Joint Sparse Graph OFDM using Low Density Parity Check

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Abstract - Increasing demand of high data rate applications at low cost, wireless communication is the key area of research, the solution to this problem is orthogonal frequency division multiplexing (OFDM). Multiple access and forward error correction (FEC) technique like OFDM with low density parity check (LDPC) and low density signature is the best choice over classical OFDM. This paper proposes joint sparse graph OFDM using low density parity check coding by combining joint sparse graph OFDM transmission techniques. The different low complexity transceiver structure joint multiuser detection and FEC decoding and shown numerical. In this paper propose JSG OFDM using LDPC coding that achieves significantly better BER (bit error rate) performance than JSG OFDM for several different system configurations.

Keywords— OFDM, Joint Sparse Graph, LDPC.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) technique is a multicarrier transmission technique which being recognized as an excellent method for high speed bidirectional wireless data communication. For high speed wireless communication OFDM is the best choice and multicarrier transmission and used in many standards such as worldwide interoperability for microwave access (WIMAX), 4th generation partnership project long term evolution, Digital Audio and Video broadcasting and wireless local area network (WLAN) [6-7]. Feature of the OFDM are controlled overlapping of bands, maximum spectral efficiency. Joint sparse graph OFDM is based on a joint sparse graph (JSG) which combines multiple access and sparse coding [5]. The linear detectors are the minimum mean square error (MMSE) detector which implements the linear mapping [10-11] and the decorrectator invert the channel matrix [12]. Non linear detectors successive interference cancellation is highly dependent on the interference cancelation detector [13]. In OFDM system multiple antennas are useful for providing transmit and receive diversity to overcome fading [14]. Due to low density signature (LDS) structures each data symbol is only spread over a limited number of chips in LDS OFDM [16]. Low density parity check code (LDPC) is a linear error correcting code, a method of transmitting a message over a noisy transmission channel. LDPC codes constructed using sparse bipartite graph [9]. LDPC decoding may be performed at greater speeds [2]. Message passing LDPC decoder though a single joint graph is message passing to do joint detection decoding. Block code are usually decoded using hard decision algorithms. For every input and output signal a hard decision is made whether it corresponds to a one or a zero bits. Convolution codes are typically decoded using soft decision algorithms like message passing algorithms. The higher error correction is performance than hard decision decoding. In LDPC codes larger girth improves the computational and bit error rate performance [3-8]. LDPC codes are finding increasing use in applications requiring reliable and highly efficient information transfer over bandwidth. LDPC codes better performance and lower decoding complexity, linear block codes whose parity check matrix has a low density of one’s that is sparse drawback of LDPC codes is their apparently high encoding complexity [20]. In digital communication QPSK is a higher order modulation scheme. Because of its advanced noise immunity, bandwidth efficiency and simpler circuitry it has generally used in OFDM. In this paper considering the system model and joint multiuser detection and decoding schemes as proposed in the previous work [1], this paper provide the performance analysis of given system using LDPC and present the performance analysis to reduce BER and results are obtained using numerical calculation on MATLAB. The rest of the paper is organized as follows: In section II, Illustrate the given system model of JSG-OFDM. In section III, Discussed different detection schemes. In section IV, JSG-OFDM using LDPC present. In section V, MATLAB results are given, finally section VI concludes the letter.

II. SYSTEM MODEL OF JSG-OFDM

JSG-OFDM transmitting number of K users to the same base station and each user are equipped with a single antenna. The block diagram of JSG-OFDM system is shown in fig.1.

![Block diagram for JSG OFDM Transmitter](image)

Let the number of chips gain to be N, each user has a data vector of M data symbols and J be the number of parity check equations in the LDPC code. Fig.1. shown transmitter the function are number of users, encoding, and mapping, data symbol are subsequently OFDM modulation is modulate the chips into subcarrier frequencies and multiplied with a random...
sequence of chips. The spreading signature has low density by the use of zero padding which means a large number of chips in the sequence are zeros is main differencnce JSG-OFDM transmitter. Each user’s generated chip is transmitter over an orthogonal subcarrier and each subcarrier is only used by a limited number of symbols that may belong to different users. Each user, transmitting on given subcarrier will experience interference from only a small number of other user’s data symbols.

