Application of the Affine Theorem to an Orthotropic Rectangular Reinforced Concrete Slab Having a Interior Corner Opening

Abstract - An attempt has been made to apply the affinity theorem to determine collapse load of two-way orthotropic slab with an interior corner opening with one long side continuous and other three sides simply supported slab (OLC) and one long side simply supported and other three sides continuous slab (OLD). Keeping in view the basic principles of yield line theory, all possible admissible yield line patterns are considered for the given configuration of the slab subjected to uniformly distributed load (udl). A computer program has been developed to solve the virtual work equations derived in this paper. Illustration of above methodology has been brought out with numerical examples. Relevant tables and charts for given data and the governing admissible failure patterns of the slab for different sizes of openings are presented using the affine theorem. In this paper, authors also present the transformation of orthotropic slab into an equivalent isotropic slab using the affine theorem. The analysis is carried out with aspect ratio of opening quite different to that of the slab.

Key Words: aspect ratio, interior corner opening, configuration, affinity theorem, orthotropic slab, uniformly distributed load, ultimate load, ultimate moment and virtual work equations.

INTRODUCTION

Openings in slabs are usually required for plumbing, fire protection pipes, heat and ventilation ducts and air conditioning. Larger openings that could amount to the elimination of a large area within a slab panel are sometimes required for stairs and elevators shafts. For newly constructed slabs, the locations and sizes of the required openings are usually predetermined in the early stages of design and are accommodated accordingly. Such two way slabs subjected to uniformly distributed load and supported on various edge conditions are being analyzed by using yield-line method as suggested by Johansen, K.W. Many researchers (Goli, H.B. et al, Rambabu, K. et al, Islam, S. et al, Zaslavsky, Aron, Siva Rama Prasad et al, Sudhakar, K.J. et al) adopted the yield-line analysis and virtual work method in deriving the virtual work equations of the rectangular reinforced concrete solid slabs subjected to uniformly distributed load and supported on various edge conditions. Johansen, K.W 1 also presented the analysis of orthogonal solid slabs implicitly to that of an equivalent isotropic slab by using “Affine Theorem” provided the ratio of negative to positive moments is same in the orthogonal directions. Various design charts are presented by Islam, S. et al 4 for continuous slab (CS) and simply supported (SS) slabs with equal openings, i.e. ratio of openings and aspect ratio are same. Whereas in this paper similar charts are presented for two edge conditions with aspect ratio of slab and opening different.

Methodology

The method of determining collapse loads based on principle of virtual work has proved to be a powerful tool for a structural engineer, despite it gives an upper bound value. The work equations are formed by equating the energy absorbed by yield lines and the work done by the external load of the orthogonal rectangular slab with interior corner openings where a small virtual displacement is given to the slab. The same principle was also used by Islam, S. et al in their paper. In other words, the work equation is given by

\[
\iint W_{ul} \delta(x, y) dxdy = \sum (m_{ul,x} \theta_y y_0 + m_{ul,y} \theta_x x_0)
\]

(1)
where \( W_{ult} \) is the ultimate load per unit area of slab, \( \delta(x,y) \) is the virtual displacement in the direction of the loading at the element of area of dimensions \( dx,dy \), \( m_{ult,x} \) and \( m_{ult,y} \) are the yield moments per unit width in the x and y directions, \( \theta_x \) and \( \theta_y \) are the components of the virtual rotation of the slab segments in the x and y directions and \( x_0 \) and \( y_0 \) are the projected length of the yield lines in x and y directions of slab. The equation (1) contains terms \( C_1, C_2, C_3 \) and \( C_4 \) which define the positions of the node points of the yield lines. The values of \( C_1, C_2, C_3 \) and \( C_4 \) to be used in the equation are those which give the minimum load to cause failure. A computer program has been written to find the values of \( C_1, C_2, C_3 \) and \( C_4 \) (in terms of \( r_1, r_2, r_3 \) and \( r_4 \)) corresponding to minimum load carrying capacity of the slab. For definitions of various parameters refer notations. Johansen has proved that the yield line theory is an upper bound method, so care has been taken to examine all the possible yield line patterns for TAC slab to ensure that the most critical collapse mode is considered otherwise the load carrying capacity of the slab will be overestimated.

