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A Comparative Analysis of DFT and Elliptic DFT in Digital Signal Processing

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Abstract — The Discrete Fourier Transform (DFT) is an important mathematical tool widely used in digital signal processing, image processing, and frequency analysis. This paper focuses on the mathematical properties and applications of two types of Elliptic Discrete Fourier Transforms (EDFT), namely EDFT Type I and EDFT Type II. Unlike the conventional DFT, which is based on complex exponential functions and circular rotations, the EDFT is formulated in the real space using block matrices whose elements are generated by real 2×2 matrices. The main objective of this study is to investigate the mathematical construction and fundamental properties of the two types of EDFT. First, the real-space representation of the conventional DFT is introduced. Then, the mathematical constructions of EDFT Type I and Type II are presented. EDFT Type I is constructed by using a specific elliptic generating matrix, whereas EDFT Type II is developed through projection matrices based on two linearly independent unit vectors. Their mathematical properties, including periodicity, matrix representation, inverse transformation, and orthogonality under a suitable weighted inner product, are discussed. Furthermore, numerical examples are provided to illustrate the transformation matrices and to compare the mathematical structures of DFT and EDFT Types. Finally, potential applications of EDFT in signal analysis, signal filtering, image enhancement, and image processing are discussed. The results provide a unified algebraic framework for Fourier-type transforms with elliptic orbits; their empirical performance in signal and image processing remains to be evaluated.

Keywords — Discrete Fourier Transform, Elliptic Discrete Fourier Transform, EDFT Type I, EDFT Type II, Digital Signal Processing, Image Processing.

I. Introduction

The Discrete Fourier Transform (DFT) is one of the fundamental mathematical tools used in digital signal processing and image processing. It converts a finite sequence of data from the time domain into the frequency domain. The traditional DFT is defined in the complex space and is based on complex exponential functions. In the traditional DFT, multiplication by complex exponential coefficients can be interpreted geometrically as rotations in the complex plane. These rotations move points around circles. Therefore, the conventional DFT is closely related to circular motion. Grigoryan [2] introduced a more general approach by representing the DFT in an extended real space. In this representation, each complex coefficient is replaced by a real two-by-two matrix. This matrix representation makes it possible to replace ordinary circular rotations with transformations that move points along ellipses. Based on this idea, two types of Elliptic Discrete Fourier Transforms (EDFT), namely EDFT Type I and EDFT Type II, introduced. Both transformations are constructed by using two-by-two matrices whose powers satisfy the N-th root of identity property. The EDFT extends the classical DFT by replacing its circular rotations with parameterized transformations in real space, yielding alternative representations of discrete signals that may prove useful in signal processing, image processing, filtering, and enhancement. This paper formulates the DFT in real space and expresses it in block-matrix form, introduces elliptic transformations, constructs EDFT Type I and Type II, analyzes their mathematical properties, and discusses their potential applications.

The real-space formulation is an important concept in the construction of the Elliptic Discrete Fourier Transform (EDFT). In the conventional Discrete Fourier Transform (DFT), a discrete signal is represented in the complex vector space $\mathbb{C}^N$. Each complex number consists of a real part and an imaginary part. These two components can be represented by a two-dimensional real vector. Therefore, the complex signal can be rewritten as a real vector in the extended space $\mathbb{R}^{2N}$. In this representation, complex multiplication is replaced by real matrix multiplication. This approach allows the DFT to be expressed in terms of block matrices of size 2×2. The real-space formulation provides a mathematical foundation for extending the conventional DFT to elliptic transformations. By replacing the circular rotation matrices with suitable elliptic matrices, the EDFT can be constructed in the real vector space $\mathbb{R}^{2N}$.

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Complex Space Representation

The traditional DFT is defined in the complex vector space $\mathbb{C}^N$. A complex signal of length N can be written as

$$f = (f_0, f_1, \dots, f_{N-1})^T \qquad (1)$$

where each component is a complex number of the form

$$f_n = r_n + j\, i_n. \qquad (2)$$

The DFT is given by

$$F_p = \sum_{n=0}^{N-1} f_n\, e^{-j\frac{2\pi np}{N}}. \qquad (3)$$

Using Euler's formula, $e^{-j\theta} = \cos\theta - j\sin\theta$, the DFT can be expressed in terms of real and imaginary components.

