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A Class of Gauged Inter Quantile Deviation Control Charts

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***Abstract*— Control charts are widely used in statistical quality control to monitor and detect shifts in the process variation. A class of control charts based on gauged inter quantile deviation (GIQD) is proposed for monitoring process variability. This class allows for the flexibility in selecting control charts based on the deviation of a pair of quantiles such as percentiles, deciles or quartiles. The proposed class of GIQD control charts are evaluated under various distributions. Its performance is analyzed and optimality of the class is discussed. An illustrative example is provided to demonstrate its practical applicability.**

***Keywords*— average run length, control limits, inter quantile deviation, process variation, robust, scale parameter**

I. INTRODUCTION

Control charts play a vital role in statistical process control (SPC). They detect the unfavorable changes in an ongoing process with regard to quality characteristics. The process parameters such as process location, process variance, process standard deviation, etc represent various entities of the production process. As consistency of the process is ensured by process variability, the control chart for process variation gain prominence in SPC. The details of Shewhart's range (R), sample standard deviation (S) and sample variance (S^2) control charts for assessing process variability are discussed in Montgomery (2019). These control charts rely on the assumption that, the process variables follow normal distribution. However, in reality this assumption need not hold. For instance, in manufacturing industries, tool wear often exhibits non-normal distributions. As tools deteriorate over time, the measurements of dimensional accuracy such as the diameter or surface finish of a machined part may follow a skewed distribution or a symmetric distribution other than normal distribution. Similarly, quality characteristic such as the tensile strength of paper do not follow normal distribution. According to Korteoja et al. (1998), this quality characteristic is adequately captured by Laplace distribution. Hence, the control charts for process variation are inevitable under non normality.

Numerous control charts under the assumption of normality have been extensively developed in the literature to monitor process variation. Control charts based on one sided cumulative sum (CUSUM) and the Shewhart control chart with warning limits based on R was discussed by Page (1963). Run length distribution of R, S and S^2 control charts were derived by Chen (1998). Shewhart's R and S control charts based on runs rules was discussed by Lowry et al. (1995). CUSUM control chart based on R to detect shift in process standard deviation (sd) was studied by Acosta-Mejia (1998). Modified Shewhart S control chart to improve the detection of small shifts in process sd was suggested by Klein (2000). The probability limits for S control chart was provided by Ryan (2011). The modified R control chart for monitoring the process variance was proposed by Khoo and Lim (2005). Control charts based on prior information for process variation due to Menzefricke (2007, 2010), Bhat and Gokhale (2014, 2016, 2017), Saghir et al. (2020) and Bhat and Malagavi (2021) are given under normal model.

Unlike parametric control charts, nonparametric control charts are distribution free and do not rely on specific assumptions about the underlying distribution. Some of the nonparametric control charts for process sd are due to Amin et al. (1995) based on the sign statistic, Abu and Abdullah (2000) on Downton estimator (D) due to Downton (1966), Riaz and Saghir (2007) on Gini's mean difference and Zombade and Ghute (2014) on Sukhatme and Mood test statistics. Also, the control charts due to Das (2008) based on Ansari-Bradley two sample rank sum test, Abu (2008) on median absolute deviation (MAD), Riaz and Saghir (2009) on mean absolute deviation from the median exist for process sd. Abbasi and Miller (2012) compared control charts based on eight different estimators. Rajmanya and Ghute (2014) developed a robust control chart by proposing a synthetic D control chart.

In this paper, we propose a class of control charts to monitor process variability based on gauged inter quantile deviation $(GIQD)$ scaled by a positive constant. As quantile based control charts are robust, they are useful in situations when process variables deviate from normality. These control charts have advantages when data contains outliers and process has heavy tailed or skewed distributions. This motivates the current study to introduce a class of control charts utilizing the difference of two equidistant quantiles from the centre of the data gauged by a constant to provide a robust and effective control chart for monitoring process scale.

The paper is organized in 6 sections. In section 2 and section 3, we introduce respectively, the concept of gauged IQD $(GIQD)$, its properties and class of control charts based on $GIQD$. Section 4 deals with performance evaluation of control charts and its optimality. Section 5 provides an illustrative example and section 6 contains conclusions based on our findings.

II. GIQD AND ITS PROPERTIES

Suppose $X_1,\ X_2,...,X_n$ is a random sample of size n from a continuous distribution $F(x)$ with location parameter μ and scale parameter λ. When X_i, $i = 1,2,...,n$ are arranged in ascending order, the $X_{(i)}$ is known as i^{th} order statistic.

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Let ζ_p and z_p denote respectively the p^{th} population and sample quantiles of $F(x)$, then according to Gibbons and Chakraborti (2020), ζ_p is given by

$$\zeta_p = \inf\{x: F(x) \geq p\} \qquad 0 < p < 1 \tag{1}$$

and z_p is given by $X_{(i)}$,

$$\text{where } i = \begin{cases} np\,, & if\ np\ is\ an\ integer \\ [np]+1\,, & if\ np\ is\ not\ an\ integer\,, \end{cases} \tag{2}$$

$[x]$ is the largest integer $\leq x$ and $0 < p < 1$.

According to Cramer (1946), suppose z_{p_1} and z_{p_2} are respectively the $p_1{}^{th}$ and $p_2{}^{th}$ sample quantiles, then

$$\begin{pmatrix} z_{p_1} \\ z_{p_2} \end{pmatrix} \sim N\left(\begin{pmatrix} \zeta_{p_1} \\ \zeta_{p_2} \end{pmatrix}, \begin{pmatrix} \frac{p_1(1-p_1)}{nf^2(\zeta_{p_1})} & \frac{p_2(1-p_1)}{nf(\zeta_{p_2})f(\zeta_{1-p_1})} \\ \frac{p_1(1-p_2)}{nf(\zeta_{p_1})f(\zeta_{1-p_2})} & \frac{p_2(1-p_2)}{nf^2(\zeta_{p_2})} \end{pmatrix} \right) \tag{3}$$

where N represents normal distribution and $f\left(\zeta_{p_i}\right), i = 1, 2$ is the probability density function (pdf) of the distribution evaluated at ζ_{p_i}.

The GIQD based on the deviation of two equidistant quantiles with g as gauged constant is given by

$${}_gD_p = \frac{z_{1-p} - z_p}{g}, \quad 0 < p < \frac{1}{2}, \quad 0 < g \leq n \tag{4}$$

is an estimator of the scale parameter. The selection of various values of p and g results in the flexibility of ${}_gD_p$ in its adjustability as it contains various members of this class of estimators.

For $g = 1$, we have IQD given by

$${}_1D_p = z_{1-p} - z_p \tag{5}$$

and for $g = 2$, we have semi IQD ($SIQD$) given by

$${}_2D_p = \frac{z_{1-p} - z_p}{2}. \tag{6}$$

Similarly, for various values of g, we get various $GIQD$ estimators.

For a specified g and $p = \frac{1}{4}$, $GIQD$ simplifies to the gauged inter quartile range ($GIQR$). Specifically, when $g = 1$, it reduces to inter quartile range (IQR) and when $g = 2$ it becomes semi inter quartile range ($SIQR$).

Similarly, for fixed g and $p = \frac{j}{10}$ with $j = 1, 2, 3, 4$, we get j^{th} gauged inter decile deviation ($GIDD_j$). When $j = 1$, we have $GIDD_1$ representing 1^{st} $GIDD$. Similarly, for $j = 2, 3, 4$, we get 2^{nd}, 3^{rd} and 4^{th} $GIDDs$ respectively given by $GIDD_2$, $GIDD_3$ and $GIDD_4$. When $g = 1$, we have IDD_j and when $g = 2$, we get semi IDD_j ($SIDD_j$). Also, the IDD_1 is known as inter decile range (IDR) and $SIDD_1$ is known as semi inter decile range ($SIDR$).

Further, on similar grounds, for specific g and $p = \frac{k}{100}$ with $k = 1, 2, \ldots, 49$, the corresponding estimator is k^{th} gauged inter percentile deviation ($GIPD_k$). When $g = 1$ and $g = 2$, we respectively have IPD_k and semi IPD_k ($SIPD_k$). Also, IPD_1 and $SIPD_1$ are respectively known as inter percentile range (IPR) and semi inter percentile range ($SIPR$).

As IQD estimator is a robust measure of scale parameter, the class of GIQD estimators contain the members that are robust to $lp(n-1)$ outliers on either side of the partition values where l is the index of quantile deviation. That is, if $p = \frac{1}{4}$, $l = 1$, then $GIQR$ is resistant to $\frac{n-1}{4}$ outliers. If $p = \frac{1}{10}$, $l = j = 1, 2, 3, 4$ then $GIDD_1$ is robust to $\frac{n-1}{10}$ outliers where as $GIDD_2$ is robust to $\frac{n-1}{5}$ outliers and so on. Similarly, if $p = \frac{1}{100}$, $l = k = 1, 2, \ldots, 49$ then $GIPD_1$ and $GIDP_{12}$ are respectively robust to $\frac{n-1}{100}$ and $3\left(\frac{n-1}{25}\right)$ outliers. Likewise, various members of $GIQD$ are resistant to various numbers of outliers. Hence, the class of $GIQD$ estimators provide a wide range of adaptable robust measures which are useful in distinct situations.

From (2) and (3), ${}_gD_p$ asymptotically follows normal distribution with mean $\frac{\zeta_{1-p} - \zeta_p}{g}$ and variance $\sigma^2_{{}_gD_p}$.

