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A Color Prediction Model for Additive and Subtractive Mixing of Colors

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Abstract

Additive and subtractive mixing of colors are the two processes of color mixing applied to various natural and manmade phenomena either in a separate or combined way. The present work is a definitive approach to build a mathematical model using the theory of set for the aforesaid systems to circumvent the problems of color mixing that may arise.
Keywords: additive, subtractive, mixing, color, primaries

1. Symbols

U	Universal set
∅	Null set
∪	Union of sets
∩	Intersection of sets
+	Arithmetical addition
-	Arithmetical subtraction, Difference of sets
R	Red
G	Green
B	Blue
C	Cyan
M	Magenta
Y	Yellow
K	Black
W	White
AM	Additive Mixing
SM	Subtractive Mixing

2. INTRODUCTION

The theory of AM and SM of colors has been treated by the various scientist in different times in various ways. In 1893 Joly produced the first additive color screens. In 1869 du Huron and C.Clos made the first color prints. They used three different transfers of yellow, magenta and cyan dyes to paper. Grassman (1854) developed a set of eight axioms that define trichromatic color matching and that serve as a basis for quantitative color measurements. The Beer's law predicts results of subtractive color mixing that hold for only one wavelength at a time. Kubelka-Munk equation forms the basis of virtually all color-mixing calculations in opaque systems. Center of gravity law predicts colors in XYZ/Yxy system in case of AM. Kueppers differentiates eleven color-mixing laws. The authors like, Letouzey, Pateman et al and Field have dealt AM and SM of colors with their own notational systems for color prediction. The present work is an approach to establish a mathematical model with the help of the algebra of sets that has been used here to remold the existing system to explain it in an easier way and to create avenues for further work.

3. EXISTING COLOR THEORIES:

1) Additive system: AM is a process of color synthesis where two or more colored lights are added to produce a new one.

Examples of AM are:

i) Superimposition of beams of light on a diffuser or simultaneous projection of two or more beams of light on the same area of a white screen.

ii) Viewing colors in succession at a frequency high enough to remove all sense of flicker.

iii) Viewing them in adjacent areas that are too small to resolve (as in TV display tubes).

iv) Human physiological color vision.

v) Chromatic threads of various colors in a fabric can be seen individually through a magnifying glass, but not with naked eye. In naked eye we see the color that is obtained by AM of individual thread colors.

vi) If the sectors of different chromatic colors are arranged on gyroscopic disk, they cannot be distinguished anymore individually as soon as the disk has reached a certain rotating speed.

The Additive system starts with darkness and achieves white. Additive primaries are red, green and blue. AM of other colors of light cannot produce these colors. AM of two additive primaries produces third color, i.e. by AM of green and blue light cyan is produced. Similarly red and blue produces magenta, and red and green produces yellow. AM of all the three produces white.

2) Subtractive system: SM is a process of color synthesis where one or more colors is subtracted/ removed (by the way of absorption) from white to produce a new one (as dictated by the reflected/ transmitted color of light).

Examples of SM are:

i) By superimposing dots of translucent colors (as in color printing).

ii) By having dyes in successive thin layers (as in modern color photography).

iii) By mixing pigments together in a suitable vehicle (as artists often do).

iv) When light passes through filter or reflects from colored surface.

The subtractive system starts with white (white paper illuminated by white light) and achieves black. Cyan, magenta and yellow are subtractive primaries. Cyan is obtained by subtracting red from white light, like that magenta and yellow is obtained by subtracting green and blue from white light respectively.
Ideal white colored surface reflects all the colors of light absorb nothing where as ideal black colored surface absorbs all the colors of light, reflects nothing.

4. APPLYING SET THEORY TO COLOR THEORIES:

From the definition of ideal white and black it is obvious that ‘White’ can be taken equivalent to ‘Universal set’ i.e. U=W, consisting of red, green and blue as elements, whereas ‘Black’ can be taken equivalent to ‘Null set’ i.e. ∅=K.

Here ‘Union of sets’ is used for ‘additive mixing’ and ‘Intersection of sets’ is used for ‘subtractive mixing’ of colors, as these operations suit perfectly to AM and SM of colors.

