Abstract—This paper presents a comparison of Wavelet Transform technique and Fast Fourier Transform technique in the estimation of power system harmonics and interharmonics. Fourier analysis converts a signal in time domain to frequency domain. Fast Fourier Transformation performs the same conversion but with a faster rate. Now-a-days, Wavelet Transformation is one of the most popular candidates of time-frequency transformation. Because Wavelet Transformation can provide time & frequency information simultaneously and it is suitable for the analysis of non-stationary signal. To investigate these methods, a number of studies have been performed using simulated signals. The analysis of the voltage waveform of a 2 level PWM converter supplying an Induction motor has been investigated employing these two methods with the same sampling period.

Index Terms— CWT, DFT, FFT, Harmonics , Interharmonics

I. INTRODUCTION

An ideal power system is defined as the system where a perfect sinusoidal voltage signal is seen at load-ends. In reality, however, such idealism is hard to maintain [2]. It is because the widespread applications of electronically controlled loads have increased the harmonic distortion in power system voltage and current waveforms. As power semiconductors are switched on and off at different points on the voltage waveform, damped high-frequency transients are generated. If the switching occurs at the same points on each cycle, the transient becomes periodic [1]. Harmonic frequencies in the power grid are a frequent cause of power quality problems. Harmonics in power systems result in increased heating in the equipment and conductors, misfiring in variable speed drives, and torque pulsations in motors. So, estimation and reduction of harmonics is very important. Many algorithms have been proposed for the evaluation of harmonics. The design of harmonic filters relies on the measurement of harmonic distortion [1]. Harmonics state estimation (HSE) techniques have been used since 1989 for harmonics analysis in power systems. Many mathematical methods have been developed over the years. It is proved that by using only partial or selected measurement data, the harmonic distortion of the actual power system can be obtained effectively. In this paper, the performances of Fast Fourier Transform (FFT) and Wavelet Transform (WT) technique have been compared in estimating power system harmonics.

FFT is an algorithm for calculation of the Discrete Fourier Transform (DFT). Usually, a Power spectra indicates the frequencies containing the power of the signal. The frequencies can be estimated by distributing the value of the power as a function of frequency, where power is considered as the average of the square of the signal. In the frequency domain, this is equivalent to the square of FFT’s magnitude. It is suitable for stationary signals only.

In WT method, Continuous Wavelet Transform(CWT) is applied to the signal. The Morlet wavelet is applied as the mother wavelet to estimate the frequencies of the signal. It is suitable for the analysis of non-stationary signal.

The principles of these two methods are explained in section II and III, the experimental results are given in section IV and V and conclusion is given in the final section.

II. FAST FOURIER TRANSFORM THEORY

Let \(x_0, x_1, \ldots, x_{N-1}\) be a vector of complex numbers. The DFT is defined by the equation-

\[
X_k = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{-j2\pi kn/N}, \quad k=0,1,\ldots,N-1
\]

(1)
Evaluation by this definition, directly requires $O(N^2)$ operations as there are $N$ outputs of $X_k$, and each output requires a sum of $N$ no. of terms. An FFT is a method to compute the same results in $O(N \log N)$ operations. To estimate the frequencies, the periodogram is obtained. The periodogram computes the power spectra of the entire input signal, i.e.

$$\text{Periodogram} = \frac{|F(\text{signal})|^2}{N} \quad (2)$$

Where $F(\text{signal})$ is the fourier transform of the signal and $N$ is the normalization factor. The spectrum power is maximum at the frequencies present in the signal.

### III. WAVELET TRANSFORM THEORY

In this approach the signal is subjected Continuous Wavelet Transform to estimate the harmonics and interharmonics.

#### A. Continuous Wavelet Transform

The CWT of a continuous, square-integrable function $x(t)$ at scale $a > 0$ and translational value $b \in \mathbb{R}$ is expressed by the following integral-

$$\text{CWT}(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t) \psi^* \left( \frac{t-b}{a} \right) dt \quad (3)$$

Where $\frac{1}{\sqrt{|a|}}$ is the normalization factor, $\psi(t)$ is called mother wavelet which is a continuous function both in time domain and frequency domain. The main purpose of the mother wavelet is to provide a source function to generate the daughter wavelets which are simply the translated and scaled version of mother wavelet.