That is,

$${}_gD_p \sim N\left(\frac{\zeta_{1-p} - \zeta_p}{g}, \sigma^2_{{}_gD_p}\right) \tag{7}$$

where $\sigma^2_{{}_gD_p} = var\left({}_gD_p\right)$

$$\begin{aligned} &= \frac{1}{g^2}\left(var(z_p) + var(z_{1-p}) - 2cov(z_p, z_{1-p})\right) \\ &= \frac{1}{g^2}\left(\frac{p(1-p)}{nf^2(\zeta_p)} + \frac{(1-p)p}{nf^2(\zeta_{1-p})} - \frac{2p^2}{nf(\zeta_p)f(\zeta_{1-p})}\right) \\ &= \frac{p}{ng^2}\left(\frac{(1-p)}{f^2(\zeta_p)} + \frac{(1-p)}{f^2(\zeta_{1-p})} - \frac{2p}{f(\zeta_p)f(\zeta_{1-p})}\right). \end{aligned} \tag{8}$$

III. CLASS OF CONTROL CHARTS BASED ON GIQD

In this section, we propose a class of Shewhart type control charts based on GIQD to monitor process scale parameter. The proposed class of control charts is given in terms of its control limits, viz.

$$UCL_{{}_gD_p} = E\left({}_gD_p\right) + 3\sigma_{{}_gD_p}, \quad CL_{{}_gD_p} = E\left({}_gD_p\right) \text{ and } LCL_{{}_gD_p} = E\left({}_gD_p\right) - 3\sigma_{{}_gD_p} \tag{9}$$

where $UCL_{{}_gD_p}$, $CL_{{}_gD_p}$ and $LCL_{{}_gD_p}$ respectively being the upper control limit, center line and lower control limit of ${}_gD_p$ control chart. The $E\left({}_gD_p\right)$ and $\sigma_{{}_gD_p}$ are respectively mean and sd of ${}_gD_p$ control chart. The width (w) of the control chart is given by

$$\begin{aligned} w_{{}_gD_p} &= UCL_{{}_gD_p} - LCL_{{}_gD_p} \\ &= 6\sigma_{{}_gD_p}. \end{aligned} \tag{10}$$

The ${}_gD_p$ control charts are developed for light tailed distribution viz. Uniform (U) distribution, skewed

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distribution viz. Exponential (E) distribution, medium tailed distributions such as Normal (N), Logistic (LG) distributions and heavy tailed distributions like Laplace (L), Cauchy (C) distributions. These distributions (D) are modified such that, the scale parameter of the distributions is their sd.

Here, the control limits of the ${}_gD_p$ control chart is derived under normal distribution, viz. $N(\mu, \lambda^2)$ with pdf

$$f(x) = \frac{1}{\lambda\sqrt{2\pi}} e^{-\frac{1}{2\lambda^2}(x-\mu)^2} -\infty < x, \mu < \infty;\ \lambda > 0. \quad (11)$$

The cdf is

$$F(x) = \frac{1}{2}\left(1 + erf\left(\frac{x-\mu}{\lambda\sqrt{2}}\right)\right) \quad (12)$$

where erf is the error function defined as $erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt$.

Taking $F(x) = p$ and substituting ζ_p for x in (11), we get

$$p = \frac{1}{2}\left(1 + erf\left(\frac{\zeta_p-\mu}{\lambda\sqrt{2}}\right)\right)$$

$$\Longrightarrow \quad \mathrm{erf}\left(\frac{\zeta_p-\mu}{\lambda\sqrt{2}}\right) = 2p - 1$$

$$\Longrightarrow \quad \frac{\zeta_p-\mu}{\lambda\sqrt{2}} = erf^{-1}(2p-1)$$

Hence, $\zeta_p = \mu + \lambda\sqrt{2}erf^{-1}(2p-1)$. (13)

Similarly, $\zeta_{1-p} = \mu + \lambda\sqrt{2}erf^{-1}(1-2p)$. (14)

Also, by (11), (13) and (14)

$$f(\zeta_p) = \frac{1}{\lambda\sqrt{2\pi}} e^{-\frac{1}{2\lambda^2}\left(\mu+\lambda\sqrt{2}erf^{-1}(2p-1)-\mu\right)^2}$$

$$= \frac{1}{\lambda\sqrt{2\pi}} e^{-\left(erf^{-1}(2p-1)\right)^2} \quad (15)$$

and

$$f(\zeta_{1-p}) = \frac{1}{\lambda\sqrt{2\pi}} e^{-\frac{1}{2\lambda^2}\left(\mu+\lambda\sqrt{2}erf^{-1}(1-2p)-\mu\right)^2}$$

$$= \frac{1}{\lambda\sqrt{2\pi}} e^{-\left(erf^{-1}(1-2p)\right)^2}. \quad (16)$$

Hence from (7), the

$$E({}_gD_p) = \frac{\mu+\lambda\sqrt{2}erf^{-1}(1-2p)-\left(\mu+\lambda\sqrt{2}erf^{-1}(2p-1)\right)}{g}$$

$$= \frac{\sqrt{2}\lambda\left(erf^{-1}(1-2p)-erf^{-1}(2p-1)\right)}{g} \quad (17)$$

and $\sigma^2_{{}_gD_p}$ is obtained by substituting (15) and (16) in (8). Hence,

$$\sigma^2_{{}_gD_p} = \frac{p}{g^2 n}\left(\frac{1-p}{\left(\frac{1}{\lambda\sqrt{2\pi}}e^{-\left(erf^{-1}(2p-1)\right)^2}\right)^2} + \frac{1-p}{\left(\frac{1}{\lambda\sqrt{2\pi}}e^{-\left(erf^{-1}(1-2p)\right)^2}\right)^2} - \frac{2p}{\left(\frac{1}{\lambda\sqrt{2\pi}}e^{-\left(erf^{-1}(2p-1)\right)^2}\right)\left(\frac{1}{\lambda\sqrt{2\pi}}e^{-\left(erf^{-1}(1-2p)\right)^2}\right)}\right) \quad (18)$$

$$= \frac{2\pi p\lambda^2}{g^2 n}\left(\frac{1-p}{\left(e^{-\left(erf^{-1}(2p-1)\right)^2}\right)^2} + \frac{1-p}{\left(e^{-\left(erf^{-1}(1-2p)\right)^2}\right)^2} - \frac{2p}{\left(e^{-\left(erf^{-1}(2p-1)\right)^2}\right)\left(e^{-\left(erf^{-1}(1-2p)\right)^2}\right)}\right)$$

Taking square root of (18),

$$\sigma_{{}_gD_p} = \frac{\lambda}{g}\left(\frac{2\pi p}{n}\right)^{1/2}\left(\frac{1-p}{\left(e^{-\left(erf^{-1}(2p-1)\right)^2}\right)^2} + \frac{1-p}{\left(e^{-\left(erf^{-1}(1-2p)\right)^2}\right)^2} - \frac{2p}{\left(e^{-\left(erf^{-1}(2p-1)\right)^2}\right)\left(e^{-\left(erf^{-1}(1-2p)\right)^2}\right)}\right)^{1/2} \quad (19)$$

Similarly, $E({}_gD_p)$, $\sigma^2_{{}_gD_p}$ are obtained for other distributions and furnished in Exhibit 1.

Exhibit 1: $E({}_gD_p)$ and $\sigma^2_{{}_gD_p}$ under various distributions

D	$E({}_gD_p)$	$\sigma^2_{{}_gD_p}$
U	$\frac{2\sqrt{3}\lambda(1-2p)}{g}$	$\frac{24\lambda^2 p(1-2p)}{ng^2}$
E	$\frac{\lambda\log\left(\frac{1-p}{p}\right)}{g}$	$\frac{\lambda^2(1-2p)}{ng^2 p(1-p)}$
LG	$\frac{\sqrt{3}\lambda}{g\pi}\log\left(\frac{(1-p)^2}{p^2}\right)$	$\frac{6\lambda^2(1-2p)}{ng^2 p(1-p)^2\pi^2}$
L	$\frac{-\sqrt{2}\lambda\log(2p)}{g}$	$\frac{\lambda^2}{ng^2}\left(\frac{1-2p}{p}\right)$
C	$\frac{2\lambda}{g}\tan\left(\pi\left(\frac{1}{2}-p\right)\right)$	$\frac{p}{ng^2}\left(\frac{1-p}{T^2}+\frac{1-p}{T'^2}-\frac{2p}{TT'}\right)_*$

$* T = \frac{1}{\pi\lambda\left[1+\left(\tan\left(\pi\left(p-1/2\right)\right)\right)^2\right]}$, $T' = \frac{1}{\pi\lambda\left[1+\left(\tan\left(\pi\left(1/2-p\right)\right)\right)^2\right]}$

Since it is not feasible to obtain control charts based on all members of the proposed class, considering $g = 2$, for various values of p, we obtain the control limits based on some members of this class.

The $E({}_2D_{0.25})$ and $\sigma_{{}_2D_{0.25}}$ under normal distribution is obtained as

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$$E({}_2D_{0.25}) = \frac{\sqrt{2}\lambda\left(erf^{-1}(0.5) - erf^{-1}(-0.5)\right)}{2}$$
$$= \frac{\sqrt{2}\lambda(0.4769-(-0.4769))}{2}$$
$$= 0.6745\lambda$$

and

$$\sigma_{{}_2D_{0.25}} = \frac{\lambda}{2}\left(\frac{2*0.25*\pi}{n}\right)^{1/2}\left(\frac{0.75}{0.6345} + \frac{0.75}{0.6345} - \frac{0.5}{0.6345}\right)^{1/2}$$

$$= \frac{\lambda}{2\sqrt{n}}(1.5707)^{1/2}\left(\frac{1}{0.6345}\right)^{1/2}$$
$$= \frac{\lambda}{2\sqrt{n}}(1.2533)(1.2554)$$
$$= \frac{0.7867\lambda}{\sqrt{n}}.$$

The control limits of ${}_2D_{0.25}$ control chart which is $SIQR$ control chart under normal distribution is given by

$$UCL_{{}_2D_{0.25}} = 0.6745\lambda + 3\left(0.7867\frac{\lambda}{\sqrt{n}}\right)$$
$$= \left(0.6745 + \frac{2.3604}{\sqrt{n}}\right)\lambda \quad (20)$$
$$CL_{{}_2D_{0.25}} = 0.6745\lambda \quad (21)$$
$$LCL_{{}_2D_{0.25}} = 0.6745\lambda - 3\left(0.7867\frac{\lambda}{\sqrt{n}}\right)$$
$$= \left(0.6745 - \frac{2.3604}{\sqrt{n}}\right)\lambda. \quad (22)$$

On similar grounds, the mean and sd of the $SIQR({}_2D_{0.25})$, $SIDD\ ({}_2D_{0.1}, {}_2D_{0.2}, {}_2D_{0.3}, {}_2D_{0.4})$ and $SIPD\ ({}_2D_{0.01}, \ldots, {}_2D_{0.49})$ estimators are derived under various distributions. These values which are useful in obtaining control limits for some members of the class along with their widths are provided in Exhibit 2.

