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A Comparative Study of Methods to Find Natural Frequencies of 2DoF Systems

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Abstract*:*A number of dynamic systems can be modeled as two degree of freedom systems. The present paper deals with a comparative study of dynamic analysis of such systems,mathematically, by numerical methods (FDM and FEM) and by experimentation. The aim is to investigate the error inprediction of natural frequencies of a typical system by computational methods. It is found that the predictions can be done within an error of 10%.

Keywords—*vibrations,twodegree of freedom systems, numerical methods, FEM,natural frequencies*

I. INTRODUCTION

Natural frequencies of a dynamic system are critical parameters of the system. To avoid resonance, it is necessary to keep the excitation frequencies at a distance from the natural frequencies[1]. Natural frequencies can be calculated by a number of techniques. However, each technique may have some errors in prediction. In this paper we have investigated the errors occurring while using various such techniques. Knowing such errors may prove to be of help, to the designer of the system, for an initial design.

The system considered in this paper, is a two degree of freedom(2DoF) system (two masses and two springs) with displacement in only one direction. Many real life systems can be modeled as two masses and two spring systems. e.g. asystem comprising of an automobile engine - suspension – chassis – tires, a marine engine mounted on double stage mounting[4](Fig.1), a hammer press with a foundation with an inertia block (Fig.2) and so on. Calculation of the natural frequencies of the concerned system is discussed further.

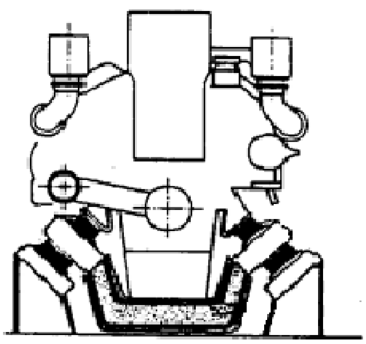


Fig. 1.Double Stage Marine Engine Foundation.

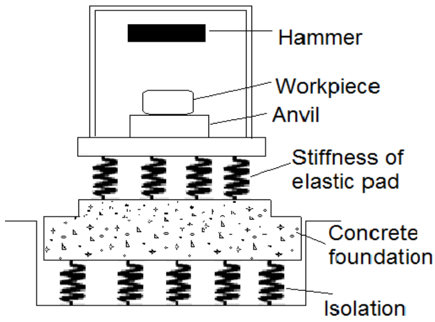


Fig 2. Two Stage Foundation for a Hammer Press.

A number of trials have been carried out for calculating the natural frequencies by changing the parameters of the system viz. two masses and stiffness of the springs. The results are then compared.

II. METHODS OF CALCULATING NATURAL FREQUENCIES OF THE SYSTEM

The common methods to find the natural frequencies of the system in use are:

*1. Mathematical Modeling:*The system can be modeled mathematically and its equations of motion can be achieved. Further the characteristic equation can be found and solved. The roots of the characteristic equation can be used to get the natural frequencies of the system.

2. Modal Analysis:

a. A program can be written in MATLAB or similar software, to find the Eigen values and Eigen vectors of the system. The mass matrix and stiffness matrix are to be provided as inputs. The roots of the Eigen values give the natural frequencies[1].

b.A modal analysis can be performed in any FEM software to extract the mode shapes and the modal frequencies. These frequencies are the natural frequencies. This analysis is similar to the Eigen value and Eigen Vector calculation.

3. Frequency Response Function:

a.Asimulation of the system can be performed in MATLAB SIMULINK as shown in Fig. 4. The input is excitation force with varying frequency i.e. a linear up-chirp signal. The output is plotted in frequency domain. The highest values of amplitudes of the plotrepresent the natural frequencies. Such a plot is known as the Frequency Response Function (FRF) of the system.

b. Harmonic Analysis can also be performed in software like ANSYS, ABAQUS, where a sinusoidal exciting force($\boldsymbol{F_0}$sinωt)is given to the system with varying frequency. The output is plotted in frequency domain. The plot is the FRF of the system. The highest values of amplitudes of the plotrepresent the natural frequencies.

4. ExperimentalAnalysis:

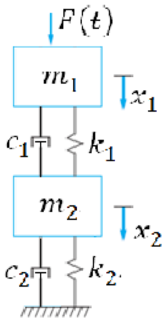
The FRF of the system can be obtained by impact testing the system, where in the system is given an impact force and the response of the system is sensed through an accelerometer[3].The energy in an impact force is distributed over the frequency domain[2].The response signal is processed in an FFT analyzer which gives the FRF of the system. The peaks in the FRF occur at the natural frequencies.

