A Comparison between Differential Evolution and Simulated Annealing for Order Reduction of Transformer Linear Section Model

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***Abstract*—In the present work, a comparative study between Differential Evolution (DE) and Simulated Annealing (SA) for order reduction of transformer linear section model has been carried out. DE is stochastic, population-based algorithm while SA is a local search algorithm, inspired by the process of physical annealing associated with solids. Reduced order model of transformer linear section is obtained such that reduced order model approximates the original higher order transformer model and maintains the stability of the original system. Both mentioned algorithms are based on the minimization of the integral square error (ISE) between the transient responses of original higher order and the reduced low order transformer models. Firstly the poles are determined by dominant pole retention technique and then the zeros of reduced order transformer model are obtained using DE and SA in order to minimize the ISE between high order and low order models of transformer. The transient response parameters have also been compared along with ISE.**

Keywords- Order reduction, DE, SA, Dominant pole, Integral Square Error.

I INTRODUCTION

Generally, physical systems are described by differential equation of higher order for analytical purpose. Model Order Reduction (MOR) is the field that converts large model to smaller model by mathematical approaches. The obtained reduced order model defines the original system behavior accurately without loss of any important information [1-2]. The exact analysis of these systems are monotonous, expensive and complicated [3]. So, Order reduction is motivated for simplifying, analyzing, synthesizing the systems. Also for reducing computational and hardware complexity of practical systems MOR is used. In this paper, DE and SA in association with dominant pole retention technique are compared for transformer model [2], [4] and [5].

Differential Evolution is used to optimize the real parameters, introduced by Storn and Price [6]. DE can be used to find approximate solutions to the problems having objective function non differentiable, non linear, noisy and have many local minima. DE is an Evolutionary algorithm and its main stages are initialization, mutation, recombination and selection. Initialization is the process of defining lower and upper limits of each parameter and then randomly selects initial values of parameters. Mutation is the process of obtaining donor vector from parent vector. Then recombining donor with parent vector, a trial vector is obtained through recombination process [7]. The trial vector is compared with parent vector for the selection of parent vector for next generation. The last three stages are conducted until some stopping condition met. The flow chart is shown in Fig. 1.

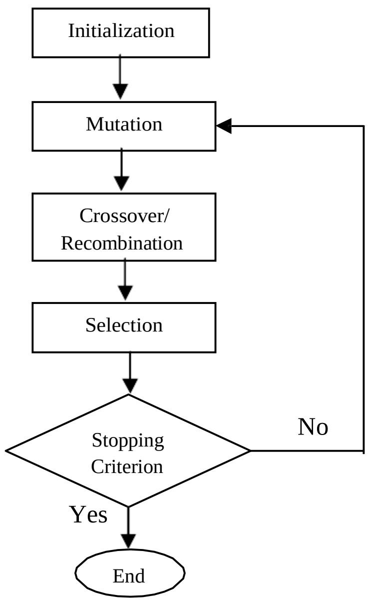

Fig. 1: Flow chart of DE algorithm.

Simulated Annealing is a local search method proposed by Kirkpatrick Gelatt and Vecchi in 1983. SA is a local search algorithm, inspired by the process of physical annealing associated with solids. Simulated annealing (SA) is a random-search technique which exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure (the annealing process) and the search for a minimum in a more general system [8]. In annealing process a crystalline solid is heated and then allowed to cool by decreasing the temperature of the environment in steps until it achieves its most regular possible crystal lattice configuration. The final configuration results in a solid with superior structural integrity if the cooling schedule is sufficiently slow. In SA a trial configuration is obtained by randomly generated perturbation of the current configuration of the solids. If the energy level of trial configuration is less than that of the current configuration, the trial configuration is accepted and becomes the current configuration. If the energy level of trial

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configuration is greater than or equal to that of the current configuration then the trial configuration is accepted as current configuration with probability proportional to $\exp(-\Delta E / T)$ where ΔE is difference in energy levels between trial configuration and current configuration [8]. The flow chart of SA [9-10] is shown in Fig. 2.