**Formulation of Virtual Work Equations**

There is several possible yield line patterns associated with different edge conditions of the slab. For the OLC edge condition of slab, the possible admissible failure yield line patterns are twenty. For the OLD edge conditions of slab, the possible admissible failure yield line patterns are twenty. These admissible failure yield line patterns are obtained basing on the yield line principle of Johansen. For the given configuration of the slab, these twenty failure patterns and corresponding equations have been investigated depending upon the support condition of the slab using a computer program.

The orthogonal reinforced rectangular slab having interior corner opening with the given configuration and the yield criteria are shown in notation Fig.5. The slab is subjected to uniformly distributed load (\( W_{ult} \)). Note that the slab is not carrying any load over the area of the opening.

The generalized virtual work equations for continuous edge (CS) condition of slab are derived for the predicted possible admissible failure yield line patterns using the virtual work equation. To get the equations for other edge conditions of the slab, modification should be carried out in the numerators of the equations of each failure patterns. For OLC slab \( I_1 = I_2 = I_3 = 0 \) or \( I_1 = I_2 = I_3 = 0 \), for OLD slab \( I_2 = 0 \) or \( I_1 = 0 \).

**Virtual Work Equations for Two Adjacent sides Continuous (TAC) Slab**

Twenty possible failure patterns are predicted for two edge conditions of the slab. The governing failure pattern for different edge conditions and for different data is presented in Table-1. Let \( \delta \) be the virtual displacement at a & b (Fig. 1), for the considered failure Pattern-1 of aslab. Three unknown dimensions \( C_1, C_2, C_3 \) are necessary to define the yield line propagation completely. All other admissible failure patterns are as shown in APPENDIX-A.
PATTERN 1:

The external work done by segment A:

\[
W_{a} C_{1} = W_{a} C_{2} + W_{a} C_{3} = \frac{1}{2} C_{1} y_{1} x_{1} + C_{3} x_{1} + \frac{1}{2} C_{1} x_{1} x_{1}
\]

The external work done by segment B:

\[
W_{b} C_{y} = W_{b} C_{y} = \frac{1}{2} C_{2} y_{2} + C_{2} y_{2} + \frac{1}{2} C_{2} x_{2}
\]

The external work done by segment C:

\[
W_{c} C_{y} = W_{c} C_{y} = \frac{1}{2} C_{3} y_{3} + C_{3} y_{3} + \frac{1}{2} C_{3} x_{3}
\]

The external work done by segment D:

\[
W_{d} C_{y} = W_{d} C_{y} = \frac{1}{2} C_{4} y_{4} + C_{4} y_{4} + \frac{1}{2} C_{4} x_{4}
\]

Total work done = work done by segment’s (A+B+C+D)

\[
W_{a} = \frac{1}{2} C_{1} y_{1} x_{1} + C_{3} x_{1} + \frac{1}{2} C_{1} x_{1} x_{1} + C_{2} y_{2} + C_{2} y_{2} + \frac{1}{2} C_{2} x_{2} + C_{3} y_{3} + C_{3} y_{3} + \frac{1}{2} C_{3} x_{3} + C_{4} y_{4} + C_{4} y_{4} + \frac{1}{2} C_{4} x_{4}
\]