Real-Space Representation

Instead of using complex numbers directly, each complex component can be represented by a two-dimensional real vector:

$$\bar{f}_n = \begin{pmatrix} r_n \\ i_n \end{pmatrix}. \qquad (4)$$

Consequently, the entire complex signal can be represented as a real vector in $\mathbb{R}^{2N}$:

$$\bar{f} = \begin{pmatrix} r_0 \\ i_0 \\ r_1 \\ i_1 \\ \vdots \\ r_{N-1} \\ i_{N-1} \end{pmatrix} \in \mathbb{R}^{2N}. \qquad (5)$$

This representation separates the real and imaginary parts of the signal into pairs of real components. The complex multiplication by a DFT coefficient can then be replaced by a real matrix multiplication. For an angle θ, the corresponding matrix is

$$T(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

Thus, the multiplication of a complex number by $e^{-j\theta}$ is equivalent to

$$\begin{pmatrix} \mathrm{Re}\left(ze^{-j\theta}\right) \\ \mathrm{Im}\left(ze^{-j\theta}\right) \end{pmatrix} = T(\theta) \begin{pmatrix} \mathrm{Re}(z) \\ \mathrm{Im}(z) \end{pmatrix}.$$

For the DFT, the angle is $\theta_k = \frac{2\pi k}{N}$. Therefore, the real-space representation provides a matrix-based formulation of the conventional DFT. It also provides the mathematical foundation for constructing elliptic transformations.

III. Block Matrix Representation

The block matrix representation is an important mathematical approach for expressing the Discrete Fourier Transform (DFT) in real space. In this representation, each complex Fourier coefficient is replaced by a 2×2 real matrix. These matrices describe the rotation of the real and imaginary components of a signal. Let the basic rotation matrix be defined as

$$T = \begin{pmatrix} \cos\frac{2\pi}{N} & \sin\frac{2\pi}{N} \\ -\sin\frac{2\pi}{N} & \cos\frac{2\pi}{N} \end{pmatrix}.$$

The powers of this matrix are given by

$$T^k = \begin{pmatrix} \cos\frac{2\pi k}{N} & \sin\frac{2\pi k}{N} \\ -\sin\frac{2\pi k}{N} & \cos\frac{2\pi k}{N} \end{pmatrix}.$$

Using these rotation matrices, the real-space DFT can be expressed as an $N \times N$ block matrix, where each block has dimension 2×2. The transformation matrix is written as

$$X = \begin{pmatrix} T^0 & T^0 & \cdots & T^0 \\ T^0 & T^1 & \cdots & T^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ T^0 & T^{N-1} & \cdots & T^{(N-1)^2} \end{pmatrix}. \qquad (6)$$

Therefore, the complete transformation matrix has dimension $2N \times 2N$. The DFT in real space can then be written as $\bar{F} = X\bar{f}$, where $\bar{f}$ is the input vector and $\bar{F}$ is the transformed vector. The block matrix representation is useful because it provides a way to replace the ordinary rotation matrix with other suitable 2×2 matrices. This idea forms the mathematical basis for constructing the Elliptic Discrete Fourier Transforms (EDFT) of Types I and II.