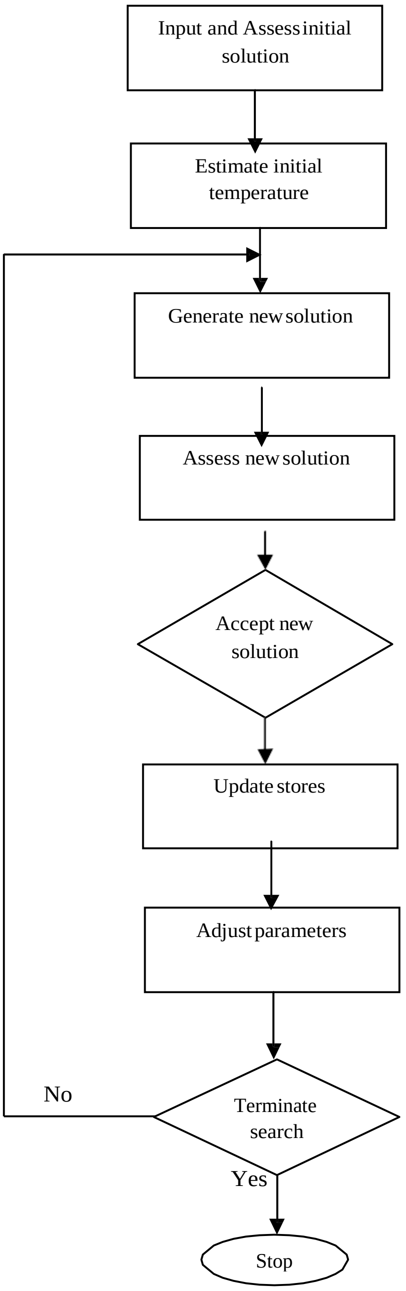

Fig. 2: Flow chart of SA algorithm.

Dominant pole retention technique has been used to determine denominator polynomial of reduced order model by taking dominant poles of higher order system. The location of the poles of a transfer function in the S-plane affects greatly the transient response of the system. So in approximation method the poles near to left half of s-plane are retained and other poles are discarded

For comparison of DE and SA transformer linear section model is taken as test system.

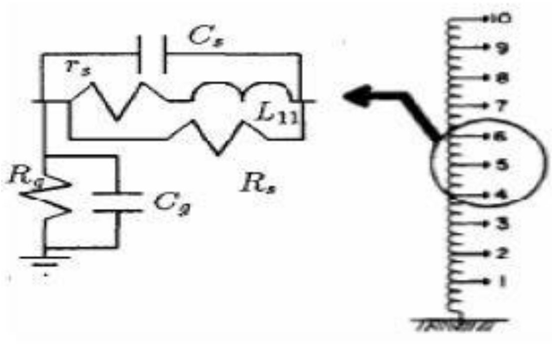

Fig. 3: Air core Transformer and its section.

Figure 3 shows the typical section of transformer model consisting a series resistance r_S , a shunt resistance R_S , a self inductance L_{11}, series capacitance C_S , parallel combination of a resistance R_g and a capacitance C_g with respect to ground. There is theory of mutual inductances between the sections of transformer model.

In this work, transformer linear section model (10 Sections) is used as assessment system having poles and zeros as mentioned below [2], [4] and [5]:

TABLE I: POLES AND ZEROS OF TRANSFORMER MODEL

Section	Poles	Zeros
1	- 3.06	-7.39
2	-10.37	-16.8
3	-19.84	-28.45
4	-31.33	-41.92
5	-44.51	-56.54
6	-58.69	-71.44
7	-72.94	-85.45
8	-86.39	-97.14
9	-97.59	-104.97
10	-105.09	---

In order to obtain reduced order transformer model, DE and SA along with dominant pole retention technique have been used.

Above mentioned techniques are simple and faster and there are few parameters to adjust.

II PROBLEM STATEMENT

Consider an n[th] order single input single output, linear time invariant system with the following transfer function:

$$G_n(s) = \frac{a_0 + a_1 s + a_2 s^2 + \ldots + a_n s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \ldots + b_n s^n} \quad (1)$$

Step 1: Calculate the dominant poles of high order model to determine the denominator polynomial of reduced order model.

The poles of the system to be:

$$-\lambda_1 < -\lambda_2 < \ldots . < -\lambda_n . \quad (2)$$

The denominator of $G_r(s)$ is obtained such that the poles of the low order system are the dominant poles of the high order model as follows:

$$-\lambda_1 < -\lambda_2 < \ldots . < -\lambda_r . \quad (3)$$

So, reduced order system's denominator is determined as:

$$(s+\lambda_1)(s+\lambda_2)\ldots.(s+\lambda_r) \quad (4)$$

Let obtained denominator is:

$$D_r(s) = \beta_0 + \beta_1 s + \beta_2 s^2 + \ldots + \beta_r s^r \quad (5)$$

Step 2: Obtain numerator of reduced order system using DE and SA, respectively by minimizing the ISE between transient responses of high order and low order models of transformer linear section model.

The Integral Square Error is given by [5] and [12]:

$$ISE = \int_0^{\infty} [y(t) - y_r(t)]^2 \, dt \quad (6)$$

So, obtain numerator by DE and SA, respectively. Let the numerator is:

$$N_r(s) = \alpha_0 + \alpha_1 s + \alpha_2 s^2 + \ldots + \alpha_r s^{r-1} \quad (7)$$

So, the reduced order model obtained is given as:

$$G_r(s) = \frac{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \ldots + \alpha_r s^{r-1}}{\beta_0 + \beta_1 s + \beta_2 s^2 + \ldots + \beta_r s^r} \quad (8)$$

III ORDER REDUCTION OF TRANSFORMER LINEAR SECTION MODEL

Consider a 10[th] order system described by transfer function having poles and zeros shown in table I:

$$G_{10}(s) = \frac{s^9 + 510.1s^8 + 1.106e5s^7 + 1.33e7s^6 + 9.690e8s^5 + 4.393e10s^4 + 1.223e12s^3 + 1.980e13s^2 + 1.652e14s + 5.211e14}{s^{10} + 529.81s^9 + 1.202e5s^8 + 1.527e7s^7 + 1.191e9s^6 + 5.892e10s^5 + 1.842e12s^4 + 3.513e13s^3 + 3.784e14s^2 + 1.965e15s + 3.330e15} \quad (9)$$