In the present paper, natural frequencies of a 2DOF system were achieved by the above methods, and the results are compared. Numbers of trials were taken by changing the system parameters to check repeatability.

III. COMPUTATIONAL METHODS

1. Modeling mathematically:

The system in consideration has two masses and two sets of springs. The equations of motion are derived using the Newton's method (free body diagram). The system and the free body diagrams are as follows (Fig. 3a and 3b):



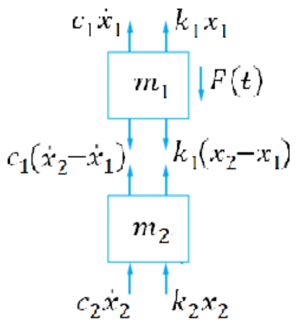


Fig: 3a **Fig. 3b**

Applying Newton's second law to mass $\boldsymbol{m_1}$& $\boldsymbol{m_2}$:

$$-k_1x_1 - c_1\dot{x}_1 + k_1(x_2 - x_1) + c_1(\dot{x}_2 - \dot{x}_1) + F(t) = m_1\ddot{x}_1$$

$$-k_2x_2 - c_2\dot{x}_2 - k_1(x_2 - x_1) - c_1(\dot{x}_2 - \dot{x}_1) = m_2\ddot{x}_2$$

Considering damping and external exciting force to be zero, we get the characteristic equation as follows:

$$m_1m_2\omega^4 - m_1k_1\omega^2 - m_2k_1\omega^2 - m_1k_2\omega^2 + k_1k_2 = 0$$

The roots of this equation give the values of the natural frequencies of the system[1].As, the system is a two mass two spring system, damping is taken as zero.

2. Modeling in MATLAB:

a. In MATLAB, to find the Eigen Values and Eigen Vectors, a program was written. The roots of the Eigen Values were taken as the Natural Frequencies and considered for comparison.

b.A model was created in SIMULINK (Fig. 4) where, the differential equations of the system were modeled such that a linear up-chirp force signalcould be given as an input and various outputs could be analyzed. This model was used to plot the frequency response of the system. The chirp signalgiven was over the frequency range, 0 Hz to 100 Hz. As the input was time based, the output signal was processed using the FFT block and a plot in Frequency domain was obtained. The amplitude of the plot is not in logarithmic scale.In Fig. 5 the plot is for the case,Mass_1=0.75kg, Mass_2=1kg, Stiffness_1=5493N/m, Stiffness_2=7848N/m.

