305	255	265	288.5
4.2	220	355	287.50	301.0	319.60
4.3	230	350	290	302	330.20
4.4	360	510	435	450	485.25
4.5	175	265	220	229	250.15
4.6	14	706	232	306.32	656.49

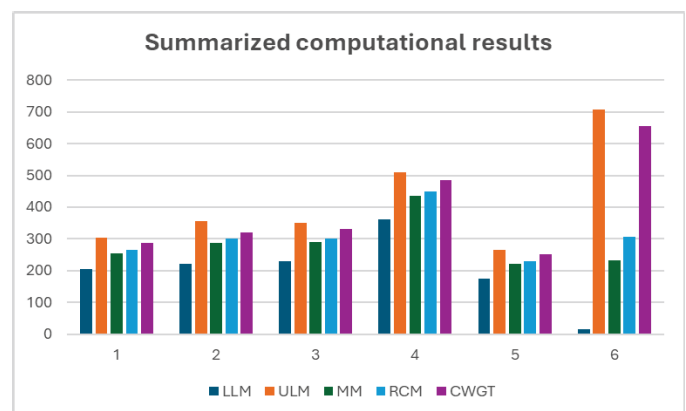


Fig 2. Position of CWGT results

The Lower Limit Method replaces each grey interval with its lower bound. As a result, the transportation model assumes the most optimistic scenario where all transportation routes operate at their minimum possible costs. From Table 1, the LLM consistently produces the smallest transportation cost values across all examples. For instance, in Example 4.2 the LLM result is 220, which is significantly lower than the other methods. However, this method has a major limitation. It ignores uncertainty entirely by assuming that the lowest possible cost will always occur. In real-world logistics systems, such assumptions are unrealistic because transportation costs are rarely guaranteed to remain at their lower bounds. Consequently, LLM may lead to overly optimistic solutions that underestimate the true transportation cost.

In contrast, the Upper Limit Method replaces each grey interval with its upper bound. This represents the most pessimistic scenario in which transportation costs always occur at their maximum values. As shown in Table 1, the ULM consistently produces the largest transportation cost values among all methods. For example, in Example 4 the ULM result is 510, which is substantially higher than the results produced by the other methods. This method guarantees feasibility under worst-case conditions; it may lead to excessively conservative transportation plans. As a result, ULM tends to overestimate the required transportation cost, which may cause inefficient resource allocation.

The Midpoint Method converts each grey interval into a deterministic value. This method represents a neutral approach between optimistic and pessimistic scenarios. From Table 1, the midpoint results lie approximately in the center of the cost range. For instance, in Example 4.3 the midpoint result is 290, which lies between the LLM value of 230 and the ULM value of 350. However, the midpoint method treats all grey intervals equally, regardless of their width. This means that transportation routes with high uncertainty receive the same weight as routes with low uncertainty. Consequently, the midpoint method does not reflect the reliability of the available cost information.

The Risk Coefficient Method introduces a parameter t that allows the decision-maker to control the level of pessimism in the cost transformation. In this study, t was used, representing a moderately conservative decision preference. As expected, the RCM results are slightly higher than the midpoint results in all examples. For example, in Example 4.2 the midpoint cost is 287.50, while the RCM cost increases to 301.00. This method provides greater flexibility because decision-makers can adjust the parameter according to their risk tolerance. However, the same risk parameter is applied uniformly to all transportation routes. Therefore, RCM still ignores differences in uncertainty between routes.

The proposed Confidence-Weighted Grey Transportation Method (CWGT) introduces route-specific confidence weights based on the interval width of grey costs. Wider intervals correspond to higher uncertainty and therefore receive lower confidence weights. The computational results show that CWGT consistently produces transportation costs that are higher than those obtained using the midpoint and risk coefficient methods but lower than those obtained using the upper limit method which is clear from Fig 3. For example: Example 2 results: MM = 287.50 RCM = 301.00 CWGT = 319.60 ULM = 355.00 This pattern indicates that the CWGT method penalizes highly uncertain transportation routes more strongly than traditional grey transformation methods. As a result, CWGT produces more conservative solutions that better reflect uncertainty in transportation costs. Furthermore, the CWGT results remain significantly lower than the upper bound solution, indicating that the method avoids excessive pessimism.