Real Block Matrix of the DFT

The DFT can be represented by a block matrix whose elements are 2×2 rotation matrices. Let

$$T = \begin{pmatrix} \cos\frac{2\pi}{N} & \sin\frac{2\pi}{N} \\ -\sin\frac{2\pi}{N} & \cos\frac{2\pi}{N} \end{pmatrix}.$$

The powers of this matrix satisfy

$$T^k = \begin{pmatrix} \cos\frac{2\pi k}{N} & \sin\frac{2\pi k}{N} \\ -\sin\frac{2\pi k}{N} & \cos\frac{2\pi k}{N} \end{pmatrix}.$$

In particular,

$$T^0 = I_2, \quad T^N = I_2.$$

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The real block matrix of the N-point DFT is

$$\bar{F}_N = \begin{pmatrix} T^0 & T^0 & T^0 & \cdots & T^0 \\ T^0 & T^1 & T^2 & \cdots & T^{N-1} \\ T^0 & T^2 & T^4 & \cdots & T^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ T^0 & T^{N-1} & T^{2(N-1)} & \cdots & T^{(N-1)^2} \end{pmatrix}.$$

Each entry of this matrix is a 2×2 block. Hence, the complete matrix has dimension $2N \times 2N$. The transformation can be written compactly as

$$\bar{F} = \bar{F}_N \bar{f}. \qquad (7)$$

This block matrix representation is important because it allows the rotation matrix T to be replaced by another suitable matrix.

Properties of the Rotation Matrix

The matrix T has the following properties: $T^m T^n = T^{m+n}$, $T^N = I_2$ and $\det(T) = 1$. Moreover, the transpose of T is its inverse: $T^T = T^{-1}$. Therefore, $T^m T^{-m} = I_2$.

These properties show that the matrix T represents a circular rotation and preserves the ordinary Euclidean length of a vector. The block matrix formulation is the starting point for the construction of the Elliptic Discrete Fourier Transform.

IV. Introduction to Elliptic Transformation

The elliptic transformation is a generalization of the conventional circular rotation used in the Discrete Fourier Transform (DFT). In the traditional DFT, the transformation is based on complex exponential functions, which can be represented geometrically as rotations around a circle in the complex plane. In the real-space formulation, these circular rotations are represented by 2×2 Givens rotation matrices. However, the same block matrix structure can be maintained by replacing the ordinary rotation matrices with suitable elliptic matrices. This idea provides the foundation for the construction of the Elliptic Discrete Fourier Transform (EDFT). Let R be a real 2×2 matrix satisfying $R^2 = -I_2$. An elliptic transformation matrix can then be defined by

$$C(\alpha) = \cos\alpha\, I_2 + \sin\alpha\, R$$

where α is a transformation parameter. Using the property $R^2 = -I_2$, the powers of the matrix $C(\alpha)$ can be expressed as

$$C(\alpha)^k = \cos(k\alpha)\, I_2 + \sin(k\alpha)\, R.$$

For an N-point transformation, choosing $\alpha = \frac{2\pi}{N}$ gives $C(\alpha)^N = I_2$. This periodicity property is important for constructing the block matrix of the EDFT. Unlike the ordinary circular rotation, an elliptic transformation can move points along elliptical orbits. Therefore, it provides a more general geometric representation of discrete Fourier transformations. Based on this concept, Grigoryan (2011) introduced two types of Elliptic Discrete Fourier Transforms, namely EDFT Type I and EDFT Type II. These transformations use different constructions of elliptic matrices and provide alternative representations of discrete signals in real space.

Circular Rotation

In the conventional DFT, the basic transformation is a rotation matrix. For a vector $\underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ a circular rotation is given by

$$T\underline{x} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}. \qquad (8)$$

The transformation preserves the Euclidean norm:

$$\| T\underline{x} \|_2^2 = \| \underline{x} \|_2^2. \qquad (9)$$

Thus, the orbit of a point under repeated applications of the rotation matrix lies on a circle.

Elliptic Transformation

An elliptic transformation generalizes the circular rotation by replacing the ordinary rotation matrix with a suitable 2×2 matrix. Let R be a real matrix satisfying $R^2 = -I_2$. For an angle α, define $C(\alpha) = \cos\alpha\, I_2 + \sin\alpha\, R$. Using the property $R^2 = -I_2$, the powers of $C(\alpha)$ satisfy

$$C(\alpha)^k = \cos(k\alpha)\, I_2 + \sin(k\alpha)\, R.$$

If $\alpha = \frac{2\pi}{N}$, then $C(\alpha)^N = I_2$.