The receiver there are three types of nodes check nodes, variable node, and parity check nodes, representing the check, the data symbol, and parity check equation of the \( k \)-th user, respectively. Variable nodes are connecting to the check nodes and parity check nodes through low density edges. The joint spares graph is arranged to process the chips from the received signal to the transmitted data [1].

### III.JMUDF FOR JSG-OFDM

Message passing algorithms (MPA) are bit probabilities using intrinsic (before an event) and extrinsic (after an event) information [4]. JMUDD is the joint sparse graph. System model is based on single antenna transmission where neither the transmitter nor receivers have multiple antennas but the JSG can be extended to multiple inputs multiple outputs (MIMO).

The generate a \( (N, K) \) linear block with a generator matrix \( G \) with \( N \) and \( K \) corresponding to size of codeword or check nodes and information word or number of users. The generator matrix \( G \) is a \( K \) by \( N \) binary matrix. The complexity of multiplying a codeword with a matrix depends on the amount of 1’s in the matrix. Sparse matrix \( H \) in the form \([P^T I]\) via the generator matrix \( G \) can be calculated as

\[
G = \begin{bmatrix} I_N & P \end{bmatrix}
\]

Where \( I_N \) identity matrix with the size \( K \) by \( P \) is a \( K \) by \( (N-K) \) matrix

The systematic generator matrix it is easy to find a systematic parity check matrix.

\[
H_s = \begin{bmatrix} P^T & I_{N-K} \end{bmatrix}
\]

Tanner graph is bipartite graph, a graph with vertices separated into two sets and edges connecting nodes from different sets

\[
S = \mathbb{C} \cup \mathbb{V}
\]

There is no edge between nodes in the same set. The tanner graph of an LDPC code with parity check matrix \( H \) has two types of nodes.

Nodes in \( V \) for each row of \( H \) are called variable nodes and \( C \) for each column of \( H \) is called check nodes. There are edges between check node \( i \) and \( j \) when \( h_{ij} = 1 \).

The check node degree \( \delta_c \) and variable node degree \( \delta_v \) the LDPC codeword can be represented as

\[
C = \{ x \in \mathbb{b}^N :HX^T = 0 \}
\]

\[
H \text{ is the parity check matrix can be written as follows}
\]

\[
h_i = \begin{bmatrix} h_{i1} & \cdots & h_{iN} \end{bmatrix}^T, \quad \text{ where every row means one check requirement, then the membership indicator function is}
\]

\[
I_c = \begin{cases} 1 & \text{if } x \in C \\ 0 & \text{else} \end{cases}
\]

The method estimated codeword \( \hat{x} \) form a received word \( y \), the detection maximum a posteriori (MAP) estimation and detection is optimal

\[
\hat{x}(y) = \arg \max_x p(x|y)
\]

Form Bayes rule,

\[
p(x|y) = \frac{p(y|x)p(x)}{p(y)}
\]

\[
\frac{p(y|x)p(x)}{p(y)}
\]

Because random codeword are used in practical, all symbols probabilities \( p(x) \) are equally likely form \( p(x) = p(y) \approx 1 \) can be transformed into

\[
\hat{x}_v(y) = \arg \max_x p(y|x)
\]

\[
\hat{x}_v(y) = \arg \max_x \prod_{x \in \mathbb{V}} p(x|y)
\]

Considering minimization of bit error probability bitwise MAP decoder estimates the codeword like

\[
\hat{x}_v(y) = [\hat{x}_{v1}(y), \hat{x}_{v2}(y), \ldots, \hat{x}_{vN}(y)]^T
\]

\[
\hat{x}(y) = \arg \max_x p(x|y)
\]