Exhibit 2: Mean, sd, w of ${}_2D_p$ for various distributions and values of p

	p \ D	U	E	N	LG	L	C
$E({}_2D_p)$ in terms of λ	0.01	1.6974	2.2976	2.3263	2.5334	2.7662	31.8205
	0.05	1.5588	1.4722	1.6449	1.6234	1.6282	6.3138
	0.10	1.3856	1.0986	1.2816	1.2114	1.138	3.0777
	0.15	1.2124	0.8673	1.0364	0.9563	0.8513	1.9626
	0.20	1.0392	0.6931	0.8416	0.7643	0.6479	1.3764
	0.25	0.866	0.5493	0.6745	0.6057	0.4901	1
$\sigma_{{}_2D_p}$ in terms of $\frac{\lambda}{\sqrt{n}}$	0.01	0.2425	4.9747	2.6264	3.8983	4.9497	222.8902
	0.05	0.5196	2.1764	1.4544	1.7410	2.1213	19.2565
	0.10	0.6928	1.4907	1.1396	1.2252	1.4142	6.5798
	0.15	0.7937	1.1716	0.9827	0.9908	1.0801	3.4925
	0.20	0.8485	0.9682	0.8749	0.8440	0.866	2.2273
	0.25	0.8660	0.8165	0.7867	0.7351	0.7071	1.5708
$w_{{}_2D_p}$ in terms of $\frac{\lambda}{\sqrt{n}}$	0.01	1.4549	29.8481	15.7586	23.3897	29.6985	1337.3414
	0.05	3.1177	13.0586	8.7264	10.4462	12.7279	115.5387
	0.10	4.1569	8.9443	6.8377	7.3511	8.4853	39.4790
	0.15	4.7624	7.0294	5.8963	5.9447	6.4807	20.9550
	0.20	5.0912	5.8095	5.2496	5.0643	5.1962	13.3641
	0.25	5.1962	4.8990	4.7203	4.4106	4.2426	9.4248

IV. PERFORMANCE OF SIQD CONTROL CHARTS

In this section, we evaluate the performance of proposed control charts in terms of power $(PW_{{}_2D_p})$, average run length $(ARL_{{}_2D_p})$, median run length $(MRL_{{}_2D_p})$ and sd of run length $(SDRL_{{}_2D_p})$.

The Power measures ability of a control chart to detect shift $'b'$ in the process scale parameter. The ARL being average number of samples required before a shift is signalled by the control chart, a higher ARL is desired when there is no shift. Conversely, when a shift occurs, a smaller ARL is preferred. The MRL is the median number of samples and behaves like ARL. The SDRL indicating the consistency captures spread in the run length distribution and smaller SDRL is beneficial when process is out of control.

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These performance measures of ${}_2D_p$ control chart are given by

$$PW_{{}_2D_p} = 1 - \beta_{{}_2D_p}, \quad (23)$$

$$\beta_{{}_2D_p} = P(LCL_{{}_2D_p} < {}_2D_p < UCL_{{}_2D_p} | \lambda_b = \lambda + b), \quad (24)$$

$$ARL_{{}_2D_p} = \frac{1}{PW_{{}_2D_p}}, \quad (25)$$

$$MRL_{{}_2D_p} = \frac{\log(0.5)}{\log\left(1 - PW_{{}_2D_p}\right)} \quad (26)$$

and $$SDRL_{{}_2D_p} = \left(ARL_{{}_2D_p}\left(ARL_{{}_2D_p} - 1\right)\right)^{1/2}. \quad (27)$$

The values of $PW_{{}_2D_p}$, $ARL_{{}_2D_p}$, $MRL_{{}_2D_p}$ and $SDRL_{{}_2D_p}$ for the distribution under consideration are determined by setting $\lambda^2 = 1$. The values of $PW_{{}_2D_p}$ and $SDRL_{{}_2D_p}$ for various values of b and n are given in Table 1. Also, $ARL_{{}_2D_p}$ and $MRL_{{}_2D_p}$ are respectively given in Figure 1 and Figure 2 for $n = 10$.

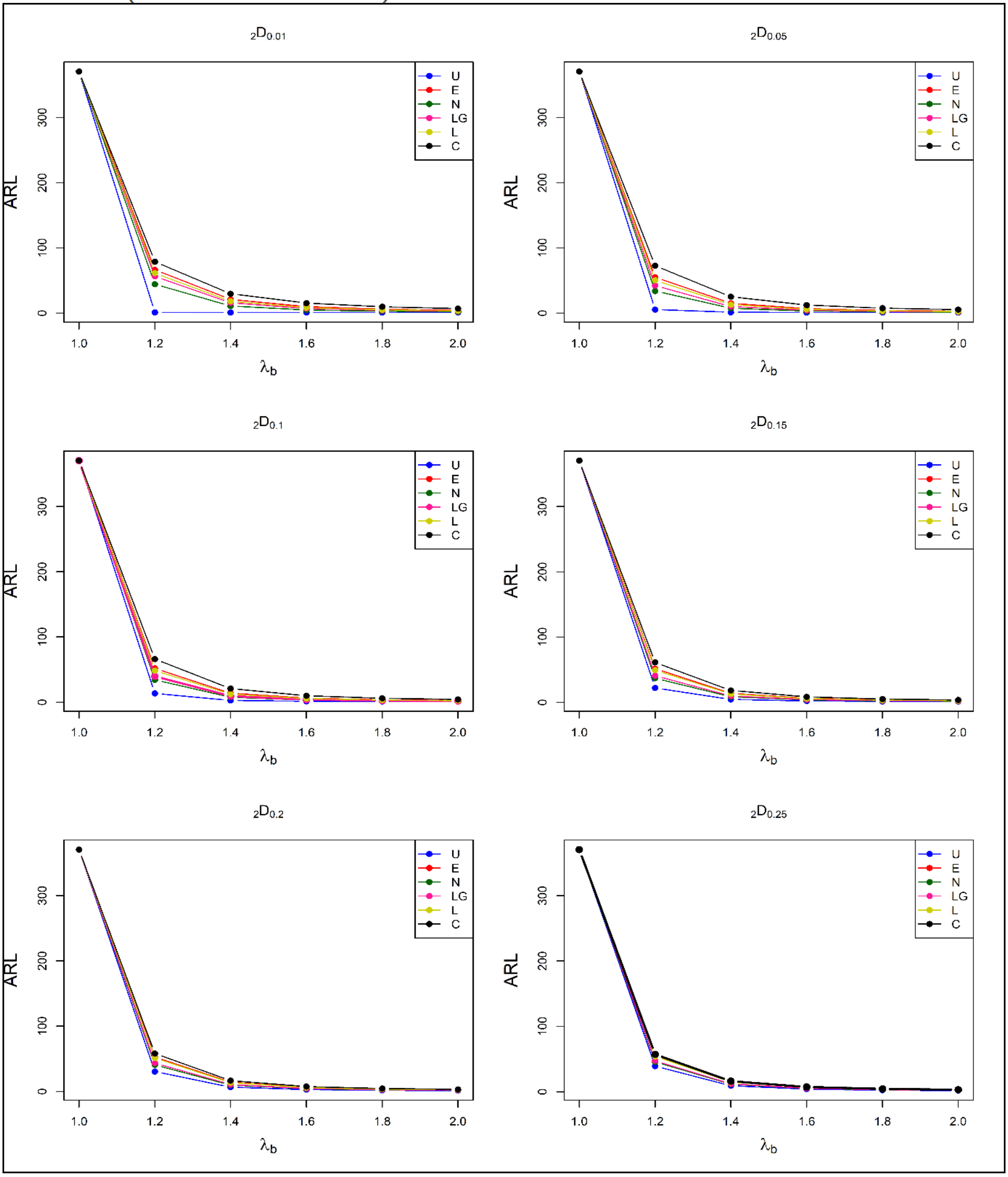

Figure 1: $ARL_{{}_2D_p}$ under various distributions

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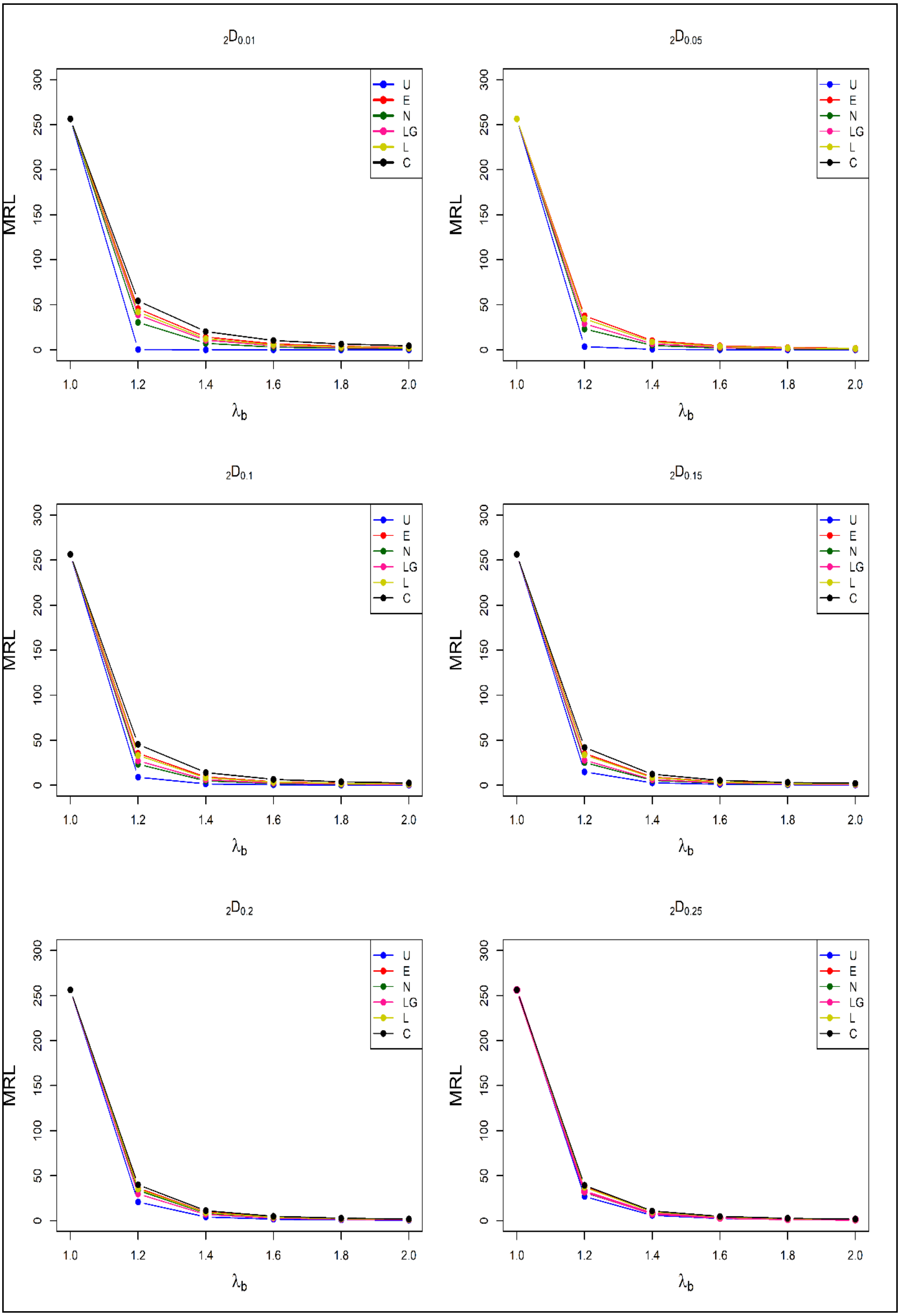

