A Comparative Review Between Bus Building Algorithms for Impedance Bus Matrix

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***Abstract*—This paper will present different proposed algorithms to build the impedance matrix and optimization method to reduce computation time by using genetic algorithm. Then the advantages and disadvantages of the algorithms are discussed.**

Index: Z-bus, Building algorithm, Optimization for Z-bus, power system.

I. INTRODUCTION

Power systems/networks is the backbone of power distribution to houses, industry and services. This rises the priority of a way to analyze the network for load flow or fault analysis. These two analyses require fast computation for the rapid increase in the network, hence the need of a better algorithm to reach the solution of the network. Impedance matrix or admittance matrix are the main ways to do these analyses. The two matrices are inverse of each other, so building one of them then find inverse is enough to have both. The impedance matrix provides more information and needed for the short circuit and unsymmetrical fault analysis. The zero, positive and negative sequence are using the Z-bus matrix. A building algorithm or an optimization to current algorithm to build the Impedance matrix is needed to reduce the time to finish the analysis.

This paper investigates the different algorithms and optimizations done to reduce time to build the Z-bus matrix and highlight the advantages and disadvantages.

The paper is structured as follows: In Section II, literature review on work about Z-bus matrix. Section III states the theory and equations of the Z-bus matrix. Section IV presents two building algorithms and an optimization method using genetic algorithm to reduce the time needed to find Z-bus. Section V gives general observations on each algorithm and optimization. Finally, Section VI provides conclusion.

II. LITERATURE REVIEW

In the literature there is many works and investigation made to find the best way to reach the Z-bus matrix and its applications in the power flow field. One method to build Z-matrix is proposed by [1]-[3]. Where they looked at the Tree-Set and Link-Set of a network to develop the branch current vs injection current of the bus matrix and the branch current vs bus voltage matrix (**BCIC)** and (**VBC)**, respectively. Then the two matrices are multiplied and Kron reduction is applied to the multiplication result to obtain Z-matrix. The work of [2] and [3] is improved by [4], which revised the calculation if there was a Current Control Voltage Source (**CCVS**) on one or more nodes in the system. This made the method more robust and general to be used even in the presence of **CCVS**. [5] proposed a way to integrate Genetic Algorithms (**GA**s) in finding the Z-bus matrix. This introduction of **GA** is to be able to find the best path in a system to minimize the calculation of Z-bus matrix. [5] tested on a 14-bus matrix to show the impact. Also, provided a theorem in reducing the number of calculation (time needed to find Z-bus). [6] investigated the impedance matrix required computation for phase transposition for short transmission lines. [6] showed multiple study cases of each of approximated Z-bus and Kron reduced impedance matrix, respectively. The results of the case studies shows that the ground resistivity has a significate effect on approximation error. The work of [7] consists of programmable strategy (code) that takes an input file describing the system layout and its values. Then the code calculates the Z-bus by the general simple building Algorithm, afterwards calculates Y-bus by inverse of Z-bus. Also, [6] and [7] highlighted that finding the Z-bus is very important to calculate Y-bus.

III. CONCEPT OF Z-BUS MATRIX

The Z-matrix is the relation between a given bus injection current and the bus voltage given as (1):

$$[Z_{matrix}] = [V_{bus}]/[I_{bus}] \qquad (1)$$

Where the diagonal and off-diagonal elements of the matrix are as follows in (2) and (3), respectively:

$$Z_{ii} = \frac{V_i}{I_i} \qquad (2)$$

$$Z_{ij} = \frac{V_i}{I_j} \qquad i \neq j \qquad (3)$$

IV. Z-BUS BUILDING ALGORITHMS AND OPTIMIZATIONS

This section shows the different algorithms and methods to obtain or reduce computation of Z-bus:

A. *Direct Building Algorithm*

The direct method is a sequential and recursive process where the Z-bus matrix building follows:

$$Z_{bus}^{new} = \begin{bmatrix} Z_{bus}^{old} & col, Z_{bus}^{old} \\ row, Z_{bus}^{old} & (Z_{bus}^{old})_{nn} + Z \end{bmatrix} \qquad (4)$$

Where this algorithm divides the addition of new values into four cases as shown in [8],

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Case 1: Addition of a line between the reference and a new bus

Case 2: Addition of a transmission line between an existing bus and a new bus

Case 3: Addition of a transmission line connecting two existing buses

Case 4: Addition of a line connecting the reference to an existing bus

Cases 3 and 4 are a loop closure since the connect to an existing bus, in these cases an intermediate matrix is introduced

$$Z^{inte} = \begin{bmatrix} Z_{bus}^{old} & col_i, Z_{bus}^{old} - col_j, Z_{bus}^{old} \\ row_i, Z_{bus}^{old} - row_j, Z_{bus}^{old} & (Z_{bus}^{old})_{ii} + (Z_{bus}^{old})_{jj} - 2(Z_{bus}^{old})_{ij} + Z \end{bmatrix} \quad (5)$$

Then Z_{bus}^{new} is found from Z^{inte} by applying bus elimination method.