Energy absorbed yield lines:

\[
m_{u} K_{x} (y_{1} + C_{1}) + m_{u} L_{y} = \frac{1}{C_{1}} + m_{u} L_{y} + m_{u} L_{y} = \frac{1}{C_{1}} + m_{u} L_{y} + m_{u} L_{y} + \frac{1}{C_{1}}
\]

\[
m_{u} K_{y} (y_{2} + C_{2}) + m_{u} L_{y} = \frac{1}{C_{2}} + m_{u} L_{y} + m_{u} L_{y} + \frac{1}{C_{2}}
\]

\[
m_{u} K_{y} (y_{3} + C_{3}) + m_{u} L_{y} = \frac{1}{C_{3}} + m_{u} L_{y} + m_{u} L_{y} + \frac{1}{C_{3}}
\]

\[
m_{u} K_{y} (y_{4} + C_{4}) + m_{u} L_{y} = \frac{1}{C_{4}} + m_{u} L_{y} + m_{u} L_{y} + \frac{1}{C_{4}}
\]
Equating total work done by the segments to energy absorbed by yield lines we get

\[
W_{ult} = \left[ K_1 r_5 \left( \frac{r_1 r_5 + r_6}{r_3} + I_1 r_5 \right) + \frac{K_1 r_5 r_5^3}{r_3} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) + I_2 r_5 \right] + \left[ K_3 r_3^2 \left( \frac{r_3^2}{r_1} \right) \left( \frac{r_1 r_5 + r_6}{r_3} + I_4 r_5 \right) \right]
\]

Equation 2 for Failure Pattern – 2

\[
W_{ult} = \left[ \frac{K_1 r_5^2}{r} + I_1 r_5 \right] + \left[ \frac{K_1 r_5}{r_3 - 1} \left( \frac{r_5 + r_7}{r_3 - 1} \right) + I_2 r_5 \right] + \left[ \frac{K_1 r_5}{r_3 - 1} \left( \frac{r_5 + r_7}{r_3 - 1} \right) + I_4 r_5 \right]
\]

Equation 3 for Failure Pattern – 3

\[
W_{ult} = \left[ \frac{K_1 r_5 \left( r_5 + r_6 \right)}{r} + I_1 r_5 \right] + \left[ \frac{K_1 r_5}{r_3 - 1} \left( \frac{r_5 + r_7}{r_3 - 1} \right) + I_2 r_5 \right] + \left[ \frac{K_1 r_5}{r_3 - 1} \left( \frac{r_5 + r_7}{r_3 - 1} \right) + I_4 r_5 \right]
\]

Equation 4 for Failure Pattern – 4

\[
W_{ult} = \left[ \frac{K_1 r_5 \left( r_5 + r_6 \right)}{r} + I_1 r_5 \right] + \left[ \frac{K_1 r_5}{r_3 - 1} \left( \frac{r_5 + r_7}{r_3 - 1} \right) + I_2 r_5 \right] + \left[ \frac{K_1 r_5}{r_3 - 1} \left( \frac{r_5 + r_7}{r_3 - 1} \right) + I_4 r_5 \right]
\]
Equation 5 for Failure Pattern – 5

\[
W_{ult} l^2 = \left[ \frac{K^1 r_l (r_6 + r_8) + I_1 r_l}{r} + \left[ \frac{K^1 r r_3}{r} \left( \frac{r_6 r_3}{r_7} + r_7 \right) + I_2 r r_3 \left( \frac{r_6 r_3}{r_7} + (r_3 - 1) \right) \right] + \left[ \frac{K^1 r^2 r_7}{r} (r_3) + I_3 r^2 \right] \right]
\]

\[
R^* \left[ \frac{r^2 r_3^2}{6 r_1} + \frac{r_6 r_3^2}{6 r_1 (r_3 - 1)^2} \right] + \left[ \frac{r_6 r_3^2}{6 r_1} \left( 1 - r_7 \right) - \frac{r_6 r_3}{r_7} \left( \frac{r_6 r_3}{r_7} + 2 (r_3 - 1) \right) \right] + \left[ \frac{r_6 r_3^2}{6 r_1} \left( 1 - r_7 \right) - \frac{r_6 r_3}{r_7} \left( \frac{r_6 r_3}{r_7} + 2 (r_3 - 1) \right) \right] + \left[ \frac{r^2 r_3^2}{6 r_1} \left( 1 - r_7 \right) - \frac{r_6 r_3}{r_7} \left( \frac{r_6 r_3}{r_7} + 2 (r_3 - 1) \right) \right]
\]