Figure 2: $MRL_{_2D_p}$ under various distributions

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From Table 1, Figures 1 and 2, we observe that, for different distributions, when there is no shift, $PW_{{}_2D_p}$ is 0.0027 while $ARL_{{}_2D_p}$, $MRL_{{}_2D_p}$ and $SDRL_{{}_2D_p}$ respectively are approximately 370, 256 and 369. For fixed b, as n increases, $P_{W_{{}_2D_p}}$ increases while $ARL_{{}_2D_p}$, $MRL_{{}_2D_p}$ and $SDRL_{{}_2D_p}$ decrease. Also, the $SDRL_{{}_2D_p}$ exhibits least value for uniform distribution followed by normal, logistic, exponential, Laplace and Cauchy distributions.

From Figures 1 and 2, we observe that, for $\lambda_b = 1.2$ and different values of p, the ARL and MRL of ${}_2D_p$ control chart is accordingly increasing from uniform distribution to normal, logistic, exponential, Laplace and Cauchy distributions.

For uniform distribution, $ARL_{{}_2D_{0.01}} < ARL_{{}_2D_{0.05}} < ARL_{{}_2D_{0.10}} < ARL_{{}_2D_{0.15}} < ARL_{{}_2D_{0.2}} < ARL_{{}_2D_{0.25}}$ for exponential distribution, $ARL_{{}_2D_{0.01}} > ARL_{{}_2D_{0.05}} > ARL_{{}_2D_{0.10}} > ARL_{{}_2D_{0.15}} < ARL_{{}_2D_{0.2}} < ARL_{{}_2D_{0.25}}$ and for normal distribution, $ARL_{{}_2D_{0.01}} > ARL_{{}_2D_{0.05}} < ARL_{{}_2D_{0.10}} < ARL_{{}_2D_{0.15}} < ARL_{{}_2D_{0.2}} < ARL_{{}_2D_{0.25}}$. Similarly for Logistic and Laplace distributions, $ARL_{{}_2D_{0.01}} > ARL_{{}_2D_{0.05}} > ARL_{{}_2D_{0.10}} < ARL_{{}_2D_{0.15}} < ARL_{{}_2D_{0.2}} < ARL_{{}_2D_{0.25}}$ and for Cauchy distribution $ARL_{{}_2D_{0.01}} > ARL_{{}_2D_{0.05}} > ARL_{{}_2D_{0.10}} > ARL_{{}_2D_{0.15}} > ARL_{{}_2D_{0.2}} > ARL_{{}_2D_{0.25}}$

Also, the $MRL_{{}_2D_p}$ follows the same pattern as $ARL_{{}_2D_p}$.

As it is observed that, the optimality of the control charts under each distribution corresponds to a specific value of p, considering $n = 10$, $\lambda_b = 1.2$, the ARL of ${}_2D_p$ control chart is plotted in Figure 3 for numerous values of p, viz. $0 < p < 1/2$. Here, the x axis represents various $SIPDs$ and y axis gives $ARL_{{}_2D_p}$. A line is dropped on x axis from the line representing minimum of $ARL_{{}_2D_p}$.

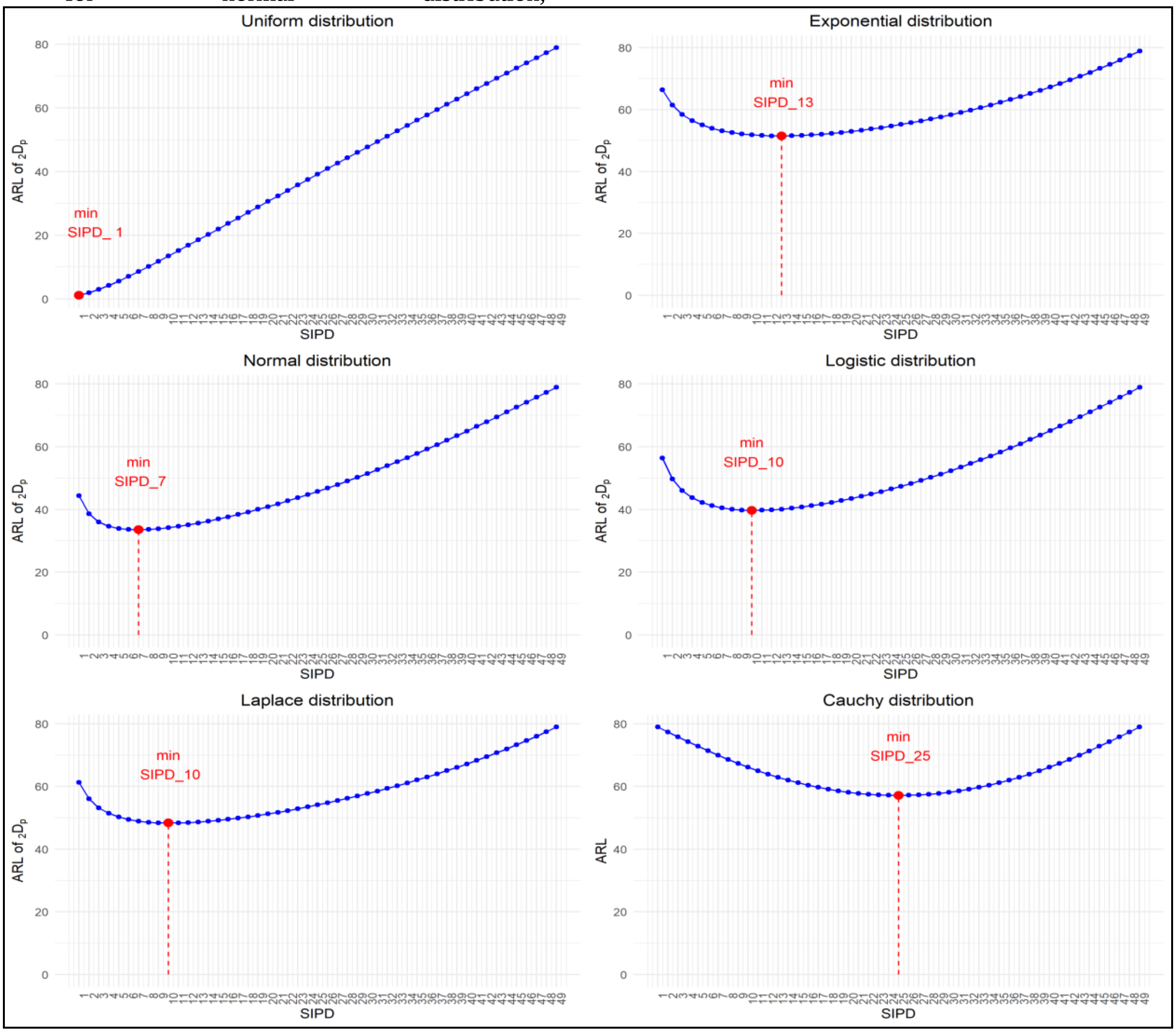

Figure 3: Optimality of ${}_2\boldsymbol{D}_p$ control charts under various distributions

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From Figure 3, we observe that, the ARL is minimum for specific quantiles of ${}_2D_p$ control charts. Under uniform distribution, the $ARL_{{}_2D_p}$ is minimum at $p = 0.01$, indicating ${}_2D_{0.01}$ control chart is optimal. Under, exponential distribution, the ARL decreases upto $p = 0.13$ and then increases, highlighting ${}_2D_{0.13}$ control chart as optimal. On similar lines, the optimal control chart is ${}_2D_{0.07}$ control chart under normal distribution, ${}_2D_{0.1}$ control chart under logistic, Laplace distributions and ${}_2D_{0.25}$ control chart under Cauchy distribution.

Considering the computations due to Abu (2008), we compare Shewhart's S control chart and MAD control chart as competitors to the proposed class of control charts. It is observed that, for $n = 5$, 10 and $\lambda_b = 1.2$, all the members of ${}_2D_p$ class of control charts perform better than S control chart under normal, logistic and Laplace distributions and better than MAD control chart under normal and Laplace distributions.

V. ILLUSTRATION

In this section, we illustrate the application of the class of ${}_2D_p$ control charts using an example due to Yang and Arnold (2016). The data represents service times (in minutes) recorded at a bank branch in Taiwan. The dataset comprises of n samples collected at m times. In Exhibit 3(a) the computations of ${}_2D_{p,i}$, s_i for $i = 1,2,\dots,10$ and their averages are carried out and in Exhibit 3(b), $\sigma_{{}_2D_p}$, $UCL_{{}_2D_p}$, $LCL_{{}_2D_p}$ and $w_{{}_2D_p}$ are computed. Also, in Exhibit 3(c), the UCL, LCL and w of various optimal control charts are obtained.

Since λ is to be estimated from the sample observations for different values of p, we calculate λ by $\hat{\lambda} = {}_2\bar{D}_{p,i}/k$, where ${}_2\bar{D}_{p,i}$ is the mean of ${}_2D_{p,i}$ and k is a varying constant for various values of p given in Exhibit 2 under various distributions. The control limits and width of S control chart is given by $UCL_S = \left(1 + \frac{3}{c_4}\sqrt{1 - c_4^2}\right)\bar{s}$, $CL_S = \bar{s}$, $LCL_S = \left(1 - \frac{3}{c_4}\sqrt{1 - c_4^2}\right)\bar{s}$ and $w_S = UCL_S - LCL_S$, where $c_4 = \left(\frac{2}{n-1}\right)^{1/2} \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$, $\bar{s} = \frac{1}{m}\sum_{i=1}^{m} s_i$ and s_i is the sd of i^{th} sample. Here $m = 10$ and $n = 10$.