Various Cases of Additive Mixing and Subtractive Mixing of Colors:

1. AM:
   1.1. AM of additive primaries
   1.2. AM of subtractive primaries (i.e. C, M & Y as additive colors)
   1.3. AM of additive primaries and subtractive primaries
2. SM:
   2.1. SM of additive primaries (i.e. R, G & B as subtractive colors)
   2.2. SM of subtractive primaries
   2.3. SM of additive primaries and subtractive primaries

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Note:

1. Single operand i.e. '+' symbol is used for both AM and SM to derive expressions in conventional systems.
2. Total eight colors are drawn in operation (Three additive primaries, three subtractive primaries and two achromatic colors viz. W and K)
3. In AM of colors the screen color should be white if light beams are to be projected onto a screen, whereas for SM of colors the substrate color should be white, for all the results to be valid.

Case-wise Derivations And Discussions:

1.1 AM of Additive Primaries:

Table-1.1.1

Sl.No.	*Derivations through sets*	*Derivations through conventional systems*	*Remarks*
1.1.1	{R}∪{G}={G}∪{R}=Y ⇒Y={R, G} i.e. red and green on AM produces yellow. Similarly, {R}∪{B}={B}∪{R}=M ⇒M={R, B} {G}∪{B}={B}∪{G}=C ⇒C={G, B}	R+G=Y R+B=M G+B=C	Colors are predictable through conventional system
1.1.2	{R}∪{R}={R}, i.e. red and red on AM produces red. Similarly, {G}∪{G}={G} {B}∪{B}={B}	R+R=2R Similarly, G+G=2G B+B=2B	Colors are predictable through conventional system
1.1.3	{R}∪{G}∪{B}=U ⇒U={R,G,B} , red, green and blue on AM produces white	R+G+B=W	Color is predictable through conventional system

The derivations in Sl.No.1.1.1 and 1.1.3 are the mathematical expressions through sets and arithmetic of whatever facts are stated in Additive system section.
The set expressions of Y, M, C and U as derived in Sl.No.1.1.1 and 1.1.3 are used as a basis for other set theory based derivations in other cases.

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1.2 AM of Subtractive Primaries

Table-1.2.1

Sl.No.	*Derivations through sets*	*Derivations through conventional systems*	*Remarks*
2.1.1	C∪M=M∪C=U, i.e. cyan and magenta on AM produces white. Similarly, C∪Y=Y∪C=U M∪Y=Y∪M=U	From equations of Sl.No. 1.1.1 of Table 1.1.1, C+M=R+G+2B Similarly, C+Y=R+B+2G M+Y=G+B+2R	Result of set derivation does not match with the result of conventional systems
2.1.2	C∪C=C, i.e. cyan on AM with cyan produces cyan. Similarly, M∪M=M Y∪Y=Y	C+C=2C Similarly, M+M=2M Y+Y=2Y	Colors are predictable through conventional system
2.1.3	C∪M∪Y=U, i.e. cyan, magenta and yellow on AM produces white.	From equations of Sl.No.1.1.1 of Table1.1.1 C+M+Y=2W	Color is predictable through conventional systems except that whiteness is doubled which is due to 2/3 reflections of individual colors.

1.3 AM of Additive Primaries and Subtractive Primaries.

Table-1.3.1 (two colors)

Sl.No.	*Derivations through sets*	*Derivations through conventional systems*	*Remarks*
1.3.1	{R}∪C=U, i.e. red and cyan on AM produces white. Similarly, {G}∪M=U {B}∪Y=U	From the definition of subtractive primaries and transposing R, G and B to the left side. C+R=W M+G=W Y+B=W	Results are same
1.3.2	{R}∪M=M, i.e. red and magenta on AM produces magenta Similarly, {R}∪Y=Y {G}∪C=C {G}∪Y=Y {B}∪C=C {B}∪M=M	From equation in Sl.No.1.1.1 of Table1.1.1 R+M=2R+B Similarly, R+Y=2R+G, G+C=2G+B, G+Y=R+2G, B+C=G+2B, B+M=R+2B	Results do not match. Colors are not predictable through conventional systems.

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The three expressions in Sl. No. 1.3.1 of the Table1.3.1 prove that complimentary colors on AM produce white. Cases of AM of three to six colors (out of R, G, B, C, M & Y) are given in the Table 1.3.3 to Table 1.3.10. Cases of AM of two to three colors (out of R, G, B, C, M &Y) with K and W are given in the Table 1.3.11.