Equation 6 for Failure Pattern – 6

\[
W_{ult} l^2 = \left[ \frac{K^1 r_l (r_6 + r_8)}{r} + I_1 r_l \right]
\]

\[
R^* \left[ \frac{r^2 r_3^2}{6 r_1} + \frac{r_6 r_3^2}{6 r_1 (r_3 - 1)^2} \right] + \left[ \frac{r^2 r_3^2}{6 r_1} \left( 1 - r_7 \right) - \frac{r_6 r_3^2}{r_7} \left( \frac{r_6 r_3^2}{r_7} + 2 (r_3 - 1) \right) \right] + \left[ \frac{r^2 r_3^2}{6 r_1} \left( 1 - r_7 \right) - \frac{r_6 r_3^2}{r_7} \left( \frac{r_6 r_3^2}{r_7} + 2 (r_3 - 1) \right) \right] + \left[ \frac{r^2 r_3^2}{6 r_1} \left( 1 - r_7 \right) - \frac{r_6 r_3^2}{r_7} \left( \frac{r_6 r_3^2}{r_7} + 2 (r_3 - 1) \right) \right]
\]

Equation 7 for Failure Pattern – 7

\[
W_{ult} l^2 = \left[ \frac{K^1 r_l (r_6 + r_8)}{r} + I_1 r_l \right]
\]

\[
R^* \left[ \frac{r^2 r_3^2}{6 r_1} + \frac{r_6 r_3^2}{6 r_1 (r_3 - 1)^2} \right] + \left[ \frac{r^2 r_3^2}{6 r_1} \left( 1 - r_7 \right) - \frac{r_6 r_3^2}{r_7} \left( \frac{r_6 r_3^2}{r_7} + 2 (r_3 - 1) \right) \right] + \left[ \frac{r^2 r_3^2}{6 r_1} \left( 1 - r_7 \right) - \frac{r_6 r_3^2}{r_7} \left( \frac{r_6 r_3^2}{r_7} + 2 (r_3 - 1) \right) \right] + \left[ \frac{r^2 r_3^2}{6 r_1} \left( 1 - r_7 \right) - \frac{r_6 r_3^2}{r_7} \left( \frac{r_6 r_3^2}{r_7} + 2 (r_3 - 1) \right) \right]
\]
Equation 8 for Failure Pattern – 8

\[
W_{ult}^2 = m_{ult} \left[ \frac{r_5^3 r_6^3}{r_1} + \frac{\beta r_r^2}{2} + \frac{r_5^3 r_6^3}{6r_1 (r_5 - 1)^2} \right] + \frac{1 - r_5 - r_5 r_8}{6r_8} \left[ 1 - r_5 + 1 \right] + \frac{(r_2 r_7 - 1)(r_5 - 1)}{2r_2 r_7} + \frac{(r_2 - 1)(r_5 - 1)}{6r_2}
\]

Equation 9 for Failure Pattern – 9

\[
W_{ult}^2 = m_{ult} \left[ \frac{K^1_{12} r_5 r_6}{r_5 r_6 (r_5 - 1) + r_5} + \frac{I_5}{r} \right] + \frac{K^1_{12} r_5 r_6}{r_5 (r_5 - 1)} \left[ r_5 + r_5 \right] + \frac{K^1_{12} r_5}{r} + \frac{I_5}{r} + \left[ K^1_{12} r_5 (r_5) + I_5 r_5 \right]
\]