Using Exhibit 3(a), 3(b), ${}_2D_p$, $p = 0.01, 0.05, 0.1, 0.15, 0.2, 0.25$ control charts are plotted in Figure 4 and using Exhibit 3(a), 3(c) the optimal control charts under various distributions along with S control chart are plotted in Figure 5. The ${}_2D_{0.01}(U)$ represents optimal $SIQD$ control chart as $SIPD_1$ under uniform distribution along with similar interpretation to other optimal control charts shown in this figure.

Exhibit 3(a): Computed values of ${}_2D_{p,i}$, s_i and their averages to data given by Yang and Arnold (2016)

p \ i	Values of ${}_2D_{p,i}$ 0.01	0.05	0.10	0.15	0.20	0.25	s_i
1	2.0628	2.0142	1.9535	1.8192	1.6640	1.4575	1.6751
2	1.8653	1.5862	1.2375	1.0445	0.8960	0.7688	1.2569
3	2.6958	2.3790	1.9830	1.6150	1.2550	0.8700	1.7713
4	3.3870	2.7552	1.9655	1.4977	1.1220	0.8213	2.1541
5	2.6811	2.4057	2.0615	1.7802	1.5170	1.2350	1.8407
6	2.7994	2.1567	1.3535	1.0647	0.9230	0.7750	1.6465
7	2.0017	1.8882	1.7465	1.4455	1.0990	0.8075	1.3902
8	1.1060	1.0502	0.9805	0.7620	0.5010	0.3750	0.7734
9	3.1138	2.8690	2.5630	1.9630	1.2790	0.9150	2.1074
10	2.2968	1.9440	1.5030	1.3280	1.2290	1.1162	1.6059
CL	2.4010	2.1049	1.7348	1.4320	1.1485	0.9141	1.6222

The control limits and width of S control chart is given by $UCL_S = 2.7836$, $LCL_S = 0.4607$ and $w_S = 2.3229$.

Exhibit 3(b): $\sigma_{{}_2D_p}$ and control limits of ${}_2D_p$ control charts for various distributions

p	D	$\sigma_{{}_2D_p}$	$UCL_{{}_2D_p}$	$LCL_{{}_2D_p}$	$w_{{}_2D_p}$	p	D	$\sigma_{{}_2D_p}$	$UCL_{{}_2D_p}$	$LCL_{{}_2D_p}$	$w_{{}_2D_p}$
0.01	U	0.1085	2.7264	2.0756	0.6508	0.15	U	0.2965	2.3214	0.5426	1.7787
	E	1.6439	7.3327	0.0000	7.3327		E	0.6117	3.2672	0.0000	3.2672
	N	0.8572	4.9726	0.0000	4.9726		N	0.4294	2.7201	0.1439	2.5762
	LG	1.1683	5.9059	0.0000	5.9059		LG	0.4692	2.8395	0.0245	2.8150
	L	1.3586	6.4767	0.0000	6.4767		L	0.5745	3.1556	0.0000	3.1556
	C	5.3183	18.3558	0.0000	18.3558		C	0.8058	3.8495	0.0000	3.8495

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0.05	U	0.2219	2.7705	1.4393	1.3312	0.20	U	0.2965	2.0381	0.2589	1.7792
	E	0.9840	5.0569	0.0000	5.0569		E	0.5073	2.6705	0.0000	2.6705
	N	0.5885	3.8705	0.3393	3.5312		N	0.3776	2.2812	0.0158	2.2653
	LG	0.7138	4.2464	0.0000	4.2464		LG	0.4015	2.3531	0.0000	2.3531
	L	0.8672	4.7065	0.0000	4.7065		L	0.4854	2.6048	0.0000	2.6048
	C	2.0301	8.1951	0.0000	8.1951		C	0.5877	2.9116	0.0000	2.9116
0.10	U	0.2743	2.5576	0.9119	1.6457	0.25	U	0.2891	1.7813	0.0469	1.7344
	E	0.7444	3.9679	0.0000	3.9679		E	0.4297	2.2032	0.0000	2.2032
	N	0.4878	3.1981	0.2714	2.9268		N	0.3372	1.9256	0.0000	1.9256
	LG	0.5548	3.3992	0.0703	3.3290		LG	0.3508	1.9666	0.0000	1.9666
	L	0.6817	3.7799	0.0000	3.7799		L	0.4171	2.1653	0.0000	2.1653
	C	1.1728	5.2531	0.0000	5.2531		C	0.4541	2.2763	0.0000	2.2763

Exhibit 3(c): Control limits and width of optimal control charts

p	0.01	0.13	0.07	0.10	0.10	0.25
D	U	E	N	LG	L	C
UCL	2.7264	3.5182	3.5822	3.3992	3.7799	2.2763
LCL	2.0756	0.0000	0.3315	0.0703	0.0000	0.0000
w	0.6508	3.5182	3.2507	3.3290	3.7799	2.2763

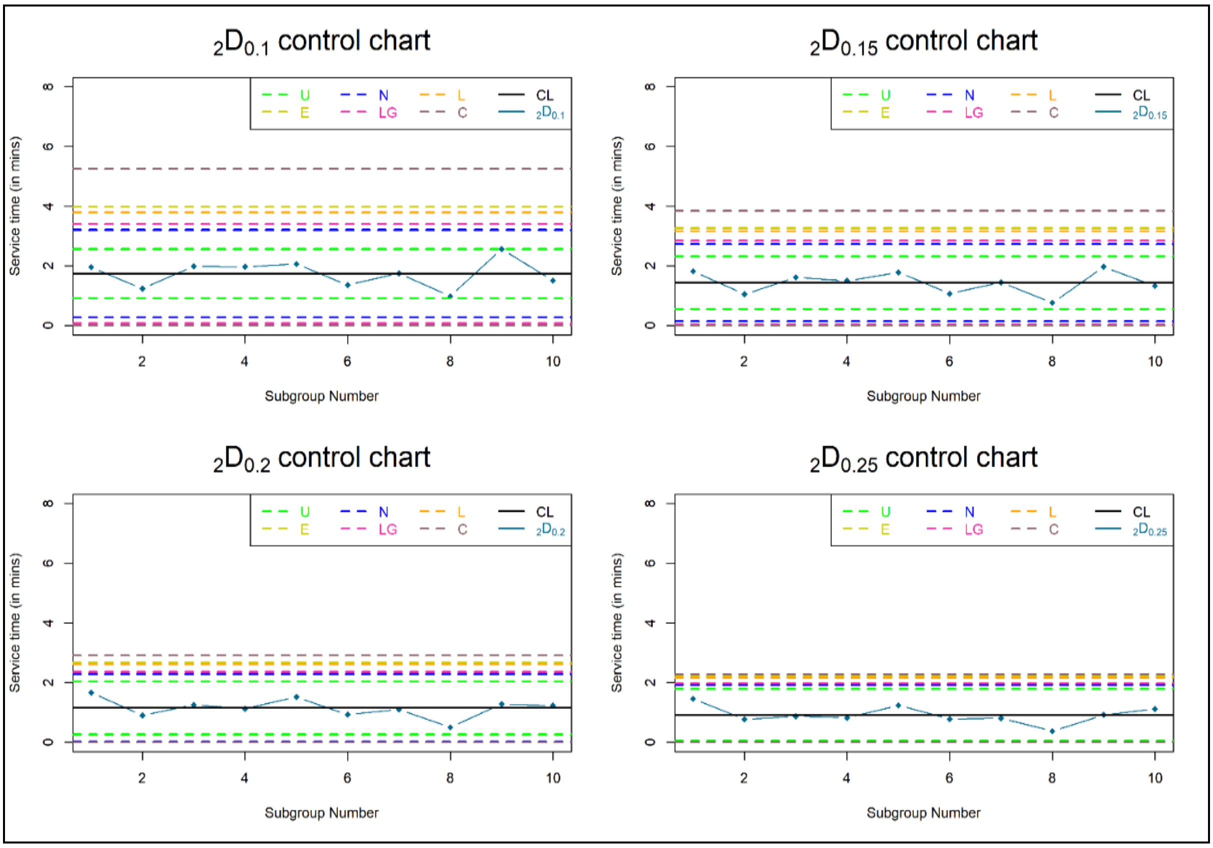

Figure 4: ${}_2D_p$ control charts for different values of p

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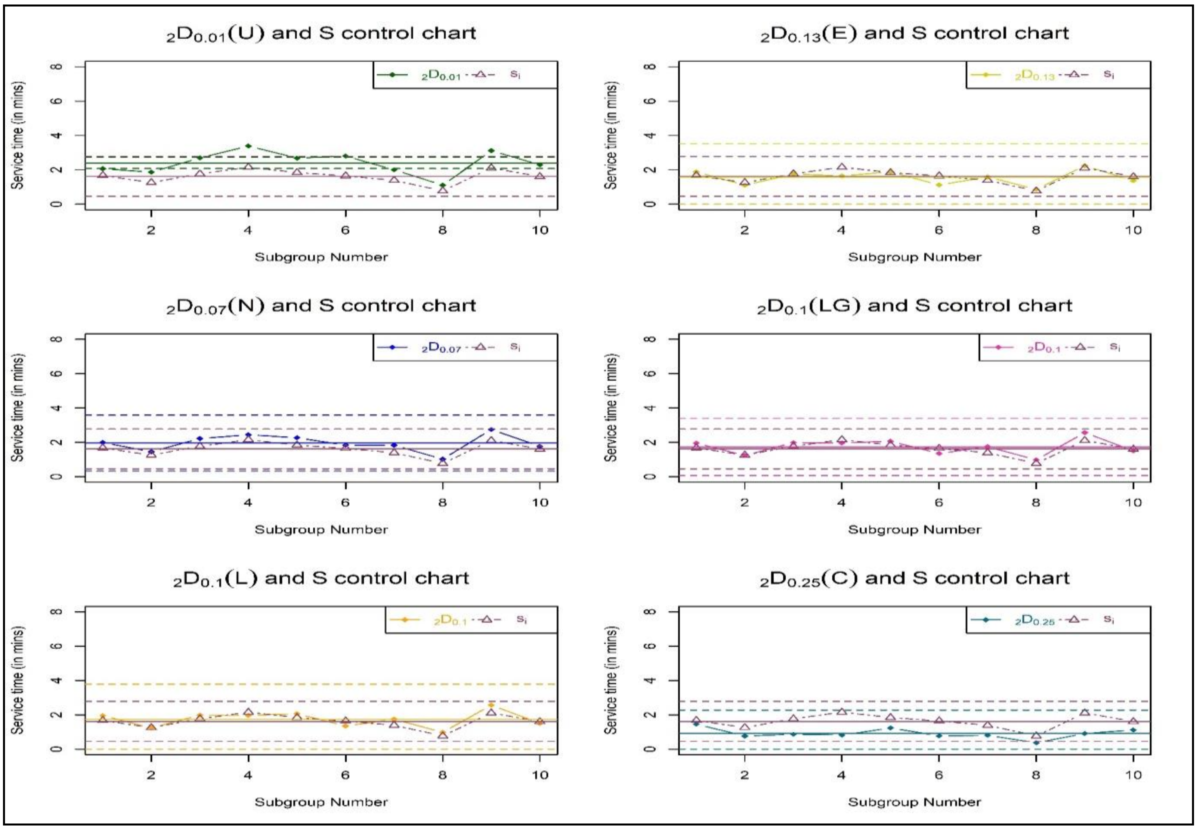

Figure 5: Optimal $_2D_p$ control charts and S control chart under various distributions

From Exhibit 3 (a), 3(b) and Figure 4, we notice that, $w_{_2D_p}$ is minimum under uniform distribution and maximum under Cauchy distribution. The $_2D_{0.15}$, $_2D_{0.2}$ and $_2D_{0.25}$ control charts indicate the process is in control. However, $_2D_{0.1}$ control chart shows out of control status for uniform distribution.