The decoder messages between variable nodes and check nodes respectively every time messages \( \mu_{\psi_{-\psi_1}}(x_{\psi_1}) \) form variable node \( \psi_1 \) to check node \( \psi_1 \) and the message \( \mu_{\psi_1\psi_{-\psi_1}}(x_{\psi_1}) \) transmitting in the opposite way are updated.

\[
\mu_{\psi_1\psi_{-\psi_1}}(x_{\psi_1}) = p(y_{\psi_1}/x_{\psi_1}) \prod_{x \in \mathbb{V}} p_{\psi_1\psi_{-\psi_1}}(x_{\psi_1})
\]

\[
\mu_{\psi_{-\psi_1}}(x_{\psi_1}) = \sum_{x \in \mathbb{V}} \psi_1(x_{\psi_1}) \mu_{\psi_{-\psi_1}}(x_{\psi_1})
\]

And

\[
b\psi_{-\psi_1}(x_{\psi_1}) = p(y_{\psi_1}/x_{\psi_1}) \prod_{x \in \mathbb{V}} \mu_{\psi_{-\psi_1}}(x_{\psi_1})
\]

\[
\mu_{\psi_{-\psi_1}}(x_{\psi_1}) = \mu_{\psi_{-\psi_1}}(x_{\psi_1}) \mu_{\psi_1\psi_{-\psi_1}}(x_{\psi_1})
\]

In the message passing algorithms, messages are often computed in the logarithmic domain. Form eq\( ^2 \) (14) and (16) in the logarithmic domain becomes

\[
\log \mu_{\psi_{-\psi_1}}(x_{\psi_1}) = \log p(y_{\psi_1}/x_{\psi_1}) + \sum_{x \in \mathbb{V}} \psi_1(x_{\psi_1}) \log \mu_{\psi_{-\psi_1}}(x_{\psi_1})
\]

\[
\log b\psi_{-\psi_1}(x_{\psi_1}) = \log p(y_{\psi_1}/x_{\psi_1}) + \sum_{x \in \mathbb{V}} \psi_1(x_{\psi_1}) \log \mu_{\psi_{-\psi_1}}(x_{\psi_1})
\]
In addition, sums can be applied by maximization. It is sums
in eq*. (15) can be replaced by max* function:
\[
\log \mu_{\mathcal{V}_n|x_n} (x_n) = \max_{\mathcal{V}_n} \left\{ \log \psi_1 (x_1) + \sum_{m \in \mathcal{V}_n \setminus \{n\}} \log \mu_{\mathcal{V}_m|x_m} (x_m) \right\}
\]
(19)
Where
\[
\max^* [L_1, L_2] = \max [L_1, L_2] - \log (1 + e^{-|L_1 - L_2|})
\]
(20)
The max function, the max* operation can be
\[
\max^* [L_1, L_2, \ldots \ldots , L_N] = ss \max^* \{\max^* [L_1, L_2, \ldots \ldots , L_{N-1}], L_N\}
\]
(21)
The messages sent on an edge contains the probabilities of 1
and 0, these two probabilities can be conveniently expressed
into log-likelihood ratio
\[
\lambda_{\mathcal{V}_n|x_n} = \log \frac{\mathcal{P}(x_n=1|x_n)}{\mathcal{P}(x_n=0|x_n)}
\]
(22)
AWGN channel, the Gaussian distribution and two
probabilities are \(\mathcal{P}(x_n=1|x_n = 1) = 0.5\) and \(\mathcal{P}(x_n=0|x_n = 0)\).
The log-likelihood ratio can be
\[
\lambda_{\mathcal{V}_n|x_n} = \frac{2\gamma_n}{\sigma^2}
\]
(23)
From eq*, (17)-(19) into the log-likelihood ratio forms the
\[
\lambda_{\mathcal{V}_n|x_n} = \log \frac{\mathcal{P}(x_n=1|x_n)}{\mathcal{P}(x_n=0|x_n)} (1) - \log \frac{\mathcal{P}(x_n=0|x_n)}{\mathcal{P}(x_n=1|x_n)} (0)
\]
(24)
So the eq*, (14)-(16) becomes variable updated nodes
\[
\lambda_{\mathcal{V}_n|x_n} = \lambda_{\mathcal{V}_n|x_n} + \sum_{k \in \mathcal{V}_n \setminus \{n\}} \psi_{k|x_n}
\]
(25)
\[
\lambda_{\mathcal{V}_n|x_n} = f \max^* (\{\lambda_{\mathcal{V}_n|x_n}\}_{n=1}^N)
\]
(26)
Where since
\[
\log \mu_{\mathcal{V}_m|x_m} (x_m) = (-1)^{1-m} \psi_{m|x_m} \frac{e^{\lambda_{\mathcal{V}_m|x_m}}}{2}
\]
\[
\max^* \left\{ \lambda_{\mathcal{V}_m|x_m} \right\}_{m=1}^N = \frac{1}{2} \sum_{m=1}^{N} \max\left( \psi_{m|x_m} + \lambda_{\mathcal{V}_m|x_m} \right)
\]
(27)
\[
\max\left( \psi_{m|x_m} + \lambda_{\mathcal{V}_m|x_m} \right) = 0 \quad \text{otherwise} \quad \psi_{m|x_m} = 0
\]
(28)
Here the definition of \(\psi_{m|x_m} = \{\psi_{m|x_m} = 0\}\) which is the \(I_{th}\)
check. If x fulfills the check \(\psi_{m|x_m} = 0\) otherwise \(\lambda_{\mathcal{V}_m|x_m} = \frac{1}{2} \sum_{m=1}^{N} \psi_{m|x_m} + \lambda_{\mathcal{V}_m|x_m} \)
(29)