This condition means that the transformation returns to the identity after N successive applications. Unlike an ordinary circular rotation, the orbit generated by $C(\alpha)$ may describe an ellipse. Therefore, the elliptic transformation provides a more general geometric representation of the traditional Fourier rotation.

Difference between Circular and Elliptic Transformations

Circular and elliptic transformations are two different mathematical approaches to describing geometric transformations. A circular transformation is based on rotation along a circular path and is represented by a conventional rotation matrix. It preserves the Euclidean length and the geometric structure of the circle. In contrast, an elliptic transformation is associated with an elliptic geometric structure and is represented by an elliptic matrix. Its mathematical behavior is governed by the properties and parameters of the corresponding matrix. These two transformations differ in their geometric representations, matrix structures, and mathematical properties. Understanding these differences provides an important mathematical background for constructing and analyzing the Elliptic Discrete Fourier Transform (EDFT). The main

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differences between circular and elliptic transformations are summarized in Table I.

TABLE I. Comparison of Circular and Elliptic Transformations

Characteristics	Circular Transformation	Elliptic Transformation
Geometric path	Circle	Ellipse
Basic matrix	Rotation matrix	Elliptic matrix
Matrix property	$T^T T = I_2$	$C^N = I_2$
Parameters	Rotation angle	Matrix parameters
Space	Real or complex	Real space

The elliptic transformation provides the mathematical foundation for the construction of the two types of Elliptic Discrete Fourier Transforms (EDFT), namely EDFT Type I and EDFT Type II, investigated in this paper.

Mathematical Construction of EDFT Type I

The Elliptic Discrete Fourier Transform (EDFT) Type I is constructed by replacing the ordinary Givens rotation matrix of the Discrete Fourier Transform with a suitable elliptic matrix. The main idea is to preserve the block matrix structure of the conventional DFT while introducing a different geometric transformation in the real space $\mathbb{R}^{2N}$. According to Grigoryan [1], the elliptic matrix is a 2×2 matrix whose powers generate a periodic group of transformations. These transformations move points along elliptical orbits rather than circular orbits.

Definition of EDFT Type I. Let $N > 1$ be an integer and define the angle $\alpha = \frac{2\pi}{N}$. The basic elliptic matrix of EDFT Type I is defined by

$$C(\alpha) = \begin{pmatrix} \cos\alpha & \cos\alpha - 1 \\ \cos\alpha + 1 & \cos\alpha \end{pmatrix}.$$

This matrix can also be expressed in the form $C(\alpha) = \cos\alpha\, I_2 + \sin\alpha\, R(\alpha)$, where

$$R(\alpha) = \begin{pmatrix} 0 & -\tan(\alpha/2) \\ \cot(\alpha/2) & 0 \end{pmatrix}.$$

Here, I_2 denotes the identity matrix of order two. The matrix $R(\alpha)$ satisfies

$$R(\alpha)^2 = -I_2.$$

Therefore, the elliptic matrix has the important property $C(\alpha)^k = \cos(k\alpha)\, I_2 + \sin(k\alpha)\, R$. For $k = N$, we obtain $C(\alpha)^N = I_2$. This property shows that the elliptic matrix generates a periodic transformation with period N.

Mathematical Construction. Using the elliptic matrix $C(\alpha)$, the EDFT Type I is constructed in the same block matrix form as the real-space DFT. Let $X(C)$ denote the transformation matrix. It is defined by

$$X(C) = \begin{pmatrix} C^0 & C^0 & C^0 & \cdots & C^0 \\ C^0 & C^1 & C^2 & \cdots & C^{N-1} \\ C^0 & C^2 & C^4 & \cdots & C^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C^0 & C^{N-1} & C^{2(N-1)} & \cdots & C^{(N-1)^2} \end{pmatrix}.$$

The (n, p)-th block of this matrix is given by

$$X_{n,p} = C^{mp}, \quad n, p = 0,1, \ldots, N-1.$$

Since every block is a 2×2 matrix, the complete transformation matrix has dimension $2N \times 2N$. Let the input signal be represented by $f = (f_0, f_1, \ldots, f_{N-1})^T \in \mathbb{R}^{2N}$, where each f_n is a two-dimensional real vector. The EDFT Type I is then defined by

$$\bar{F} = X(C)\, \bar{f}.$$

Equivalently, the n-th output component is

$$\bar{F}_n = \sum_{p=0}^{N-1} C^{mp}\, \bar{f}_p.$$

Thus, the transformation is obtained by applying the powers of the elliptic matrix to the input signal components.