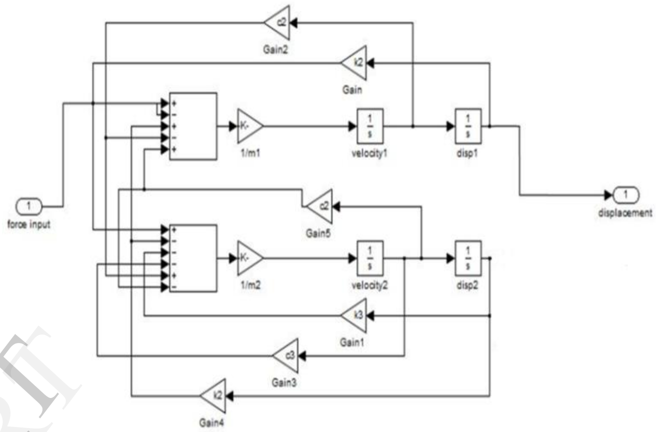


Fig. 4. System in Simulink

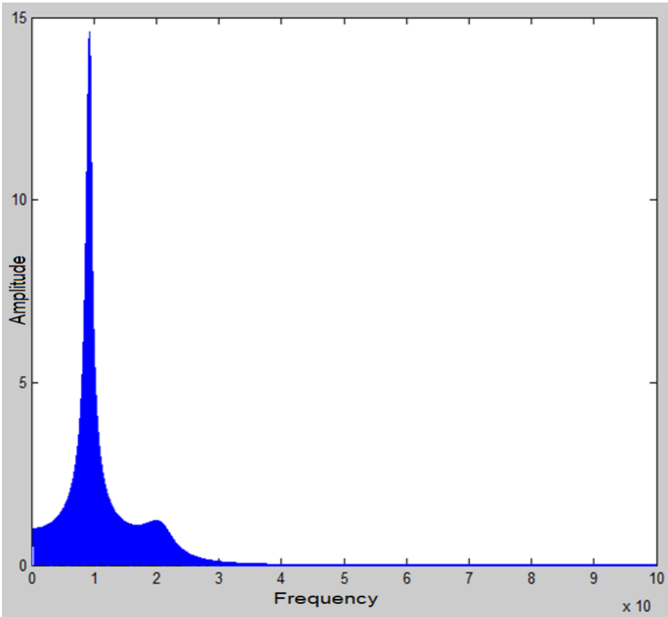


Fig. 5. FRF in Simulink

3. Modeling in ANSYS:

a. The model in ANSYS was created using discrete elements. The elements used were MASS21, a 3D mass element and COMBIN21, a Spring-Damper Element. First, the modal analysis option was selected to extract the Mode Shapes. The frequencies of the mode shapes were considered for comparison.

b. Next the Harmonic Analysis option was used and the frequency range from 0 Hz to 100 Hz was selected. The output is an FRF. The amplitude of the plot is in logarithmic scale

The summary of the analyses done above is discussed in the results section.

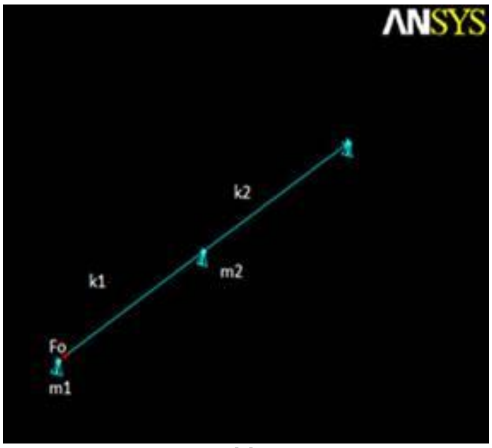


Fig. 6. Model in ANSYS

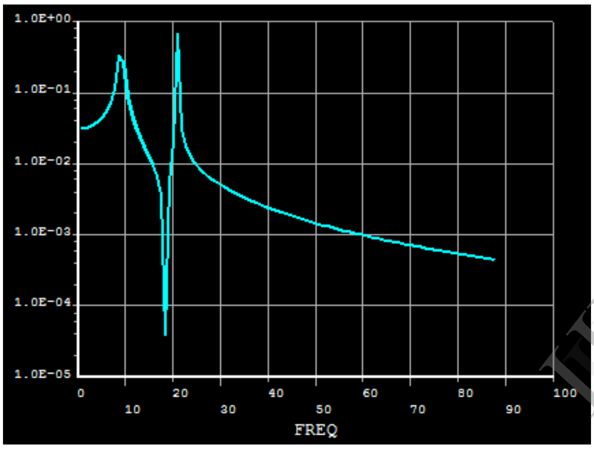


Fig. 7.FRF in Ansys as an output of harmonic analysis

The results of Modal Analysis in ANSYSand roots of Eigen vectors in MATLAB were observed to be the same. Hence, only one of the results is used for comparison for every case. Similar is the case for FRF obtained from ANSYS and SIMULINK.

IV.EXPERIMENTATION

The experimental model used is as shown in Fig 8. A combination of masses and springs was used to create the system. The masses were 0.5 kg, 0.75 kg, 1 kg, 1.25 kg, 1.5 kg, 1.75 kg, 2 kg and 2.25 kg. One stage of spring was created using four compression springs in parallel. Four such sets of springs having stiffness 5493 N/m, 7848 N/m, 11772 N/m, 14715 N/m were used. The combinations of masses and springs were selected arbitrarily for every reading.

The FFT analyzer used was Svan 958. The accelerometer was mounted on the top mass as shown in the Figure. The output of the FFT was viewed in the SvanPC++ software. A Frequency Response Curve was achieved for different combinations of the set up by using an impact hammer. The peaks of the frequency response curve were treated as natural frequencies of the system.