From Exhibit 3(c) and Figure 5, we observe that, $w_{_2D_{0.10}}$ is maximum under Laplace distribution and $w_{_2D_{0.01}}$ is minimum under uniform distribution. The optimal control charts and S control chart shows slight similarity in their patterns under exponential, normal, logistic and Laplace distributions, whereas they show slightly different pattern for uniform and Cauchy distributions. Further, using R software, it is seen that, the given data follows exponential distribution and both $_2D_{0.13}$ and S control charts show that the process is in control.

VI. CONCLUSIONS

In this section, based on our findings we record our conclusions on the proposed class of control charts.

- A class of control charts based on gauged inter quantile deviation (GIQD) for monitoring process scale parameter is proposed. It includes $GIQR$, $GIDD$ and $GIPD$ control charts as its subclasses.
- The class of control charts contains numerous members that are resistant to varying number of outliers.
- The GIQD control charts are developed under both symmetric and skewed distributions.
- For $g = 1$ and $g = 2$, the class reduces respectively to the IQD and SIQD control charts.
- For a specified shift, as sample size increases, the power of $_2D_p$ control chart increases whereas, ARL, MRL and SDRL of control chart decrease.
- Among $SIQD$ control charts, that is among $_2D_p$ control charts, the optimal control charts are identified as $_2D_{0.01}$ control chart under uniform distribution, $_2D_{0.13}$ control chart under exponential distribution, $_2D_{0.07}$ control chart under normal distribution, $_2D_{0.1}$ control chart under logistic and Laplace distributions and $_2D_{0.25}$ control chart under Cauchy distribution.
- The $_2D_p$ control charts perform better than Shewhart's S control chart under normal, logistic, Laplace distributions and outperforms MAD control charts due to Abu (2008) for normal, Laplace distributions.
- As the proposed class of control charts are robust to outliers, they are useful when process variables are taken from non-normal models, that is when underlying distributions are heavy tailed or skewed distributions.

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Appendix

Table 1: $PW_{{}_2D_p}$ and $SDRL_{{}_2D_p}$ for various values of p under different distributions

U / p	n / λ_b	$PW_{{}_2D_p}$ 5	10	15	20	$SDRL_{{}_2D_p}$ 5	10	15	20
0.01	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.5432	0.8828	0.9782	0.9967	1.244	0.388	0.151	0.058
	1.4	0.9901	1.0000	1.0000	1.0000	0.101	0.004	0.000	0.000
	1.6	1.0000	1.0000	1.0000	1.0000	0.006	0.000	0.000	0.000
0.05	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0836	0.1791	0.2865	0.3959	11.4452	5.0586	2.9477	1.9631
	1.4	0.4105	0.7149	0.8804	0.9545	1.8701	0.7470	0.3929	0.2234
	1.6	0.7391	0.9538	0.9935	0.9992	0.6911	0.2254	0.0813	0.0283
	1.8	0.9057	0.9946	0.9998	1.0000	0.3390	0.0738	0.0153	0.0029
	2.0	0.9681	0.9994	1.0000	1.0000	0.1844	0.0243	0.0029	0.0003
	2.5	0.9976	1.0000	1.0000	1.0000	0.0487	0.0019	0.0001	0.0000
0.10	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0402	0.0743	0.1134	0.1565	24.3417	12.9509	8.3029	5.8705
	1.4	0.1938	0.3685	0.5280	0.6601	4.6330	2.1562	1.3011	0.8833
	1.6	0.4217	0.6903	0.8484	0.9304	1.8031	0.8061	0.4589	0.2835
	1.8	0.6260	0.8737	0.9621	0.9895	0.9769	0.4067	0.2023	0.1035
	2.0	0.7692	0.9518	0.9912	0.9985	0.6245	0.2307	0.0948	0.0385
	2.5	0.9311	0.9953	0.9997	1.0000	0.2820	0.0691	0.0168	0.0039
	3.0	0.9763	0.9994	1.0000	1.0000	0.1578	0.0255	0.0040	0.0006
0.15	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0278	0.0455	0.0653	0.0868	35.4270	21.4626	14.8149	11.0063
	1.4	0.1225	0.2230	0.3255	0.4243	7.6454	3.9522	2.5231	1.7884
	1.6	0.2770	0.4748	0.6344	0.7539	3.0695	1.5265	0.9530	0.6581
	1.8	0.4417	0.6845	0.8322	0.9146	1.6918	0.8205	0.4923	0.3196
	2.0	0.5830	0.8200	0.9276	0.9723	1.1077	0.5174	0.2901	0.1712
	2.5	0.8027	0.9553	0.9906	0.9981	0.5533	0.2213	0.0978	0.0434
	3.0	0.8997	0.9868	0.9984	0.9998	0.3519	0.1164	0.0403	0.0138
0.20	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0221	0.0327	0.0442	0.0566	44.8395	30.1117	22.1261	17.1720
	1.4	0.0886	0.1506	0.2157	0.2818	10.7785	6.1192	4.1057	3.0078
	1.6	0.2001	0.3367	0.4618	0.5711	4.4704	2.4187	1.5886	1.1469
	1.8	0.3285	0.5221	0.6707	0.7787	2.4945	1.3239	0.8557	0.6042
	2.0	0.4501	0.6691	0.8084	0.8923	1.6477	0.8598	0.5415	0.3679
	2.5	0.6734	0.8697	0.9502	0.9815	0.8486	0.4151	0.2350	0.1385
	3.0	0.7979	0.9433	0.9847	0.9960	0.5635	0.2523	0.1255	0.0635
0.25	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0187	0.0255	0.0327	0.0402	52.8830	38.7352	30.1119	24.3417
	1.4	0.0690	0.1088	0.1506	0.1938	13.9845	8.6796	6.1192	4.6330
	1.6	0.1533	0.2465	0.3367	0.4217	6.0016	3.5220	2.4187	1.8031
	1.8	0.2544	0.3980	0.5221	0.6260	3.3939	1.9493	1.3239	0.9769
	2.0	0.3557	0.5334	0.6691	0.7692	2.2569	1.2807	0.8598	0.6245
	2.5	0.5618	0.7582	0.8697	0.9311	1.1782	0.6486	0.4151	0.2820
	3.0	0.6946	0.8671	0.9433	0.9763	0.7957	0.4205	0.2523	0.1578
E / p	n / λ_b	5	10	15	20	5	10	15	20
0.01	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0137	0.0151	0.0164	0.0178	72.3422	65.9110	60.4470	55.7492
	1.4	0.0397	0.0474	0.0553	0.0634	24.6823	20.5713	17.5628	15.2688
	1.6	0.0803	0.0999	0.1197	0.1396	11.9502	9.4962	7.8379	6.6446