Generalized Elliptic Matrix. The construction of EDFT Type I can be extended by considering a more general matrix. Let $R = \begin{pmatrix} a & b \\ c & -a \end{pmatrix}$, where the parameters satisfy $a^2 + bc = -1$. By direct multiplication, we obtain

$$R^2 = \begin{pmatrix} a^2 + bc & 0 \\ 0 & a^2 + bc \end{pmatrix} = -I_2.$$

The generalized elliptic matrix is therefore defined as

$$C = \cos\alpha\, I_2 + \sin\alpha\, R.$$

Its powers satisfy

$$C(\alpha)^k = \cos(k\alpha)\, I_2 + \sin(k\alpha)\, R.$$

Consequently, when $\alpha = \frac{2\pi}{N}$, $C^N = I_2$. Different choices of the parameters a, b, and c produce different elliptic matrices. This generalized construction provides a family of EDFT Type I transformations [2].

Inverse Transformation. The inverse transformation is an important property of EDFT Type I. To express the transformation matrix in a convenient form, let D and S be the $N \times N$ matrices defined by

$$D_{n,p} = \cos(np\alpha) \text{ and } S_{n,p} = \sin(np\alpha), \qquad (10)$$

where $n, p = 0,1, \ldots, N-1$. Using the Kronecker product, the EDFT Type I matrix can be written as

$$X(C) = D \otimes I_2 + S \otimes R. \qquad (11)$$

Its inverse matrix is given by

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$$X(C)^{-1} = \frac{1}{N}(D \otimes I_2 + S \otimes R). \qquad (12)$$

Therefore, the original signal can be recovered from the transformed signal by

$$\bar{f} = X(C)^{-1}\bar{F}.$$

This inverse formula follows from the trigonometric orthogonality relations and the property $R^2 = -I_2$.

Example of EDFT Type I. Consider the case $N = 5$. The transformation angle is $\alpha = \frac{2\pi}{5}$. Since $\cos\left(\frac{2\pi}{5}\right) \approx 0.3090$, the elliptic matrix becomes

$$C = \begin{pmatrix} 0.3090 & -0.6910 \\ 1.3090 & 0.3090 \end{pmatrix}.$$

The matrix satisfies $C^5 = I_2$. Therefore, after five successive applications of the transformation, the initial point returns to its original position. The resulting motion describes an elliptical orbit. This example illustrates the basic construction of EDFT Type I and shows how the conventional circular rotation can be generalized to an elliptic transformation.

Mathematical Construction of EDFT Type II

The Elliptic Discrete Fourier Transform (EDFT) Type II is another class of elliptic discrete transform introduced by Grigoryan [2]. Unlike EDFT Type I, which is constructed by using a parameterized elliptic matrix, EDFT Type II is based on two-dimensional unit vectors and projection operators. The main idea of EDFT Type II is to construct a 2×2 matrix whose powers generate a periodic group of transformations. These transformations provide an alternative way of representing discrete signals in the real space $\mathbb{R}^{2N}$.

Definition of EDFT Type II

Let $\underline{a} = (a_1, a_2)$, $\underline{b} = (b_1, b_2)$ be two-dimensional unit vectors. They satisfy the conditions $\| a \| = \| b \| = 1$. The inner product of these two vectors is defined by

$$s = ab^T = a_1 b_1 + a_2 b_2. \qquad (13)$$

Since both vectors have unit norm, the Cauchy-Schwarz inequality gives

$$-1 \le s \le 1.$$

For the construction of EDFT Type II, we require $1 - s^2 \neq 0$. This condition means that the two vectors are neither parallel nor antiparallel. The parameter s plays an important role in the construction of the projection matrices and the generating matrix of EDFT Type II.