B. *Tree-set and Link-set algorithm*

This algorithm proposed by [1]-[3] uses the tree set (**TS**) and the corresponding links (**LS**) of the system to obtain two matrices **BCIC** and **VBC**, respectively. Then develop the bus matrix from these two matrices. First **TS** and **LS** contains buses' currents from [Case1, Case2] and buses' currents from [Case3, Case4], respectively. A 6-bus system is shown in **Fig.1**

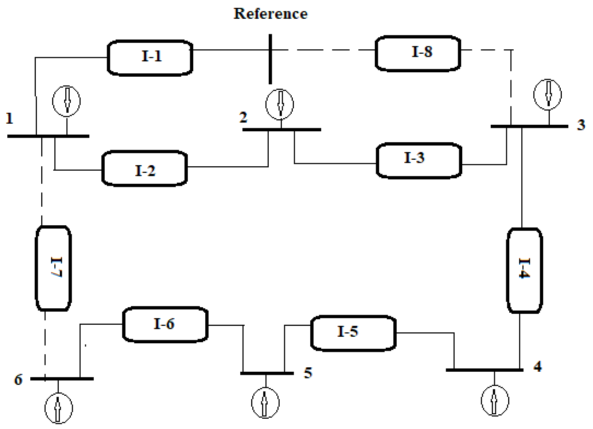

Fig 1 A 6-Bus example used for the algorithm

From **Fig.1**, **TS** and **LS** would be as follows:

$$TS = [I1, I2, I3, I4, I5, I6] \quad (6)$$

$$LS = [I7, I8] \quad (7)$$

The relation between the line current and the current injected to a bus is expressed as follows:

$$I_n = -\sum_{m=n}^{6} Ic_m \quad (m = 1,2,3,4,5,6) \quad (8)$$

Where I_n is the line current and Ic_m is the injected current. The matrix describing the relation in (8) is shown in (9):

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Ic_1 \\ Ic_2 \\ Ic_3 \\ Ic_4 \\ Ic_5 \\ Ic_6 \end{bmatrix} \quad (9)$$

Taking into consideration the effect of **LS** as expressed in (10):

$$\begin{aligned} Ic_1 &= Ic_1 + I_7 \\ Ic_3 &= Ic_3 - I_8 \\ Ic_6 &= Ic_6 - I_7 \end{aligned} \quad (10)$$

The matrix in (9) becomes as follow:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Ic_1 + I_7 \\ Ic_2 \\ Ic_3 - I_8 \\ Ic_4 \\ Ic_5 \\ Ic_6 - I_7 \end{bmatrix} \quad (11)$$

The matrix in (11) can be rewritten as follows:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} = -\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Ic_1 \\ Ic_2 \\ Ic_3 \\ Ic_4 \\ Ic_5 \\ Ic_6 \\ I_7 \\ I_8 \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} TS \\ LS \end{bmatrix} = -[BCIC] \begin{bmatrix} Ic_{bus} \\ LS \end{bmatrix} \quad (12)$$

Afterwards the development of **VBC** of **TS** is done through:

$$V_6 = V_0 - \sum_{m=1}^{6} Z_m I_m \quad (13)$$

Which the equation in (13) can be rewritten as follows:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} V_0 \\ V_0 \\ V_0 \\ V_0 \\ V_0 \\ V_0 \end{bmatrix} - \begin{bmatrix} Z_1 & 0 & 0 & 0 & 0 & 0 \\ Z_1 & Z_2 & 0 & 0 & 0 & 0 \\ Z_1 & Z_2 & Z_3 & 0 & 0 & 0 \\ Z_1 & Z_2 & Z_3 & Z_4 & 0 & 0 \\ Z_1 & Z_2 & Z_3 & Z_4 & Z_5 & 0 \\ Z_1 & Z_2 & Z_3 & Z_4 & Z_5 & Z_6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \quad (14)$$

Then by adding the effect of **LS** into (14) yields the following:

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$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_1 & Z_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_1 & Z_2 & Z_3 & 0 & 0 & 0 & 0 & 0 \\ Z_1 & Z_2 & Z_3 & Z_4 & 0 & 0 & 0 & 0 \\ Z_1 & Z_2 & Z_3 & Z_4 & Z_5 & 0 & 0 & 0 \\ Z_1 & Z_2 & Z_3 & Z_4 & Z_5 & Z_6 & 0 & 0 \\ 0 & -Z_2 & -Z_3 & -Z_4 & -Z_5 & -Z_6 & Z_7 & 0 \\ -Z_1 & -Z_2 & -Z_3 & 0 & 0 & 0 & 0 & Z_8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} \quad (15)$$

$$\begin{bmatrix} V_{bus} \\ 0 \end{bmatrix} = -[VBC]\begin{bmatrix} TS \\ LS \end{bmatrix} \quad (16)$$