Equation 10 for Failure Pattern – 10

\[
W_{ult}^2 = m_{ult} \left[ \frac{r_5^3 r_5^3}{r_1} \left( \frac{r_6 - r_6}{r_4} + \frac{I_5}{r} \right) + \left[ K^1_{12} r_5 (r_5) + I_5 r_5 \right] + \left[ K^1_{12} r_5 (r_6) + I_5 r_5 \right] \right]
\]
Equation 11 for Failure Pattern – 11

\[ W_{ult} f^2 = m_{ult} R^* \]

\[ K_1' r_1 \left( r_3 r_5 + r_3 r_6 \right) + I_1 r_1 \] + \[ K_1' r_4 \left( r_3 r_6 + \left( r_3 r_6 \right) \left( r_3 - 1 \right) \right) + I_2 r_4 \] + \[ K_1' r_3 \left( r_3 r_5 + \left( r_3 r_6 \right) \left( r_3 - 1 \right) \right) + I_1 r_5 \]

Equation 12 for Failure Pattern – 12

\[ W_{ult} f^2 = m_{ult} R^* \]

\[ K_1' r_1 \left( r_3 r_5 + r_3 r_6 \right) + I_1 r_1 \] + \[ K_1' r_4 \left( r_5 + r_5 \right) + I_2 r_4 \] + \[ K_1' r_3 \left( r_3 r_5 + \left( r_3 r_6 \right) \left( r_3 - 1 \right) \right) + I_1 r_5 \]

Equation 13 for Failure Pattern – 13

\[ W_{ult} f^2 = m_{ult} R^* \]

\[ K_1' r_1 \left( r_3 r_5 + r_3 r_6 \right) + I_1 r_1 \] + \[ K_1' r_4 \left( r_3 r_6 + \left( r_3 r_6 \right) \left( r_3 - 1 \right) \right) + I_2 r_4 \] + \[ K_1' r_3 \left( r_3 r_5 + \left( r_3 r_6 \right) \left( r_3 - 1 \right) \right) + I_1 r_5 \]
Equation 14 for Failure Pattern – 14

\[
W_{ab} = \frac{J^2}{m_{ab}} = \frac{r^2_r r_6^3 + \frac{r^2_r r_4 r_6}{6 r_1}}{1 - r_6 - \frac{r_6 r_5}{r_4} + \frac{r^2_r r_5^3}{6 r_4} + \frac{r^3_r^2 r_5^2}{6 r_4 (r_1 - 1)^2}} + \frac{r^2_r r_3}{6 r_4 (r_1 - 1)^2} + \left(1 - \frac{r_6 r_7}{r_1} \right) \frac{r_5 r_3}{6 r_6} + \frac{r^2_r^3}{6 r_6}
\]

Equation 15 for Failure Pattern – 15

\[
W_{ab} = \frac{J^2}{m_{ab}} = \frac{r^2_r r_6^3 + \frac{r^2_r r_4 r_6}{6 r_1}}{1 - r_6 - \frac{r_6 r_5}{r_4} + \frac{r^2_r r_5^3}{6 r_4} + \frac{r^3_r^2 r_5^2}{6 r_4 (r_1 - 1)^2}} + \frac{r^2_r r_3}{6 r_4 (r_1 - 1)^2} + \left(1 - \frac{r_6 r_7}{r_1} \right) \frac{r_5 r_3}{6 r_6} + \frac{r^2_r^3}{6 r_6}
\]

Equation 16 for Failure Pattern – 16

\[
W_{ab} = \frac{J^2}{m_{ab}} = \frac{r^2_r r_6^3 + \frac{r^2_r r_4 r_6}{6 r_1}}{1 - r_6 - \frac{r_6 r_5}{r_4} + \frac{r^2_r r_5^3}{6 r_4} + \frac{r^3_r^2 r_5^2}{6 r_4 (r_1 - 1)^2}} + \frac{r^2_r r_3}{6 r_4 (r_1 - 1)^2} + \left(1 - \frac{r_6 r_7}{r_1} \right) \frac{r_5 r_3}{6 r_6} + \frac{r^2_r^3}{6 r_6}
\]
Equation 17 for Failure Pattern – 17
\[
W_{ult}I_y^2 = \frac{\left[ K^1_i r_6 (y_6 + r_8) + \frac{I_i r_i}{r} \right]}{m_{ult}} + \left[ K^1_i r_4 + I_2 r r_4 \right] + \left[ \frac{K^1_i r_i (y_6 + r_8)}{r (r_i - 1)} + \frac{I_i r_i}{r (r_i - 1)} \right] + \left[ K^1_i r^2 (y_6 + r_8) + I_4 r r_5 \right]
\]