2.1 SM of Additive Primaries

Table-2.1.1

Sl.No.	*Derivations through sets*	*Derivations through conventional systems*	*Remarks*
2.1.1	{R}∩{G}={G}∩{R}=∅, i.e. red and green on SM produces black. Similarly, {R}∩{B}={B}∩{R}=∅ {G}∩{B}={B}∩{G}=∅		
2.1.2	{R}∩{R}={R}, i.e. red on SM with red produces red Similarly, {G}∩{G}={G} {B}∩{B}={B}		
2.1.3	{R}∩{G}∩{B}=∅, i.e. red, green and blue on SM produces black.		

In this Table 1.2.1 to derive through conventional systems the use of equations of Sl.No.1.1.1 will lead to same results as that of Table 1.1.1
These derivations prove why the additives primaries are not used in process color reproduction.

2.2 SM of Subtractive Primaries

Table-2.2.1

Sl.No.	*Derivations through sets*	*Derivations through conventional systems*	*Remarks*
2.2.1	C∩M=M∩C={B}, i.e. cyan and magenta on SM produces blue. Similarly, C∩Y=Y∩C={G} M∩Y=Y∩M={R}	i) RGB-R-G=B [3]* ii) C+M=W-R-G=B [16] Similarly, C&Y gives G M&Y gives R	Colors are predictable through conventional systems.
2.2.2	C∩C=C, i.e. cyan on SM with cyan produces cyan. Similarly, M∩M=M Y∩Y=Y	C+C=W-R-R=GB-R	Colors are not predictable through conventional system
2.2.3	C∩M∩Y=∅, i.e. cyan, magenta, yellow on SM produces black.	i) RGB-R-G-B=0 [3]* ii) C+M+Y=W-R-G-B=K [16]	Colors are predictable through conventional system

The conventional process color reproduction is based the derivations with Sl. No. 2.2.1 and 2.2.3

* The author Field have taken C=-R, M=-G, Y=-B and W=RGB (paper white) where as the authors Pateman et al have taken C=W-R, M=W-G and Y=W-B (this is stated in the subtractive system section while defining subtractive primaries) which are the best expression to define a subtractive color, as it shows color absorbed and the colors reflected or transmitted can also be derived from it by substituting the value of W from Sl.No.1.1.3 of Table 1.1.1 and subtracting the absorbed colors.

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2.3 SM of Additive Primaries and Subtractive Primaries.

Table-2.3.1 (two colors)

Sl.No.	*Derivations through sets*	*Derivations through conventional systems*	*Remarks*
2.3.1	$\{R\}\cap C=\varnothing$, i.e. red and cyan on SM produces black. Similarly, $\{G\}\cap M=\varnothing$ $\{B\}\cap Y=\varnothing$		Use of expressions that are derived from the definition of subtractive primaries will not produce desired result in conventional systems.
2.3.2	$\{R\}\cap M=\{R\}$, i.e. red and magenta on SM produces red. Similarly, $\{R\}\cap Y=\{R\}$ $\{G\}\cap C=\{G\}$ $\{G\}\cap Y=\{G\}$ $\{B\}\cap C=\{B\}$ $\{B\}\cap M=\{B\}$		Use of expressions that are derived from the definition of subtractive primaries or Equations in Sl.No.1.1.1 of Table1.1.1 will not produce desired result in conventional systems.

The three expressions in Sl No. 2.3.1 prove that complimentary colors on SM produce black.
Cases of SM of three to six colors (out of R, G, B, C, M & Y) are given in the Table 2.3.3 to Table 2.3.10.
Cases of SM of two to three colors (out of R, G, B, C, M &Y) with K and W are given in the Table 2.3.11.

5. INFLUENCE OF BASE COLORS (SCREEN, SUBSTRATE ETC) WHERE AM AND SM OCCURS AND COLORS OF LIGHT ON AM AND SM:

1. For AM: The General formula for Color prediction is given by:
$(S)\cap(C_1\cup C_2\cup C_3)$
This implies AM of colored light beams C_1, C_2 and C_3 that takes place onto a screen of color 'S'. Here obviously S, C_1, C_2 and C_3 are sets.