Construction of Projection Matrices. Using the vectors $\underline{a}$ and $\underline{b}$, four 2×2 matrices are defined as

$$P_{1,1} = a^T a, \quad P_{1,2} = a^T b,$$

$$P_{2,1} = b^T a, \quad P_{2,2} = b^T b.$$

Here, the superscript T denotes the transpose of a vector or matrix. These matrices satisfy the following relations:

$$P_{n,m}^T = P_{m,n},$$

$$P_{n,m}^2 = P_{n,m}, \text{ and } P_{n,m}P_{p,k} = s^{|m-p|}P_{n,k},$$

where the indices n, m, p, k take values in $\{1,2\}$. These algebraic properties are important for constructing the basic elliptic matrix of Type II.

Construction of the Matrices U and Q. Using the projection matrices, two matrices U and Q are defined by

$$U = \frac{1}{1-s^2}\left[s\left(P_{1,1} - P_{2,2}\right) - P_{1,2} + P_{2,1}\right], \qquad (14)$$

and

$$Q = \frac{1}{1-s^2}\left[P_{2,2} - sP_{2,1}\right]. \qquad (15)$$

The matrices U and Q satisfy three important algebraic properties:

$$U^2 = -I_2, \quad Q^2 = Q \text{ and } QU + UQ = U.$$

The first relation shows that U has a similar algebraic property to the matrix used in the construction of EDFT Type I. The second relation indicates that Q is idempotent. The third relation describes the interaction between these two matrices.

Construction of the Type II Elliptic Matrix. Let $N > 1$ be the order of the transformation and define

$$c = \cos\left(\frac{2\pi}{N}\right).$$

The basic matrix of EDFT Type II is defined by $H = U + 2cQ$. By using the algebraic properties of U and Q, the matrix H can be shown to satisfy

$$H^N = I_2. \qquad (16)$$

Thus, H is an N-th root of the identity matrix. Its powers form a periodic sequence of transformations:

$$I_2, H, H^2, \ldots, H^{N-1}, H^N = I_2. \qquad (17)$$

This periodicity property makes it possible to construct the block matrix of EDFT Type II.

Block Matrix Representation of EDFT Type II. Similar to the conventional DFT and EDFT Type I, the transformation matrix of EDFT Type II is constructed by using the powers of its generating matrix H. The N-block transformation matrix is defined as

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$$X(H) = \begin{pmatrix} H^0 & H^0 & H^0 & \cdots & H^0 \\ H^0 & H^1 & H^2 & \cdots & H^{N-1} \\ H^0 & H^2 & H^4 & \cdots & H^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H^0 & H^{N-1} & H^{2(N-1)} & \cdots & H^{(N-1)^2} \end{pmatrix}.$$

The (n, p)-th block of this matrix is given by

$$X(H)_{n,p} = H^{np}, \quad n, p = 0,1, \dots, N-1.$$

Since every block is a 2×2 matrix, the complete transformation matrix has dimension $2N \times 2N$. For an input signal $\bar{f} \in \mathbb{R}^{2N}$, the EDFT Type II is expressed as $\bar{F} = X(H)\bar{f}$. Equivalently, the n-th output component can be written as

$$\bar{F}_n = \sum_{p=0}^{N-1} H^{np}\ \bar{f}_p.$$

Therefore, the transformation is determined by the powers of the matrix H, which depends on the selected vectors $\underline{a}$ and $\underline{b}$.