Equation 18 for Failure Pattern – 18
\[
W_{ult}I_y^2 = \frac{\left[ K^1_i r_6 (y_6 + r_8) + \frac{I_i r_i}{r} \right]}{m_{ult}} + \left[ K^1_i r^2 r_4 + I_2 r r_4 \right] + \left[ \frac{K^1_i r_i (y_6 + r_8)}{r (r_i - 1)} + \frac{I_i r_i}{r (r_i - 1)} \right] + \left[ K^1_i r r_5 + I_4 r r_5 \right]
\]

Equation 19 for Failure Pattern – 19
\[
W_{ult}I_y^2 = \frac{\left[ K^1_i r_6 (y_6 + r_8) + \frac{I_i r_i}{r} \right]}{m_{ult}} + \left[ K^1_i r r_4 \left( r_4 + \frac{r_5 r_8}{r_i} \right) + I_2 r r_4 \right] + \left[ \frac{K^1_i r_i (y_6 + r_8)}{r (r_i - 1)} \left( r_4 + \frac{r_5 r_8}{r_i} (r_i - 1) \right) \right] + \frac{I_i r_i}{r (r_i - 1)} + \left[ K^1_i r r_5 + I_4 r r_5 \right]
\]

Equation 20 for Failure Pattern – 20
\[
W_{ult}I_y^2 = \frac{\left[ K^1_i r_6 (y_6 + r_8) + \frac{I_i r_i}{r} \right]}{m_{ult}} + \left[ K^1_i r r_4 \left( r_4 + \frac{r_5 r_8}{r_i} \right) + I_2 r r_4 \right] + \left[ \frac{K^1_i r_i (y_6 + r_8)}{r (r_i - 1)} \left( r_4 + \frac{r_5 r_8}{r_i} (r_i - 1) \right) \right] + \frac{I_i r_i}{r (r_i - 1)} + \left[ K^1_i r^2 r_6 + I_4 r r_5 \right]
\]

The respective equations for corresponding failure patterns can be obtained for OLC and OLD condition by making respective negative yield moment coefficients zero.
Minimization of the Virtual Work Equations

The value \( \frac{W_{ult}L_y^2}{m_{ult}} \) of these equations consists of the unknown non-dimensional parameters \( r_1, r_2, r_3 \) and \( r_4 \) which define the positions of the yield lines. A computer program has been developed for various values of the non-dimensional parameters \( r_1, r_2, r_3 \) and \( r_4 \) within their allowable ranges in order to find the minimum value of \( \frac{W_{ult}L_y^2}{m_{ult}} \) for the yield line failure patterns considered. In this computer program, the values of \( r_1, r_2, r_3 \) and \( r_4 \) were varied at increments of 0.1. Using the above equations, one can develop useful charts basing on orthogonality which may be used either for design or analysis in general. The governing failure pattern for different edge conditions and for different data is presented in Table-1.

EXAMPLE: One Long side Discontinuous Slab (OLD)
(Negative moment to positive moment ratio in both directions is same and unity)

Transform an orthotropic slab to an equivalent isotropic slab in which the ratio of Negative moment to positive moment in both directions is same and unity using the affine theorem.

Since \( \frac{I_1}{K_x} \frac{I_2}{K_y} = 1.0 \), the transformation of the given orthotropic slab (Fig.2 (a)) in \( X \) – direction is transformed to an equivalent isotropic slab (Fig.2 (b)) by dividing with \( \sqrt{\mu} \). This principle is illustrated in Fig.2. Using the above methodology, few numerical examples are presented in Table 2.