2. For SM: The General formula for Color prediction is given by:
$(S)\cap(C_1\cap C_2\cap C_3)\cap(L)$
This implies SM of transparent colored inks C_1, C_2 and C_3 that takes place in colored light 'L' and on colored substrate 'S'. Here obviously S, C_1, C_2, C_3 and L are sets.

6. A CASE OF COMBINED COLOR MIXING WHERE BOTH AM AND SM OCCURS:

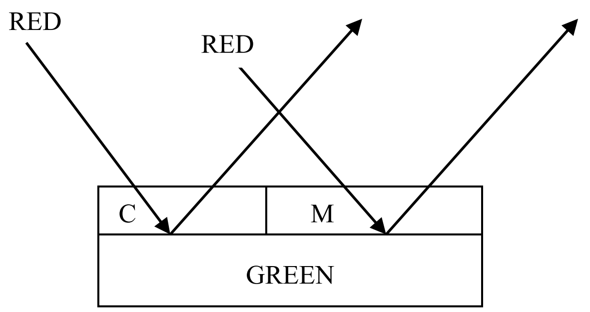

Fig: CCM1

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In the fig. CCM1 two thin bands of transparent Cyan and Magenta ink are placed over Green base and the system is illuminated by Red light.
The expression for Color is given by

$(R\cap C\cap G)\cup(R\cap M\cap G)$
$\Rightarrow\varnothing\cup\varnothing$
$\Rightarrow\varnothing$ (i.e. black)

7. RESULTS AND DISCUSSIONS

- In defining a color particularly subtractive primary color there is difference in opinion among authors as discussed under Table2.2.1, which will lead to different results.
- Two same colors on AM and SM produces no change in color (Ref. derivations with Sl. No. 1.1.2 of Table 1.1.1, Sl. No. 1.2.2 of Table 1.2.1, Sl.No.2.1.2 of Table 2.1.1 and Sl.No.2.2.2 of Table 2.2.1
- Equations in the Sl.No.1.1.1 and 1.1.3 of Table 1.1.1 and expressions that are derived from the definition of subtractive primaries, fails in most of the cases of SM of colors, which is evident from Tables 1.2.1, 2.2.1 and 3.2.1
- It is evident from the Table 1.3.3 to 1.3.10 that all in the cases of AM of 3-6 colors produces white.
- It is evident from the Tables 2.3.2 to 2.3.10 that in all the cases of SM of 3-6 colors produces black except in Table 2.3.3 where SM of C&M, C&Y and M&Y with B, G & R respectively produces B, G and R.
- From the Table 1.3.11 it is evident that result of AM of two or three colors out of R, G and B with K retains their previous result of AM of those colors without black. It is proved that AM of additive primaries starts with ‘K’ and ends with ‘W’.
- From the Table 2.3.11 it is evident that result of SM of two or three colors out of C, M and Y with W retains their previous result of SM of those colors without white. It is proved that SM of subtractive primaries starts with ‘W’ and ends with ‘K’.
- To derive the results of AM and SM of three to six colors, the author has not gone for conventional arithmetic based derivation as already seen that these system fails to give a right, prompt color prediction.

8. CONCLUSION

It has been found that conventional notational system have failed to derive correct color in

i) Some cases of AM of subtractive primaries and, additive and subtractive primaries.

ii) In most of the cases of SM of colors.

So it can be concluded that the proposed set theory based model is found to be best suited for all cases of AM and SM of colors to give a correct and prompt result i.e. color

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APPENDIX

A guideline for reading the Tables with symbols (i.e. Table 1.1.2, 1.2.2, 1.3.2, 2.1.2, 2.2.2, and 2.3.2) contained here:

R1C1	**R1C2**	**R1C3**	**R1C4**
R2C1	R2C2	R2C3	R2C4
R3C1	R3C2	R3C3	R3C4
R4C1	R4C2	R4C3	R4C4

Here in each cell R=Row, C=Column, and the numbers following R and C are the row number and column number, e.g. for cell R1C1, row and column numbers are 1 (one).
The cell R1C1 contains the operand. The remaining three cells of first column contain first group of operators and the remaining three cells of first row, contain the second group of operators. The rest of the cells of other rows and columns contain the results.