Dominant poles of above system are:

$\lambda_1 = -3.06,\ \lambda_2 = -10.37.$

Therefore, the denominator polynomial will be:

$$D(s) = s^2 + 13.43s + 31.7322 \quad (10)$$

The numerator polynomial of $G_2(s)$ is obtained by DE as:

$$N_2(s) = 0.7450s + 4.9479 \quad (11)$$

So, reduced order model by DE is:

$$G_2(s) = \frac{0.7450s + 4.9479}{s^2 + 13.43s + 31.7322} \quad (12)$$

with an ISE $= 1.4638\text{x}10^{-6}$.

The numerator polynomial of $G_2(s)$ is obtained by SA as:

$$N_2(s) = 0.7657s + 4.9786 \quad (13)$$

So, reduced order model by SA is:

$$G_2(s) = \frac{0.7657s + 4.9786}{s^2 + 13.43s + 31.7322} \quad (14)$$

with an ISE $= 2.1800\text{x}10^{-6}$.

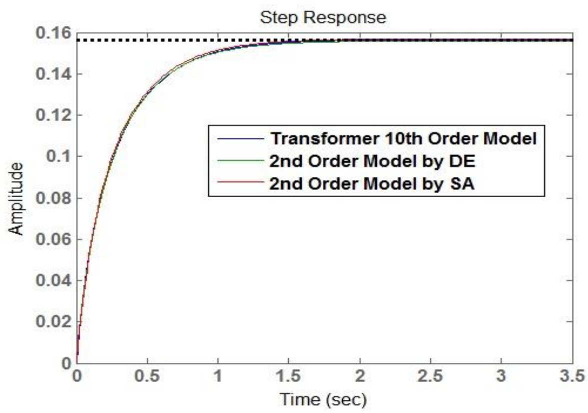

Fig. 4: Step response of high order and low order transformer models.

The step responses of high order and low order models of transformer linear section by DE and SA are shown in Fig. 4, which show that the transformer low order models are good approximation of the original high order transformer model, keeping ISE minimum.

Also, the frequency responses of high order and low order models of transformer by DE and SA are shown in Fig. 5, which are also comparable.

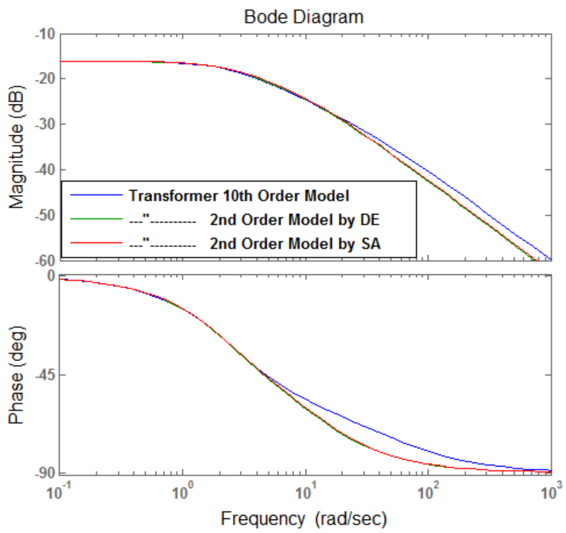

Fig. 5: Frequency response of high order and low order transformer models.

A comparison of transient response parameters of high order and low order transformer models is given in Table II, which shows that both reduced order model's parameters are identical to high order transformer model.

TABLE II: TRANSIENT RESPONSE PARAMETERS

Models	Rise time (sec.)	Settling time(sec.)	Steady state
Transformer 10th Order model	0.655	1.2	0.157
Transformer 2nd Order model by DE	0.664	1.19	0.156
Transformer 2nd Order model by SA	0.66	1.185	0.156

A comparison of ISE obtained from the above techniques has been given in Table III.

TABLE III: COMPARISON OF ISE

Algorithm	Reduced Model	ISE
DE	$\frac{0.7450s + 4.9479}{s^2 + 13.43s + 31.7322}$	$1.4638\text{x}10^{-6}$
SA	$\frac{0.7657s + 4.9786}{s^2 + 13.43s + 31.7322}$	$2.1800\text{x}10^{-6}$

IV CONCLUSIONS

In the present work, two techniques DE and SA in association with dominant pole retention technique have been compared for the order reduction of transformer linear section model. The ISE has been calculated between high and low order transformer model and found to be low for DE as compared with SA in combination with dominant pole retention technique. Both low order transformer models are good approximation of high order transformer model. Reduced order models preserve the uniqueness of original high order transformer model and also the transient specifications are found to be comparable. Hence finally it can be concluded that combination of DE with dominant pole retention technique is more accurate than SA with dominant pole retention technique. The algorithms have been implemented in MATLAB 7.11.0 and the computational time taken is about 4 seconds for both the algorithms.

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