Example of EDFT Type II. Consider two unit vectors defined by

$$\underline{a} = \frac{1}{\sqrt{5}}(1,2), \quad \underline{b} = \frac{1}{5}(3,4).$$

Both vectors satisfy the unit norm condition: $\| a \| = \| b \| = 1$. Their inner product is

$$s = ab^T = \frac{1}{\sqrt{5}}(1,2)\begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix} = \frac{11}{5\sqrt{5}}.$$

Therefore,

$$1 - s^2 = 1 - \frac{121}{125} = \frac{4}{125} \neq 0.$$

For $N = 5$, we have

$$c = \cos\left(\frac{2\pi}{5}\right) \approx 0.3090.$$

The projection matrices are first constructed from the vectors $\underline{a}$ and $\underline{b}$. Then, the matrices U and Q are obtained using their defining equations. Finally, the generating matrix is calculated from $H = U + 2cQ$. For these parameters, the resulting matrix is approximately

$$H \approx \begin{pmatrix} 0.4603 & -0.4407 \\ 2.1044 & 0.1578 \end{pmatrix}.$$

The matrix satisfies $H^5 = I_2$. This example illustrates the construction of EDFT Type II using two unit vectors. Different choices of the vectors may produce different generating matrices and different transformation representations.

Summary of EDFT Type II. EDFT Type II is constructed from two two-dimensional unit vectors and their associated projection matrices. These matrices are used to define the matrices U and Q, which satisfy specific algebraic relations. The generating matrix $H = U + 2cQ$ provides the periodic transformation required for the block matrix construction. Therefore, EDFT Type II extends the conventional DFT by introducing a parameterized elliptic transformation in real space [1].

Comparison between DFT and EDFT

The Discrete Fourier Transform (DFT) is a fundamental mathematical tool used to analyze discrete signals in the frequency domain [1]. The Elliptic Discrete Fourier Transform (EDFT) is an alternative transformation based on an elliptic mathematical construction. Although both methods can be represented in matrix form, their mathematical structures and transformation properties depend on their respective definitions.

The N-point DFT of a discrete sequence $x(n)$ is defined by

$$X(k) = \sum_{n=0}^{N-1} x\,(n)\, e^{-i2\pi kn/N}, \quad k = 0,1, \dots, N-1, \quad \text{where } i = \sqrt{-1}.$$

Its inverse transform is

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X\,(k)\, e^{i2\pi kn/N}.$$

In matrix form, the DFT can be written as $X = F_N x$, where F_N is the Fourier transformation matrix. For comparison, a general matrix representation of an EDFT can be expressed as $X_E = E_N x$, where E_N denotes the transformation matrix of the selected elliptic discrete Fourier transform. If E_N is nonsingular, the inverse transformation is given by $x = E_N^{-1} X_E$. The main difference between DFT and EDFT lies in the mathematical construction of their transformation matrices. The DFT is defined using complex exponential functions, whereas an EDFT uses the matrix structure specified by its elliptic construction. Table II summarizes some general characteristics of the two transformations.

TABLE II. Summary of general characteristics of DFT and EDFT

Characteristic	DFT	EDFT
Mathematical construction	Based on complex exponential functions	Based on an elliptic transformation construction
Transformation matrix	Fourier matrix F_N	Elliptic transformation matrix E_N

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Characteristic	DFT	EDFT
Inverse transformation	Defined by the inverse DFT formula	Depends on the inverse of E_N
Linearity	Linear	Linear when defined by a fixed matrix
Energy preservation	Depends on normalization; the normalized DFT can be unitary	Must be verified for the selected matrix
Computational cost	Direct computation requires $O(N^2)$ operations; FFT algorithms can reduce this cost	Must be verified for the selected matrix
Signal reconstruction	Reconstructs the input using the inverse DFT	Reconstructs the input if the transform matrix is invertible
Applications	Frequency analysis, filtering, and signal reconstruction	Potential applications depend on the properties of the selected EDFT

Consequently, the invertibility, normalization, energy properties, and computational cost of an EDFT must be examined according to its particular definition. From the comparison, the DFT has a well-established mathematical formulation and widely used fast algorithms. The EDFT provides an alternative transformation structure that can be investigated for specific mathematical and signal-processing problems. However, neither method should be considered superior in every situation. A fair comparison requires the same input data and clearly defined evaluation criteria, such as reconstruction error, computational cost, and numerical stability. Therefore, the choice between DFT and EDFT depends on the mathematical requirements of the problem and the properties of the particular transformation being used.