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	1.8	0.1304	0.1647	0.1986	0.2319	7.1540	5.5480	4.5073	3.7793
	2.0	0.1845	0.2336	0.2807	0.3258	4.8937	3.7478	3.0213	2.5201
	2.5	0.3152	0.3920	0.4611	0.5230	2.6249	1.9890	1.5922	1.3204
	3.0	0.4234	0.5137	0.5905	0.6556	1.7936	1.3575	1.0837	0.8950
0.05	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0153	0.0182	0.0212	0.0243	65.0639	54.5073	46.6713	40.6346
	1.4	0.0486	0.0657	0.0835	0.1018	20.0782	14.7023	11.4614	9.3051
	1.6	0.1028	0.1454	0.1882	0.2308	9.2168	6.3587	4.7880	3.8003
	1.8	0.1697	0.2414	0.3100	0.3750	5.3702	3.6074	2.6790	2.1084
	2.0	0.2405	0.3385	0.4271	0.5063	3.6228	2.4025	1.7722	1.3878
	2.5	0.4025	0.5398	0.6478	0.7320	1.9204	1.2569	0.9161	0.7072
	3.0	0.5256	0.6726	0.7753	0.8466	1.3103	0.8507	0.6113	0.4625
0.10	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0158	0.0193	0.0229	0.0267	62.8213	51.3190	43.1047	36.9589
	1.4	0.0517	0.0723	0.0937	0.1159	18.8214	13.3171	10.1552	8.1159
	1.6	0.1107	0.1614	0.2121	0.2623	8.5184	5.6743	4.1845	3.2741
	1.8	0.1833	0.2675	0.3469	0.4209	4.9302	3.1995	2.3292	1.8081
	2.0	0.2596	0.3728	0.4727	0.5596	3.3150	2.1247	1.5363	1.1858
	2.5	0.4307	0.5833	0.6977	0.7823	1.7521	1.1067	0.7881	0.5964
	3.0	0.5572	0.7154	0.8185	0.8851	1.1942	0.7456	0.5204	0.3830
0.15	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0158	0.0194	0.0230	0.0268	62.6977	51.1473	42.9160	36.7671
	1.4	0.0519	0.0727	0.0943	0.1167	18.7541	13.2453	10.0887	8.0562
	1.6	0.1112	0.1623	0.2135	0.2641	8.4815	5.6393	4.1542	3.2481
	1.8	0.1841	0.2690	0.3490	0.4234	4.9071	3.1788	2.3118	1.7933
	2.0	0.2606	0.3747	0.4752	0.5625	3.2990	2.1106	1.5246	1.1759
	2.5	0.4322	0.5856	0.7003	0.7849	1.7433	1.0991	0.7817	0.5909
	3.0	0.5589	0.7177	0.8207	0.8870	1.1882	0.7403	0.5158	0.3791
0.20	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0156	0.0189	0.0223	0.0258	63.6087	52.4224	44.3258	38.2068
	1.4	0.0506	0.0700	0.0901	0.1108	19.2549	13.7856	10.5916	8.5098
	1.6	0.1079	0.1557	0.2036	0.2511	8.7571	5.9035	4.3843	3.4470
	1.8	0.1784	0.2582	0.3339	0.4047	5.0799	3.3355	2.4447	1.9065
	2.0	0.2528	0.3606	0.4567	0.5411	3.4195	2.2172	1.6141	1.2520
	2.5	0.4207	0.5681	0.6806	0.7654	1.8092	1.1567	0.8304	0.6328
	3.0	0.5461	0.7007	0.8041	0.8725	1.2336	0.7807	0.5505	0.4092
0.25	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0152	0.0181	0.0211	0.0242	65.1988	54.7036	46.8948	40.8681
	1.4	0.0484	0.0654	0.0829	0.1010	20.1561	14.7908	11.5465	9.3836
	1.6	0.1023	0.1445	0.1868	0.2289	9.2607	6.4032	4.8279	3.8355
	1.8	0.1689	0.2399	0.3079	0.3722	5.3981	3.6340	2.7023	2.1286
	2.0	0.2394	0.3365	0.4244	0.5031	3.6424	2.4207	1.7879	1.4013
	2.5	0.4008	0.5371	0.6447	0.7288	1.9312	1.2667	0.9246	0.7146
	3.0	0.5237	0.6699	0.7725	0.8441	1.3177	0.8576	0.6173	0.4679
N / p	n / λ_b	5	10	15	20	5	10	15	20
0.01	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0173	0.0225	0.0280	0.0337	57.1888	43.8985	35.2420	29.1813
	1.4	0.0608	0.0913	0.1234	0.1565	15.9457	10.4376	7.5896	5.8680
	1.6	0.1332	0.2065	0.2789	0.3488	6.9906	4.3136	3.0453	2.3134
	1.8	0.2212	0.3384	0.4442	0.5377	3.9886	2.4038	1.6781	1.2646
	2.0	0.3115	0.4622	0.5859	0.6848	2.6637	1.5865	1.0985	0.8197
	2.5	0.5038	0.6866	0.8052	0.8805	1.3982	0.8154	0.5482	0.3926
	3.0	0.6358	0.8092	0.9015	0.9497	0.9491	0.5398	0.3482	0.2361
0.05	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0206	0.0294	0.039	0.0492	48.1046	33.4628	25.1451	19.8302

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	1.4	0.0798	0.1319	0.1867	0.2428	12.0142	7.0627	4.8300	3.5833
	1.6	0.1794	0.2974	0.4083	0.5087	5.0484	2.8187	1.8838	1.3777
	1.8	0.2963	0.4698	0.6105	0.7195	2.8311	1.5498	1.0224	0.7360
	2.0	0.4098	0.6139	0.7551	0.8483	1.8750	1.0121	0.6555	0.4592
	2.5	0.6278	0.8283	0.9234	0.9667	0.9718	0.5002	0.2996	0.1886
	3.0	0.7572	0.9172	0.9726	0.9912	0.6506	0.3137	0.1701	0.0949
0.10	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0205	0.0292	0.0387	0.0487	48.3318	33.7033	25.3659	20.0272
	1.4	0.0793	0.1307	0.1848	0.2403	12.1034	7.1329	4.8846	3.6272
	1.6	0.1781	0.2948	0.4047	0.5045	5.0907	2.8487	1.9062	1.3954
	1.8	0.2942	0.4663	0.6062	0.7152	2.8559	1.5668	1.0350	0.7461
	2.0	0.4071	0.6101	0.7512	0.8449	1.8917	1.0235	0.6641	0.4661
	2.5	0.6246	0.8252	0.9213	0.9655	0.9808	0.5066	0.3045	0.1925
	3.0	0.7543	0.9152	0.9716	0.9907	0.6570	0.3183	0.1735	0.0974
0.15	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0195	0.0271	0.0352	0.0438	50.8631	36.4506	27.9267	22.3358
	1.4	0.0733	0.1180	0.1650	0.2135	13.1268	7.9582	5.5365	4.1546
	1.6	0.1638	0.2671	0.3660	0.4578	5.5818	3.2053	2.1753	1.6084
	1.8	0.2714	0.4277	0.5594	0.6663	3.1448	1.7690	1.1867	0.8669
	2.0	0.3778	0.5673	0.7064	0.8047	2.0875	1.1595	0.7671	0.5492
	2.5	0.5896	0.7891	0.8946	0.9485	1.0867	0.5819	0.3629	0.2393
	3.0	0.7215	0.8900	0.9577	0.9840	0.7316	0.3726	0.2149	0.1285
0.20	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0182	0.0244	0.0310	0.0379	54.3076	40.3980	31.7346	25.8527
	1.4	0.0661	0.1027	0.1411	0.1808	14.6126	9.2258	6.5684	5.0060
	1.6	0.1463	0.2327	0.3169	0.3970	6.3138	3.7649	2.6082	1.9561
	1.8	0.2430	0.3778	0.4961	0.5971	3.5804	2.0881	1.4308	1.0632
	2.0	0.3406	0.5096	0.6420	0.7426	2.3841	1.3741	0.9320	0.6832
	2.5	0.5425	0.7353	0.8500	0.9165	1.2469	0.6997	0.4556	0.3154
	3.0	0.6753	0.8492	0.9313	0.9692	0.8438	0.4572	0.2814	0.1812
0.25	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0170	0.0219	0.0269	0.0322	58.2664	45.2570	36.6372	30.5294
	1.4	0.0589	0.0874	0.1172	0.1481	16.4671	10.9318	8.0159	6.2334
	1.6	0.1286	0.1973	0.2653	0.3315	7.2606	4.5413	3.2305	2.4667
	1.8	0.2135	0.3242	0.4252	0.5153	4.1530	2.5356	1.7833	1.3510
	2.0	0.3011	0.4448	0.5645	0.6620	2.7767	1.6754	1.1691	0.8781
	2.5	0.4896	0.6676	0.7866	0.8647	1.4594	0.8637	0.5872	0.4254
	3.0	0.6209	0.7928	0.8882	0.9404	0.9917	0.5741	0.3764	0.2597
LG / p	n / λ_b	5	10	15	20	5	10	15	20
0.01	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0150	0.0177	0.0205	0.0233	66.0300	55.9253	48.2958	42.3402
	1.4	0.0473	0.0631	0.0794	0.0962	20.6414	15.3505	12.0893	9.8875
	1.6	0.0995	0.1388	0.1783	0.2177	9.5361	6.6860	5.0840	4.0628
	1.8	0.1640	0.2306	0.2945	0.3554	5.5735	3.8043	2.8520	2.2592
	2.0	0.2326	0.3240	0.4075	0.4829	3.7657	2.5372	1.8891	1.4892
	2.5	0.3906	0.5207	0.6251	0.7083	1.9989	1.3297	0.9794	0.7625
	3.0	0.5120	0.6532	0.7548	0.8274	1.3643	0.9015	0.6560	0.5021
0.05	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0179	0.0237	0.0298	0.0362	55.4188	41.7258	33.0503	27.0913
	1.4	0.0640	0.0982	0.1340	0.1711	15.1166	9.6754	6.9432	5.3202
	1.6	0.1411	0.2223	0.3019	0.3781	6.5674	3.9668	2.7676	2.0857
	1.8	0.2344	0.3623	0.4760	0.5743	3.7328	2.2040	1.5209	1.1362
	2.0	0.3292	0.4912	0.6206	0.7209	2.4883	1.4520	0.9927	0.7328
	2.5	0.5275	0.7169	0.8336	0.9037	1.3032	0.7422	0.4894	0.3435
	3.0	0.6602	0.8344	0.9207	0.9625	0.8830	0.4876	0.3059	0.2011