Operations are as follows:
<First operator> <operand> <second operator>=<result>
e.g. (R2C1) ∪ (R1C2)=(R2C2), where '∪' is the operand and, cell R2C1, R1C2 and R2C2 contains first operator, second operator and results respectively
i.e. the results are placed in a cell whose row number is same as that of the first operator and column number is same as that of the second operator.

Table 1.1.2 (Tabulated symbolic form of case 1.1, Sl. No. 1.1.3 derivation is not included in the Table)

∪	**R**	**G**	**B**
R	R	Y	M
G	Y	G	C
B	M	C	B

Table-1.2.2 (Tabulated symbolic form of case 1.2, Sl. No. 1.2.3 derivation is not included in the Table)

∩	**R**	**G**	**B**
R	R	K	K
G	K	G	K
B	K	K	B

Table-1.2.2 (Tabulated symbolic form of case 2.1, Sl. No. 2.1.3 derivation is not included in the Table)

∪	**C**	**M**	**Y**
C	C	W	W
M	W	M	W
Y	W	W	Y

Table-2.2.2 (Tabulated symbolic form of case 2.2, Sl. No. 2.2.3 derivation is not included in the Table)

∩	**C**	**M**	**Y**
C	C	B	G
M	B	M	R
Y	G	R	Y

Table-1.3.2 (Tabulated symbolic form of case 3.1)

∪	**C**	**M**	**Y**
R	W	M	Y
G	C	W	Y
B	C	M	W

Table-2.3.2 (Tabulated symbolic form of case 3.2)

∩	**C**	**M**	**Y**
R	K	R	R
G	G	K	G
B	B	B	K

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Table 1.3.3 (three colors)

∪	**C∪M**	**C∪Y**	**M∪Y**
R	W	W	W
G	W	W	W
B	W	W	W

Table 1.3.4 (three colors)

∪	**C**	**M**	**Y**
R∪G	W	W	W
R∪B	W	W	W
G∪B	W	W	W

Table 1.3.5 (four colors)

∪	**C∪M**	**C∪Y**	**M∪Y**
R∪G	W	W	W
R∪B	W	W	W
G∪B	W	W	W

Table 1.3.6 (four colors)

∪	**C**	**M**	**Y**
R∪G∪B	W	W	W

Table 1.3.7 (four colors)

∪	**R**	**G**	**B**
C∪M∪Y	W	W	W

Table 1.3.8 (five colors)

∪	**C∪M**	**C∪Y**	**M∪Y**
R∪G∪B	W	W	W

Table 1.3.9 (five colors)

∪	**R∪G**	**R∪B**	**G∪B**
C∪M∪Y	W	W	W

Table 1.3.10 (six colors)

∪	**R∪G∪B**
C∪M∪Y	W

Table 1.3.11

∪	**C∪M**	**C∪Y**	**M∪Y**	**R∪G**	**R∪B**	**G∪B**	**C∪M∪Y**	**R∪G∪B**
K	W	W	W	Y	M	C	W	W
W	W	W	W	W	W	W	W	W

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Table 2.3.3 (three colors)

∩	C∩M	C∩Y	M∩Y
R	K	K	R
G	K	G	K
B	B	K	K

Table 2.3.4 (three colors)

∩	C	M	Y
R∩G	K	K	K
R∩B	K	K	K
G∩B	K	K	K

Table 2.3.5 (four colors)

∩	C∩M	C∩Y	M∩Y
R∩G	K	K	K
R∩B	K	K	K
G∩B	K	K	K

Table 2.3.6 (four colors)

∩	C	M	Y
R∩G∩B	K	K	K

Table 2.3.7 (four colors)

∩	R	G	B
C∩M∩Y	K	K	K

Table 2.3.8 (five colors)

∩	C∩M	C∩Y	M∩Y
R∩G∩B	K	K	K

Table 2.3.9 (five colors)

∩	R∩G	R∩B	G∩B
C∩M∩Y	K	K	K

Table 2.3.10 (six colors)

∩	R∩G∩B
C∩M∩Y	K

Table 2.3.11

∩	C∩M	C∩Y	M∩Y	R∩G	R∩B	G∩B	C∩M∩Y	R∩G∩B
K	K	K	K	K	K	K	K	K
W	B	G	R	K	K	K	K	K