IV. JSG-OFDM USING LDPC
A low density parity check (LDPC) codes is a linear block
error correcting code, a technique of transmitting a message
over a noisy transmission channel. LDPC code is constructed
using a sparse bipartite graph. Advantage of LDPC codes is
decoding of low complexity. A practical objection to the use
of LDPC codes is that for large block lengths, their encoding
complexity is high.

Fig. 2. LDPC code Tanner graph representation.
Performance of 200% Loaded System

- JSG-OFDM Scen-1, Joint Detection and Decoding
- JSG-OFDM Scen-2, Joint Detection and Decoding
- JSG-OFDM Scen-3, Joint Detection and Decoding
- JSG-OFDM Scen-3, 10 OFDM Symbols/Frame

BER vs Eb/N0 (dB)

Fig. 3. Performance of 200% Loaded System

Performance of 300% Loaded System

- JSG-OFDM Scen-1, Joint Detection and Decoding
- JSG-OFDM Scen-2, Joint Detection and Decoding
- JSG-OFDM Scen-3, Joint Detection and Decoding
- JSG-OFDM Scen-3, 10 OFDM Symbols/Frame

BER vs Eb/N0 (dB)

Fig. 4. Performance of 300% Loaded System

SNR vs Channel Capacity

Fig. 5. SNR vs Channel Capacity

Channel Comparison

- AWGN Channel
- ITU Pedestrian Channel

BER vs Number of Iterations

Fig. 6. Channel Comparison

Performance of Different User

- Worst User
- Overall
- Best User

BER vs Eb/N0 (dB)

Fig. 7. Performance of Different User

Channel Comparison

- 300% Loaded JSG-OFDM
- 200% Loaded JSG-OFDM

BER vs Eb/N0 (dB)

Fig. 8. Channel Comparison for 200% and 300% Loaded
VI. CONCLUSION

The proposed work called JSG OFDM using LDPC. It has been shown via numerical calculations on MATLAB platform that proposed work provides significant BER performance improvement almost nearer to zero over scenario JSG OFDM for several different configurations. As research on this concept has been going on, we can improve its performance further especially for more complex modulation like QAM by using advanced detection techniques.

REFERENCE

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