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0.10	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0186	0.0252	0.0322	0.0396	53.3033	39.2212	30.5833	24.7788
	1.4	0.0681	0.1069	0.1477	0.1899	14.1678	8.8374	6.2482	4.7398
	1.6	0.1512	0.2423	0.3308	0.4144	6.0922	3.5919	2.4729	1.8468
	1.8	0.2510	0.3920	0.5144	0.6175	3.4479	1.9892	1.3545	1.0016
	2.0	0.3512	0.5263	0.6611	0.7615	2.2937	1.3076	0.8805	0.6413
	2.5	0.5561	0.7515	0.8640	0.9270	1.1981	0.6633	0.4267	0.2916
	3.0	0.6889	0.8619	0.9400	0.9743	0.8096	0.4311	0.2607	0.1645
0.15	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0183	0.0245	0.0312	0.0381	54.1858	40.2541	31.5931	25.7202
	1.4	0.0664	0.1032	0.1419	0.1819	14.5582	9.1779	6.5286	4.9729
	1.6	0.1469	0.2338	0.3186	0.3991	6.2866	3.7434	2.5913	1.9424
	1.8	0.2440	0.3795	0.4983	0.5995	3.5641	2.0759	1.4213	1.0555
	2.0	0.3419	0.5116	0.6443	0.7449	2.3730	1.3658	0.9256	0.6780
	2.5	0.5441	0.7373	0.8518	0.9178	1.2409	0.6952	0.4520	0.3124
	3.0	0.6770	0.8508	0.9324	0.9698	0.8396	0.4540	0.2788	0.1791
0.20	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0176	0.0230	0.0287	0.0347	56.4363	42.9660	34.2954	28.2746
	1.4	0.0621	0.0942	0.1278	0.1626	15.5892	10.1063	7.3070	5.6276
	1.6	0.1365	0.2131	0.2885	0.3611	6.8077	4.1622	2.9234	2.2132
	1.8	0.2268	0.3485	0.4577	0.5533	3.8778	2.3164	1.6091	1.2081
	2.0	0.3189	0.4745	0.6007	0.7004	2.5876	1.5277	1.0520	0.7815
	2.5	0.5138	0.6996	0.8176	0.8908	1.3570	0.7834	0.5224	0.3710
	3.0	0.6462	0.8202	0.9100	0.9555	0.9205	0.5171	0.3297	0.2207
0.25	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0167	0.0211	0.0257	0.0306	59.5360	46.8937	38.3441	32.1972
	1.4	0.0568	0.0829	0.1103	0.1385	17.0983	11.5461	8.5538	6.6991
	1.6	0.1233	0.1868	0.2499	0.3115	7.5915	4.8277	3.4664	2.6636
	1.8	0.2047	0.3079	0.4030	0.4889	4.3556	2.7022	1.9176	1.4621
	2.0	0.2891	0.4244	0.5391	0.6344	2.9164	1.7878	1.2594	0.9531
	2.5	0.4729	0.6447	0.7635	0.8443	1.5352	0.9246	0.6369	0.4674
	3.0	0.6032	0.7726	0.8711	0.9277	1.0442	0.6173	0.4121	0.2899
L / p	n / λ_b	5	10	15	20	5	10	15	20
0.01	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0143	0.0163	0.0183	0.0204	69.2212	60.8109	54.0807	48.5777
	1.4	0.0433	0.0548	0.0666	0.0787	22.6019	17.7511	14.5113	12.2004
	1.6	0.0894	0.1183	0.1474	0.1766	10.6797	7.9386	6.2632	5.1368
	1.8	0.1464	0.1962	0.2448	0.2919	6.3124	4.5695	3.5501	2.8829
	2.0	0.2075	0.2774	0.3430	0.4041	4.2893	3.0644	2.3634	1.9100
	2.5	0.3519	0.4564	0.5455	0.6212	2.2879	1.6156	1.2357	0.9907
	3.0	0.4671	0.5854	0.6784	0.7512	1.5627	1.0999	0.8360	0.6640
0.05	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0161	0.0199	0.0239	0.0280	61.6723	49.7392	41.3804	35.2154
	1.4	0.0534	0.0759	0.0993	0.1235	18.2035	12.6656	9.5573	7.5816
	1.6	0.1150	0.1700	0.2250	0.2791	8.1820	5.3592	3.9136	3.0418
	1.8	0.1906	0.2813	0.3663	0.4446	4.7204	3.0135	2.1734	1.6762
	2.0	0.2697	0.3906	0.4960	0.5863	3.1691	1.9985	1.4315	1.0971
	2.5	0.4453	0.6051	0.7217	0.8055	1.6726	1.0385	0.7310	0.5474
	3.0	0.5733	0.7362	0.8383	0.9017	1.1393	0.6977	0.4796	0.3477
0.10	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0165	0.0207	0.0251	0.0297	60.2675	47.8548	39.3598	33.1995
	1.4	0.0556	0.0805	0.1064	0.1333	17.4704	11.9167	8.8827	6.9863
	1.6	0.1204	0.1809	0.2412	0.3003	7.7888	5.0023	3.6119	2.7860
	1.8	0.1998	0.2987	0.3903	0.4737	4.4770	2.8041	2.0007	1.5313
	2.0	0.2824	0.4127	0.5244	0.6181	3.0003	1.8567	1.3153	0.9997

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	2.5	0.4634	0.6313	0.7496	0.8317	1.5808	0.9619	0.6675	0.4933
	3.0	0.5930	0.7604	0.8605	0.9195	1.0758	0.6437	0.4341	0.3087
0.15	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0163	0.0203	0.0245	0.0289	60.8935	48.6882	40.2486	34.0826
	1.4	0.0546	0.0784	0.1032	0.1288	17.7941	12.2442	9.1760	7.2442
	1.6	0.1180	0.1760	0.2339	0.2908	7.9616	5.1577	3.7426	2.8965
	1.8	0.1957	0.2909	0.3795	0.4607	4.5837	2.8951	2.0754	1.5938
	2.0	0.2767	0.4028	0.5117	0.6040	3.0743	1.9183	1.3655	1.0417
	2.5	0.4553	0.6197	0.7374	0.8203	1.6210	0.9952	0.6950	0.5167
	3.0	0.5843	0.7498	0.8509	0.9119	1.1036	0.6672	0.4538	0.3255
0.20	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0159	0.0195	0.0233	0.0272	62.4021	50.7385	42.4678	36.3124
	1.4	0.0524	0.0736	0.0957	0.1186	18.5940	13.0751	9.9319	7.9156
	1.6	0.1122	0.1645	0.2168	0.2684	8.3940	5.5567	4.0829	3.1868
	1.8	0.1859	0.2725	0.3540	0.4295	4.8525	3.1300	2.2707	1.7584
	2.0	0.2632	0.3792	0.4812	0.5694	3.2609	2.0775	1.4970	1.1525
	2.5	0.4360	0.5913	0.7065	0.7910	1.7226	1.0812	0.7667	0.5780
	3.0	0.5631	0.7231	0.8259	0.8914	1.1739	0.7277	0.5051	0.3697
0.25	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0154	0.0185	0.0217	0.0249	64.4512	53.6222	45.6694	39.5925
	1.4	0.0494	0.0675	0.0862	0.1056	19.7280	14.3080	11.0846	8.9590
	1.6	0.1049	0.1497	0.1946	0.2392	9.0202	6.1618	4.6122	3.6459
	1.8	0.1733	0.2484	0.3200	0.3874	5.2457	3.4895	2.5768	2.0200
	2.0	0.2457	0.3478	0.4396	0.5210	3.5355	2.3221	1.7032	1.3283
	2.5	0.4101	0.5518	0.6618	0.7464	1.8726	1.2134	0.8787	0.6746
	3.0	0.5343	0.6846	0.7878	0.8580	1.2773	0.8203	0.5848	0.4392
C / p	n / λ_b	5	10	15	20	5	10	15	20
0.01	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0125	0.0127	0.0128	0.0129	79.2212	78.4387	77.6703	76.9155
	1.4	0.0328	0.0336	0.0343	0.0350	29.9453	29.2935	28.6677	28.0663
	1.6	0.0626	0.0645	0.0663	0.0682	15.4556	14.9970	14.5636	14.1532
	1.8	0.0989	0.1023	0.1056	0.1089	9.5965	9.2664	8.9574	8.6675
	2.0	0.1386	0.1435	0.1484	0.1533	6.6987	6.4502	6.2189	6.0030
	2.5	0.2386	0.2471	0.2554	0.2637	3.6563	3.5119	3.3783	3.2542
	3.0	0.3282	0.3389	0.3495	0.3598	2.4975	2.3992	2.3081	2.2234
0.05	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0131	0.0137	0.0144	0.0151	75.9728	72.2865	68.9140	65.8171
	1.4	0.0359	0.0398	0.0436	0.0476	27.3331	24.6437	22.4061	20.5162
	1.6	0.0706	0.0804	0.0903	0.1002	13.6607	11.9261	10.5632	9.4648
	1.8	0.1131	0.1306	0.1480	0.1653	8.3231	7.1378	6.2363	5.5280
	2.0	0.1595	0.1849	0.2099	0.2344	5.7479	4.8820	4.2351	3.7337
	2.5	0.2741	0.3159	0.3556	0.3932	3.1082	2.6184	2.2578	1.9813
	3.0	0.3729	0.4241	0.4714	0.5150	2.1238	1.7891	1.5421	1.3521
0.10	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0138	0.0151	0.0165	0.0179	72.1628	65.6088	60.0600	55.3037
	1.4	0.0399	0.0478	0.0560	0.0642	24.5581	20.3942	17.3642	15.0638
	1.6	0.0808	0.1009	0.1212	0.1416	11.8726	9.3955	7.7323	6.5407
	1.8	0.1312	0.1665	0.2012	0.2353	7.1020	5.4838	4.4422	3.7167
	2.0	0.1858	0.2361	0.2843	0.3303	4.8561	3.7026	2.9763	2.4773
	2.5	0.3173	0.3957	0.4661	0.5290	2.6038	1.9642	1.5677	1.2973
	3.0	0.4259	0.5180	0.5959	0.6617	1.7792	1.3404	1.0667	0.8789
0.15	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0144	0.0164	0.0184	0.0205	69.1167	60.6457	53.8803	48.3572
	1.4	0.0434	0.0550	0.0670	0.0792	22.5351	17.6654	14.4221	12.1134
	1.6	0.0897	0.1189	0.1484	0.1779	10.6399	7.8927	6.2187	5.0955

	1.8	0.1469	0.1973	0.2464	0.2939	6.2864	4.5412	3.5235	2.8587
	2.0	0.2083	0.2789	0.3451	0.4067	4.2708	3.0448	2.3453	1.8936
	2.5	0.3531	0.4585	0.5483	0.6243	2.2776	1.6049	1.2259	0.9818
	3.0	0.4686	0.5877	0.6811	0.7540	1.5557	1.0925	0.8291	0.6578
0.20	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0148	0.0172	0.0197	0.0222	67.1718	57.6370	50.2890	44.4601
	1.4	0.0458	0.0600	0.0746	0.0897	21.3247	16.1610	12.8898	10.6403
	1.6	0.0958	0.1312	0.1669	0.2026	9.9290	7.1017	5.4671	4.4067
	1.8	0.1576	0.2180	0.2764	0.3325	5.8254	4.0562	3.0771	2.4576
	2.0	0.2235	0.3071	0.3843	0.4549	3.9435	2.7101	2.0416	1.6228
	2.5	0.3766	0.4979	0.5975	0.6787	2.0967	1.4233	1.0618	0.8351
	3.0	0.4960	0.6296	0.7290	0.8025	1.4315	0.9666	0.7141	0.5539
0.25	1.0	0.0027	0.0027	0.0027	0.0027	369.8980	369.8980	369.8980	369.8980
	1.2	0.0149	0.0175	0.0202	0.0229	66.5070	56.6356	49.1186	43.2116
	1.4	0.0467	0.0618	0.0774	0.0934	20.9245	15.6830	12.4156	10.1930
	1.6	0.0979	0.1356	0.1735	0.2114	9.6982	6.8557	5.2394	4.2017
	1.8	0.1613	0.2253	0.2869	0.3458	5.6771	3.9068	2.9431	2.3392
	2.0	0.2288	0.3170	0.3978	0.4712	3.8388	2.6075	1.9508	1.5430
	2.5	0.3847	0.5112	0.6137	0.6962	2.0390	1.3678	1.0127	0.7918
	3.0	0.5053	0.6435	0.7442	0.8173	1.3919	0.9280	0.6795	0.5230

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