Fig. 8. Experimental setup

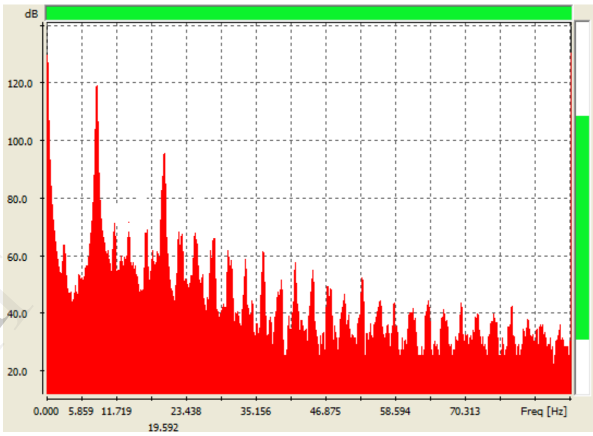


Fig. 9 FRF from FFT Analyzer (experimental result)

V. RESULTS AND DISCUSSION

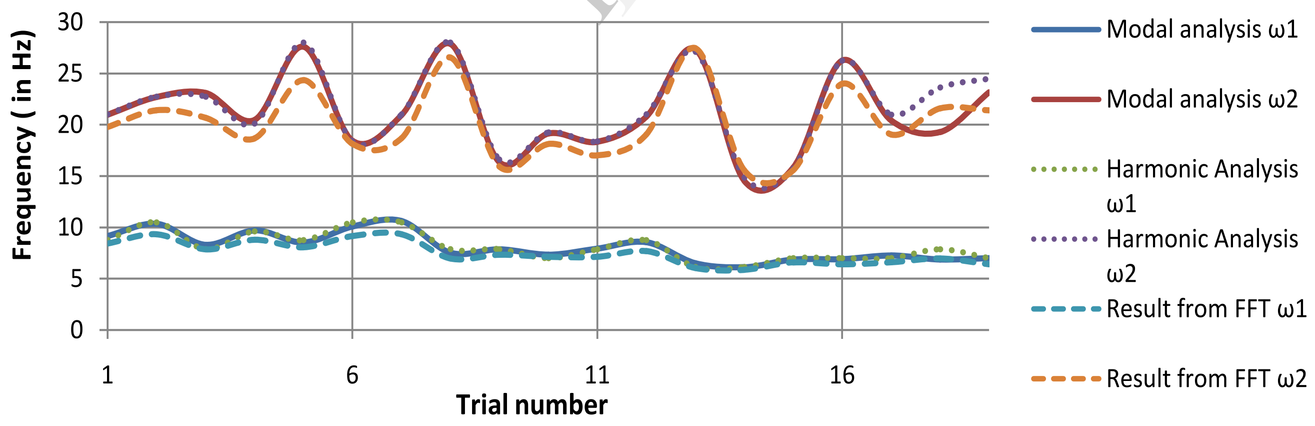
The Result Table and Graphs are shown below. Graph 1 shows the relation between the outputs from various methods. Graph 2 shows the percentage errors from the same. The comparative study is based on these graphs. Here, the errors are calculated on the basis of experimental results.

The results from the computational modal analysis and computational harmonic analysismatch within a percentage error of 1%. These errors may be discretization errors and round off errors, introduced while performing the harmonic analysis computationally. Reducing this error will increase the computation cost[5].

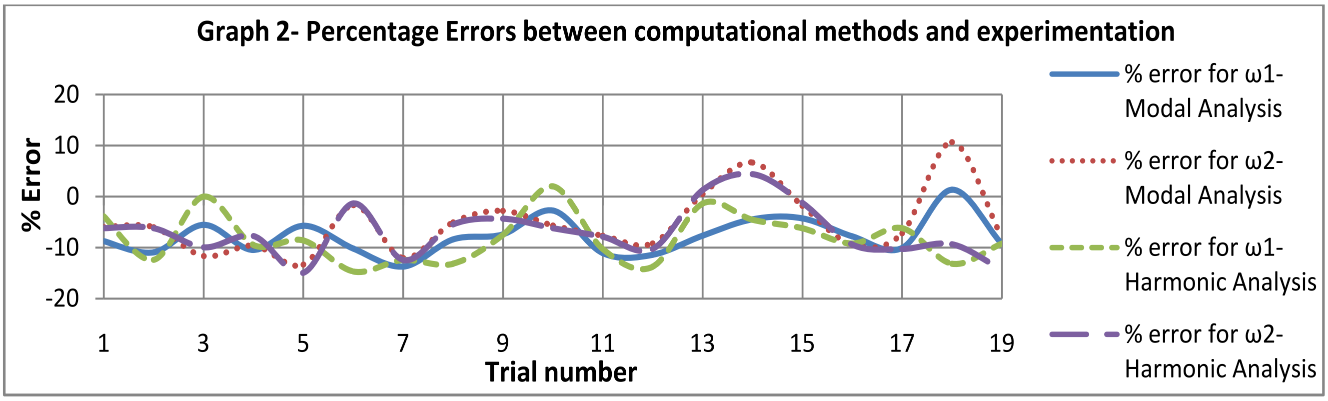
The average percentage error between the experimental results and the computational results is around 8 %, whereas the maximum error between them is 15%.These are numerical errors and there may bea number of reasons for these errors. Firstly, any real dynamic system has infinite degrees of freedom. However, to model the system computationally, it is necessary to approximate the system to have finite degrees of freedom, two in this case.Secondly, the springs were assumed to be massless and masses were assumed to be point masses.Also, their centers of gravity were assumed to move in the same vertical axis of motion. In addition, damping due to air resistance and structural damping in masses were assumed to be zero while modeling the system computationally. These assumptions made for the sake of computational modeling introduce the numerical errors.