Table 3 shows the strength and the failure pattern for OLD based on the principle \( \mu = r^2 \) for different values of coefficient of orthotropy(\( \mu \)) and their corresponding orthogonal affine moment coefficients.

Note: In the case of SS, TLC and TSC edge conditions of the slab; the affine theorem cannot be applied because the negative moment is not present along one of the edges of the given slab. Therefore one can design the slab as orthotropic using any available computer program.

Preparation of Charts

Design charts have been prepared for One Long side Discontinuous slab (OLD) slabs. Charts 1-4 are shown in Appendix-B. Charts 1-4 are plotted for \( \frac{W_{ult}L_y^2}{m_{ult}} \) versus Opening (\( \alpha = \beta \)) for various ratios of K1, K2 and \( r \) for different CS conditions. The values of \( r \) are taken between 1 and 3. The K1 values are plotted varying between 1 and 5 thus ensuring that greater yield moment is in the direction of short span. This is in accordance with elastic theory distribution of bending moments. The values of K2 are plotted for 1 and 2 for One Long side Discontinuous slab (OLD). While preparing the design charts, the least value of \( \frac{W_{ult}L_y^2}{m_{ult}} \) given by thirteen failure patterns is considered. Using these charts one can directly design or do analysis of a slab with anterior corner opening. In addition, a special charts 5-6 is presented for various values of openings which can be used for transforming an orthogonal slab to equivalent isotropic slab for the governing failure patterns only.

A. Analysis Problem: (OLC)

Determine the safe uniformly distributed load on a rectangular two way slab with interior corner opening all sides continuous slab (Fig.3(a)) for the following data:

A slab 6m X 4m with interior corner opening of size 1.2m X 0.4m at a distance of 1.2m from long edge and 0.8m from short edge is reinforced with 10mm diameter bars @ 200mm c/c perpendicular to long span and 8mm diameter bars @ 150mm c/c perpendicular to short span. Two meshes are used one at top long side continuous and one at bottom, thickness of the slab is 120mm. Characteristic strength of concrete is 20MPa and yield stress of steel is 415MPa.
Solution:

According to IS 456:2000, $m_{ult} = 0.87 f_y A_{st} \sqrt{z}$, where $z = d \left(1 - \frac{f_y A_{st}}{f_{ck} b d^2} \right)$ ----------- (2)

Assuming effective depth of slab in short span direction=100.00 mm
effective depth of slab in long span direction=90.00 mm
Area of the steel perpendicular to long span=374 mm$^2$
Area of the steel perpendicular to short span=314 mm$^2$
The ultimate moments in short and long span directions can be found using the expression (2).
Therefore, $m_{ult}$ parallel to long span=13.489 kNm/m
$m_{ult}$ parallel to short span=10.192 kNm/m

For aspect ratio of slab, $r = \frac{6.0}{4.0} = 1.5$ and taking $m_{ult} = 13.489$ kNm/m,
The orthogonal coefficients (Fig.3(b)) will be $K'_x = 0.755$, $I_1 = I_3 = 0$ and $K'_y = I_2 = 1.0$, $L_y = 0$. With these orthogonal coefficients and for $\alpha = 0.2$, $\beta = 0.1$, $r = 1.5$, $C_2 = 1.2$ m, $C_6 = 0.8$ m;
Twenty predicted failure patterns are evaluated by using computer program to find the governing failure pattern, and the final results are as follows.

$W_{ult} L_y^2 = 37.83424$ and the failure pattern is 7

$W_{ult} = 16.5895 kN/m^2$ 
$W_{ult} = 1.5(W_{ll}+W_{dl}) = 16.5895 kN/m^2$ 
$W_{ll} = \frac{16.5895}{1.5} - 3.5 = 7.56 kN/m^2$
The intensity of live load on the slab is 7.56 kN/m$^2$

B. Design Problem: (OLD)

Design all sides continuous slab of 5.0 m X 2.5 m with interior corner openings of size 1.5 m X 0.75 m at a distance of 0.5 m from long edge and 0.5 m from short edge to carry a uniformly distributed live load of 4.5 kN/m$^2$. Use M20 mix and Fe 415 grade steel.