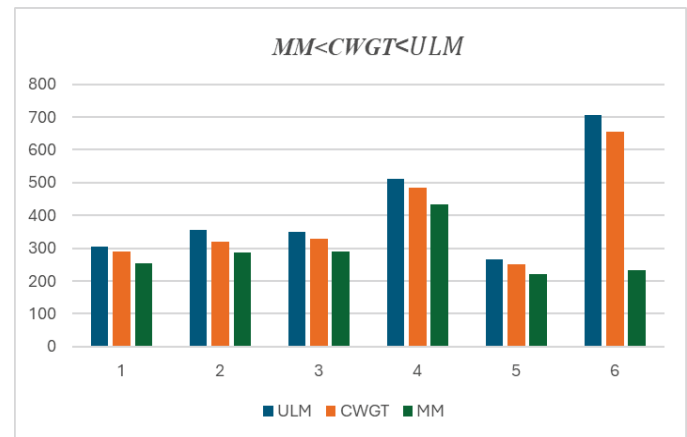


Fig 3. Position of CWGT results

Overall Comparison The numerical results reveal a clear ordering among the five methods:

$$LLM < MM < RCM < CWGT < ULM$$

This ordering reflects increasing levels of conservatism in cost estimation. *LLM* provides the most optimistic cost estimates. *ULM* represents the worst-case scenario. *MM* represents a neutral estimate. *RCM* introduces moderate risk adjustment. *CWGT* incorporates uncertainty width to produce reliability-aware solutions. Because *CWGT* considers both the expected cost and the uncertainty associated with each route, it provides a balanced and realistic transportation solution.

The experimental results demonstrate the following hierarchy of decision behavior:

<i>LLM</i>	Optimistic
<i>MM</i>	Neutral
<i>RCM</i>	Controlled risk
<i>CWGT</i>	Uncertainty-aware
<i>ULM</i>	Worst case

CWGT improves decision reliability because it considers interval width, penalizes high uncertainty routes, avoids unrealistic optimism, avoids excessive pessimism. Thus, CWGT provides a balanced transportation planning strategy under uncertainty.

VI. CONCLUSIONS

This research introduced a Confidence-Weighted Grey Transportation Method (CWGT) to address transportation issues characterized by interval grey costs. The proposed method involves uncertainty width through route-specific confidence weights, that's different from traditional grey transportation methods. This lets the method tell the difference between transportation routes that have a lot of uncertainty and those that

don't. The six numerical examples show that CWGT strikes a good balance between cost-effectiveness and reliability in the face of uncertainty. So, the suggested CWGT method is a dependable and useful tool for helping people make decisions about transportation planning when there is uncertainty, especially in logistics settings where transportation costs can change a lot. Subsequent research may expand this methodology to encompass multi-objective grey transportation challenges, extensive transportation networks, and a hybrid grey-fuzzy transportation model.

Availability of data and materials: The authors have confirmed that the data supporting the findings are available upon request from the corresponding author.

Ethical issues: Not applicable.

Conflicts of Interest: The authors of this paper assert that there are no conflicts of interest, regarding this study.

REFERENCES:

- [1] Monge, G. (1781). Mémoire sur la théorie des déblais et des remblais. *Mem. Math. Phys. Acad. Royale Sci.*, 666-704.
- [2] Kantorovich, L. V. (1942). On the translocation of masses. In *Dokl. Akad. Nauk. USSR (NS)* (Vol. 37, pp. 199-201).
- [3] Hitchcock, F. L. (1941). The distribution of a product from several sources to numerous localities. *Journal of mathematics and physics*, 20(1-4), 224-230.
- [4] Cosma, O., Pop, P. C., & Dănciulescu, D. (2020). A novel matheuristic approach for a two-stage transportation problem with fixed costs associated to the routes. *Computers & Operations Research*, 118, 104906.
- [5] Hossain, M. M. (2020). Improved Average Penalty Cost (IAPC) Method to Obtain Initial Basic Feasible Solution of Transportation Problem. *Glob J Sci Front Res*, 20(F8), 23-36.
- [6] Edokpia, R. O., & Amiolemhen, P. E. (2016). Transportation cost minimization of a manufacturing firm using genetic algorithm approach. *Nigerian Journal of Technology*, 35(4), 866-873.
- [7] Fulkerson, D. R. (1961). A network flow computation for project cost curve. *Management Science*, 7(2), 167-178.
- [8] Ghassemi Tari, F. (2016). A hybrid dynamic programming for solving fixed cost transportation with discounted mechanism. *Journal of Optimization*, 2016(1), 851-921.
- [9] Girmay, N., & Sharma, T. (2013). Balance an unbalanced transportation problem by a heuristic approach. *International Journal of Mathematics and its applications*, 1(1), 12-18.
- [10] Juman, Z. A. M. S., & Hoque, M. A. (2015). An efficient heuristic to obtain a better initial feasible solution to the transportation problem. *Applied Soft Computing*, 34, 813-826.
- [11] Karagul, K., & Sahin, Y. (2020). A novel approximation method to obtain initial basic feasible solution of transportation problem. *Journal of King Saud University-Engineering Sciences*, 32(3), 211-218.
- [12] Klingman, D., & Russell, R. (1975). Solving constrained transportation problems. *Operations Research*, 23(1), 91-106.
- [13] Mondal, R. N., Hossain, M. R., & Uddin, M. K. (2012). Solving transportation problem with mixed constraints. In *Proceedings of the 2012 international conference on industrial engineering and operations management Istanbul, Turkey*, 3,(6).
- [14] Pandian, P., & Natarajan, G. (2010). A new approach for solving transportation problems with mixed constraints. *Journal of Physical Sciences*, 14, 53-61
- [15] Rashid, F., Khan, A. R., & Uddin, M. S. (2021). Mixed constraints cost minimization transportation problem: an effective algorithmic approach. *American Journal of Operational Research*, 11(1), 1-7.
- [16] Damcı-Kurt, P., Dey, S. S., & Küçükyavuz, S. (2015). On the transportation problem with market choice. *Discrete Applied Mathematics*, 181, 54-77.
- [17] Maity, G., & Kumar Roy, S. (2016). Solving a multi-objective transportation problem with nonlinear cost and multi-choice demand. *International Journal of Management Science and Engineering Management*, 11(1), 62-70.
- [18] Christi, M. A., & Kalpana, I. (2016). Solutions of multi objective fuzzy transportation problems with non-linear membership functions. *International Journal of Engineering Research and Application*, 6(11), 52-57.
- [19] Yeola, M. C., & Jahav, V. A. (2016). Solving multi-objective transportation problem using fuzzy programming technique parallel method. *International Journal of Recent Scientific Research*, 7(1), 8455-8457.
- [20] Azad, S. M. A. K., Hossain, M. B., & Rahman, M. M. (2017). An algorithmic approach to solve transportation problems with the average total opportunity cost method. *International Journal of Scientific and Research Publications*, 7(2), 266-270.
- [21] Sharma, G., Abbas, S. H., & Gupta, V. K. (2012). Optimum solution of Transportation Problem with the help of Zero Point Method. *International Journal of Engineering Research & Technology*, 1(5), 1-5.
- [22] Gupta, A., & Kumar, A. (2012). A new method for solving linear multi-objective transportation problems with fuzzy parameters. *Applied Mathematical Modelling*, 36(4), 1421-1430.
- [23] Patel, J. G., & Dhodiya, J. M. (2017). Solving multi-objective interval transportation problem using grey situation decision-making theory based on grey numbers. *International Journal of Pure and Applied Mathematics*, 113(2), 219-233.
- [24] Kowalski, K., Lev, B., Shen, W., & Tu, Y. (2014). A fast and simple branching algorithm for solving small scale fixed-charge transportation problem. *Operations Research Perspectives*, 1(1), 1-5.
- [25] Murugesan, S., & Kumar, B. R. (2013). New optimal solution to fuzzy interval transportation problem. *International Journal of Engineering Science and Technology*, 3(1), 188-192.
- [26] Pandian, P., Natarajan, G., & Akilbasha, A. (2016). Fully rough integer interval transportation

problems. *International Journal of Pharmacy and Technology*, 8(2), 13866-13876.

- [27] Pandian, P., Natarajan, G., & Akilbasha, A. (2018). Fuzzy interval integer transportation problems. *International Journal of Pure and Applied Mathematics*, 119(9), 133-142.
- [28] Palanivel, M., & Suganya, M. (2018). A new method to solve transportation problem-Harmonic Mean approach. *Engineering Technology Open Access Journal*, 2(3), 1-3.
- [29] Roy, S. K., & Mahapatra, D. R. (2014). Solving solid transportation problem with multi-choice cost and stochastic supply and demand. *International Journal of Strategic Decision Sciences (IJSDS)*, 5(3), 1-26.
- [30] Porchelvi, R. S., & Anitha, M. (2018). On solving multi objective interval transportation problem using fuzzy programming technique. *International Journal of Pure and Applied Mathematics*, 118(6), 483-491.
- [31] Ju-Long, D. (1982). Control problems of grey systems. *Systems & control letters*, 1(5), 288-294.
- [32] Deng, J. (1988). Grey forecasting and decision-making. Huazhong University of Science and Technology Press.
- [33] Bai, G., Mao, J., & Lu, G. (2004). Grey transportation problem. *Kybernetes*, 33(2), 219-224.
- [34] Palancı, O., Alparslan Gök, S. Z., Olgun, M. O., & Weber, G. W. (2016). Transportation interval situations and related games. *Or Spectrum*, 38(1), 119-136.
- [35] Nasser, S. H., DARVISHI, S. D., & YAZDANI, C. A. (2017). A new approach for solving grey assignment problems. 15-28.
- [36] Pourofoghi, F., Saffar, A. J., & Darvishi, S. D. (2019). A new approach for finding an optimal solution for grey transportation problem. *Int. J. Nonlinear Anal. Appl.* 83-95.