VI. RESULT TABLE

Trial number	M1	M2	K1	K2	Computational Methods				Experimentation		% error for ω1-Modal Analysis	% error for ω2-Modal Analysis	% error for ω1-Harmonic Analysis	% error for ω2-Harmonic Analysis
					Modal analysis		Harmonic Analysis		Result from FFT					
	kg	kg	N/m	N/m	ω1	ω2	ω1	ω2	ω1	ω2				
1	0.75	1	5493	7848	9.1599	20.9654	8.75	21	8.423	19.775	-8.749	-6.020	-3.882	-6.195
2	0.75	1	5493	11772	10.3587	22.7056	10.5	22.75	9.338	21.423	-10.931	-5.987	-12.444	-6.194
3	0.75	1	7848	5493	8.3111	23.1066	7.875	22.75	7.874	20.691	-5.551	-11.675	-0.013	-9.951
4	0.75	1	7848	11772	9.7082	20.4768	9.625	20.125	8.789	18.677	-10.459	-9.636	-9.512	-7.753
5	0.75	1	11772	5493	8.5185	27.6106	8.75	28	8.057	24.353	-5.728	-13.377	-8.601	-14.976
6	0.75	2	5493	14715	10.0891	18.43	10.5	18.375	9.155	18.127	-10.203	-1.672	-14.691	-1.368
7	0.75	2	7848	14715	10.6192	20.9297	10.5	21	9.338	18.677	-13.720	-12.061	-12.444	-12.438
8	1.25	0.5	7848	5493	7.5421	27.8929	7.875	28	6.958	26.55	-8.395	-5.058	-13.179	-5.461
9	1.25	2	5493	11772	7.8683	16.3727	7.875	16.625	7.324	15.93	-7.432	-2.779	-7.523	-4.363
10	1.75	1.5	5493	14715	7.338	19.1549	7	19.25	7.141	18.127	-2.759	-5.671	1.975	-6.195
11	1.75	2	7848	14715	7.9326	18.3421	7.875	18.375	7.141	17.029	-11.085	-7.711	-10.279	-7.904
12	1.75	2	11772	14715	8.5696	20.7946	8.75	21	7.69	19.043	-11.438	-9.198	-13.784	-10.277
13	1.75	0.5	7848	5493	6.5045	27.334	6.125	27.125	6.042	27.466	-7.655	0.481	-1.374	1.242
14	1.75	2	5493	7848	6.1196	14.5267	6.125	14.875	5.859	15.564	-4.448	6.665	-4.540	4.427
15	1.75	2	5493	11772	6.8732	15.8408	7	15.75	6.592	15.564	-4.266	-1.778	-6.189	-1.195
16	2.25	1	14715	7848	6.9114	26.2568	7	26.25	6.409	23.987	-7.839	-9.463	-9.221	-9.434
17	2.25	1.5	7848	14715	7.2587	20.4128	7	21	6.592	19.043	-10.114	-7.193	-6.189	-10.277
18	2.25	1.5	7848	11772	6.8664	19.3009	7.875	23.625	6.958	21.606	1.316	10.669	-13.179	-9.345
19	2.25	1	7848	11772	7.0096	23.156	7	24.5	6.409	21.423	-9.371	-8.089	-9.221	-14.363



Graph 1- Comparison of Natural frequencies achieved by different methods



VII. CONCLUSION

It can be concluded that the computational methods used for the two degree freedom system can be relied upon for a prima face prediction of natural frequencies within an expected error of 10%. This would help the designer of such systems to use these methods for an initial guess upto an accuracy of 90%. Thus, the cost of prototype manufacturing and testing would be saved in such cases. A further increase in accuracy will increase the cost of computation.

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