Next The Z-bus matrix of the system is found as follows:

$$\begin{bmatrix} V_{bus} \\ 0 \end{bmatrix} = [VBC][BCIC]\begin{bmatrix} Ic_{bus} \\ LS \end{bmatrix} = \begin{bmatrix} Z_{new}^{11} & Z_{new}^{12} \\ Z_{new}^{21} & Z_{new}^{22} \end{bmatrix}\begin{bmatrix} Ic_{bus} \\ LS \end{bmatrix} \quad (17)$$

Finally, Kron reduction is applied to (17) yields that the matrix to be:

$$[V_{bus}] = [Z_{new}^{11} - Z_{new}^{12}(Z_{new}^{22})^{-1}Z_{new}^{21}][Ic_{bus}] = [Z_{matrix}]\,[Ic_{bus}] \quad (18)$$

This work is improved by [4] as they proposed a solution for systems containing **CCVS**. The line/branch having **CCVS** with a control parameter, that control parameter is multiplied by column having CCVS in it. On the other hand, the matrix of **BCIC** is the same.

C. Genetic algorithm to optimize path

This optimization proposed by [5] is using genetic algorithm to find the optimal path to calculate the Z-bus matrix to minimize the number of computations needed.
The first generation of the genetic algorithm is randomly generated, but should have these two requirements:

1) The first element (gen) of each chromosome should be a generator that is added between a new bus and the reference bus
2) Each chromosome should meet the condition that each bus must have an indirect or direct connection to the reference bus.

After producing the first generation a cost function is defined to evaluate all the chromosomes of the current generation. [5] proposed the following equation (19) – (22) representing the 4 cases of bus addition, respectively.

$$t_j = (n+1)^2 t_R \quad (19)$$

$$t_j = (n+1)^2 t_R + t_a \quad (20)$$

$$t_j = t_M(n+n^2)^2 t_d + n^2 t_m + t_a + n^2 t_R \quad (21)$$

$$t_j = (n+1)^2 t_m + (n+1)^2 t_M + 2t_a + t_d + n^2 t_R \quad (22)$$

Where n is the order of the Z-bus matrix and

t_R = Replacement time needed

t_m = Subtraction time needed

t_a = Addition time needed

t_d = Division time needed

t_M = Multiplication time needed

Using the equation from (19)-(22) the cost/fitness function is defined as follows:

$$CostF(i) = -\sum_{j=1}^{L} t_j \quad (23)$$

For the selection of the chromosome to propagate the tournament without replacement method is used. This method eliminates the low fitness valued chromosomes.
The Cross over between the chromosomes of different parents is not used in [5].
The mutation in genetic algorithm is used by [5]. To ensure the produced generation, that have a mutation still satisfies the two requirements of the first generation, the following steps are proposed:

Step 1: x and y are randomly generated in the range [2, L].
Step 2: The gene at y is saved in temporary variable
Step 3: the genes from x to y-1 are shifted one position to the right
Step 4: The gene saved in temporary is then saved at position x

Finally, the time needed to get the Z-bus matrix is the time needed to finish the algorithm in addition to the time to build the matrix. According to the results of [5] the time to finish the algorithm and find the optimal path is very minimal and negligible compared to the time needed to build the matrix. The minimum time needed for random path to build Z-bus and finding optimal path then building Z-bus according to $CostF(i)$ are 7500 and 6000, respectively.

V. OBSERVATIONS ON EACH METHOD

The direct method algorithm can be used to program a general code as done by [7], but the algorithm suffers from needing more memory and computational power compared to the other two methods. The tree set algorithm results in [2] shows that it needs less memory and computational power compared to the direct method as the system/network gets bigger, where the comparison of proposed algorithm of 4-bus system and 16-bus system are 0.42 and 0.2 of the amount of computations needed of the direct method, respectively. Then the method is improved to include **CCVS** by [4], making the algorithm more appealing. Finally, the use genetic algorithm to find optimal path proved that it has effect on computation as it reduced to time needed to calculate the Z-bus matrix by 20%. But genetic algorithm in [5] is only applied on the direct building algorithm. It would be beneficial to apply it on other algorithms as well. But the genetic algorithm suffers from the need to be applied from scratch with any addition of line or bus into the network.

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VI. CONCLUSION

Different Algorithms were discussed, the direct Z-bus matrix building algorithm and the tree-set and link-set algorithm. The direct method provides is more general and easier to program, but the tree-set is faster method that is not recursive and can be generalized to include branches with **CCVS**. Applying genetic algorithm to the direct building method produce good results in improving the algorithm efficiency by 20%.

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