#### B. Harmonics and Interharmonics Estimation

To estimate the harmonics and interharmonics, CWT is applied to the signal. The Morlet wavelet is selected to be the mother wavelet. It is defined in time domain as follows [3]:

$$\psi(t) = \exp(j \omega_0 t - 0.5 t^2) \quad (4)$$

Where $\omega_0 = 2\pi f_{\omega_0}$; $f_{\omega_0}$ is frequency of Morlet wavelet. The relationship between scale and frequency in CWT is given by:

$$f_a = \frac{f_{\omega_0}}{a} \Delta \quad (5)$$

where $a=\text{scale}$, $f_a=\text{frequency corresponding to the scale } a$, $\Delta=\text{sampling period}$. The table showing the scales and their corresponding frequencies is first determined and then the scalograms are obtained for the signal at different scales for the estimation. The maximum energy points represent the scales corresponding to the frequencies present in the signals. Order of harmonics and interharmonics can be found from the following expression as [3]:

$$\text{order of harmonics} = \frac{\text{Harmonic frequency}}{\text{system frequency}} \quad (6)$$

### IV. Experiments with Simulated Waveform

The first signal considered is given by

$$x(t) = 100\cos(2\pi 40t) + 50\cos(2\pi 217t) + 40\cos(2\pi 760t) + k_\text{e}(t) \quad (7)$$

where $e(t)$ is a white Gaussian noise of zero mean and variance equal to 1. The signal to noise ratio (SNR) is 10. To investigate the methods, several experiments have been performed with the waveform described by (7). Sampling frequency is taken as 2000 Hz for both the methods.

#### A. FFT

FFT estimates the frequencies present in (7) as shown in the following figures.
The CWT is applied to the signal with Morlet as the mother wavelet and with the sampling frequency of 2000 Hz.

**B. Wavelet Transform**

The CWT is applied to the signal with Morlet as the mother wavelet and with the sampling frequency of 2000 Hz.
V. SIMULATION OF A FREQUENCY CONVERTER

A PWM converter with modulation frequency of 1080 Hz supplying a 4 pole, 3 hp asynchronous motor (U=220 V) is simulated in simulink. The simulated converter has a modulation index of 0.92. The output voltage waveform of the converter corrupted with noise having zero mean value & unity variance is taken for analysis. Fig. 9 shows the noise corrupted voltage waveform at the converter output for the frequency 60 Hz and estimation of harmonics for this signal using FFT.

A. FFT

The simulated voltage signal and its FFT estimation are shown in Fig.9. The signal is sampled with frequency 6400 Hz. The first two cycles (in red) of the waveform are considered for this estimation. FFT estimates the major frequencies as 60, 960, 1200, 1770, 2100, 2220, 2370, 2910, 3000, 3030 and 3120 Hz.

B. Wavelet Transformation

Continuous Wavelet Transform is applied to the voltage signal with sampling frequency 6400 Hz. Again, Morlet wavelet is considered as mother wavelet. The first 512 samples are taken for this analysis.

Some of the major frequencies estimated by CWT are shown below.
The estimation of Harmonics and Interharmonics present in a power system has been investigated using FFT and Continuous Wavelet Transform for different test signals with same sampling period. It is observed that Continuous Wavelet Transform provides more accurate results compared to FFT. The analysis indicates that Continuous Wavelet Transform is a powerful tool for analyzing non-stationary signals and identifying transient phenomena in electrical systems.
transform is not as accurate as FFT in estimating frequencies in case of stationary signal. Distinct Estimation of higher order frequencies is difficult with Continuous Wavelet Transform because the frequency decreases exponentially with scale in case of CWT as shown in the figures (6) and (11). In general, the techniques best suited for estimation of frequency of stationary signal is based on FFT. However, wavelets, though not specifically dedicated to this type of analysis, can recover some of the spectral information. In case of non-stationary signal, Wavelet analysis can estimate the frequencies as well as instants of occurrence of the frequencies.

REFERENCES


Wavelet Transform estimates the major frequencies as- 59.77, 1040, 2031.3, 2071.7, 2131.5 , 2241.4, 2872.9, 3076.9, 3322.7 Hz.