Applications of EDFT in Digital Signal Processing

Digital Signal Processing (DSP) is concerned with the analysis, transformation, filtering, and reconstruction of discrete-time signals. Discrete Fourier Transform methods are widely used in DSP because they represent signals in a transform domain that can provide useful information about their frequency components. The Elliptic Discrete Fourier Transform (EDFT) is an alternative discrete transformation based on an elliptic mathematical construction. Its potential applications depend on the properties of the particular transformation matrix, the corresponding inverse transform, and the computational algorithm. Let x be a discrete signal of length N. The EDFT can be represented in matrix form as $X_E = E_N x$, where E_N is the transformation matrix and X_E is the transformed coefficient vector. When the matrix E_N is invertible, the signal can be reconstructed using $x = E_N^{-1} X_E$. This representation provides a mathematical framework for investigating the use of EDFT in signal analysis, reconstruction, filtering, and other numerical signal-processing tasks.

Signal Reconstruction. Signal reconstruction is the process of recovering an original signal from its transformation coefficients. This is important in digital signal processing when a signal has been transformed for analysis or processing.

Digital Filtering. Digital filtering is used to modify a signal by suppressing unwanted components or emphasizing selected components. Transform-domain filtering generally consists of transforming the input signal, modifying its coefficients, and reconstructing the output signal.

Noise Reduction. Noise reduction is an important application of digital signal processing. A measured signal may contain unwanted noise that affects the accuracy of subsequent analysis. A discrete noisy signal can be expressed as

$$x(n) = s(n) + \mu(n),$$

where $s(n)$ is the desired signal and $\mu(n)$ is the noise component. Applying an EDFT gives $X_E = E_N x$. A suitable processing procedure may modify selected transform coefficients to reduce the influence of noise. The processed coefficients are then transformed back to the original domain using the inverse EDFT. The performance of such a method depends on the properties of the transformation, the noise model, and the coefficient-modification procedure.

Image Processing. A digital image can be represented as a two-dimensional array of pixel values. Transform methods can be used to analyze image structures and represent image data in an alternative mathematical domain. Let A be an $M \times N$ image matrix. A general two-dimensional matrix transformation can be written as

$$B = E_M A E_N^T, \qquad (18)$$

where E_M and E_N are transformation matrices of compatible dimensions. If both transformation matrices are invertible, the original image matrix can be recovered by

$$A = E_M^{-1} B \, (E_N^{-1})^T. \qquad (19)$$

This expression describes a general separable matrix-transform framework. To use it as a specific two-dimensional EDFT, the matrices must be defined according to the selected elliptic transform. Possible investigations include image representation, coefficient analysis, and reconstruction accuracy. Actual performance should be evaluated experimentally before drawing conclusions about image-processing advantages.

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Conclusion

In this paper, the mathematical structures and properties of the Discrete Fourier Transform (DFT) and two types of Elliptic Discrete Fourier Transforms (EDFT), namely EDFT Type I and EDFT Type II, have been investigated. The real-space matrix construction and block matrix representation of the conventional DFT are presented as the mathematical foundation for developing elliptic transformations.

Furthermore, the mathematical constructions of EDFT Type I and EDFT Type II are discussed using real 2×2 matrices. Type I is based on a specially defined elliptic generating matrix, whereas Type II is constructed using projection matrices associated with two linearly independent unit vectors. Their important mathematical properties, including periodicity, matrix powers, and block matrix structures, are also examined. A comparative analysis is presented to highlight the similarities and differences among DFT, EDFT Type I, and EDFT Type II in terms of mathematical construction and geometric representation. In addition, their potential applications in image processing, including frequency analysis, filtering, and image enhancement, are discussed. In conclusion, this study provides a mathematical foundation for understanding and comparing the conventional DFT and two types of EDFT. The results may serve as a basis for further research on elliptic Fourier transformations and their potential applications in digital signal and image processing.

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