Given data: Aspect ratio of slab ($r$) = $\frac{L_x}{L_y} = 5.0/2.5 = 2.0$, $aL_x = 1.5$ m, $\beta L_y = 0.75$ m.

\[ \therefore \alpha = 0.3, \beta = 0.3, C_5 = 0.5 \text{m}, C_6 = 0.5 \text{m} \]

Twenty predicted failure patterns are evaluated by using the computer program to find the governing failure pattern, by assuming $K'_x = I_1 = I_3 = 1.0$, $K'_y = I_2 = 2.0$, $L_y = 0$. 

$W_{ult} L_y^2 = 37.83424$ and the failure pattern is 8

$m_{ult}$

Unknown parameters:

$\frac{L_x}{C_1} = 4.6000$, $\frac{L_y}{C_2} = 3.55185$, $\frac{L_y}{C_3} = 2.34783$

$C_1 = 1.0869 \text{ m}, C_2 = 1.4077 \text{ m}, C_3 = 1.0648 \text{ m}$

Assuming overall thickness of slab = 110 mm
Dead load of slab = 110 X 25 = 2.75 kN
Dead loads including finishing’s = 3.5 kN/m$^2$
Total load = 8.0 kN/m$^2$
Ultimate total load = 15 X 8.0 = 120 kN/m$^2$
The orthogonal moments are 

\[ K_{1x} = I_{1} m_{zz} = 1.0 \times 2.15305 = 2.15305 \text{ kNm/m} \]

\[ K_{1y} = I_{2} m_{zz} = 2.0 \times 2.15305 = 4.3061 \text{ kNm/m} \]

Effective depth, 

\[ d = \frac{2.15305 \times 10^{-6}}{0.138 \times 20 \times 1000} = 27.9301 \text{ mm} \]

Adopt effective depth as 100 mm and overall depth as 110 mm

Area of steel along short span = 155.169 mm²

Minimum area of steel required along short span = 204.82 mm²

Use 8 mm diameter bars @ 240 mm c/c

Area of steel along long span = 84.3189 mm²

Use 6 mm diameter bars @ 300 mm c/c

Details of reinforcement are shown in Fig. 4(c).

CONCLUSIONS:

1. The virtual work equations for orthotropic slabs with unequal interior corner openings with all sides continuous whose aspect ratio of opening is different from the aspect ratio of slab subjected to udl are presented.

2. Design charts for One Longside Discontinuous slab are presented for different aspect ratios of slab.

3. Few numerical examples are presented based on theorem of VI and VII of affine theorem of Johansen, K.W. for orthotropic slabs with interior corner openings.

4. Two useful charts for affine transformation for different sizes of openings are also presented.

5. In case of One Longside Discontinuous, for \( K_2 = 1, 2 \) and \( K_1 = 1, 2, 3 \) and 5 the strength of the slab is decreasing with increase in aspect ratio when compared to solid slab.

6. The charts can be used either for analysis or design for different size of openings in the interior corner of the slab.

REFERENCES:


As per affine theorem the transformed
\( K'_{x} = I_{1} = 1, K'_{y} = I_{2} = 1, I_{4} = 0. \)
\( r = 0.2, \quad r_{6} = 0.2, \quad \alpha = 0.2, \quad \beta = 0.2, \)
\( L_{x}' = L_{y}' = 5.6/\sqrt{0.5} = 7.92 \text{m}, \)
\( r_{5} = 7.92/4 = 1.98, \quad \mu = 1.0, \quad \sum K = 4, \)
The value of \( W_{ult} L_{y}^2/m_{ult} \) obtained from chart – 5 for \( r = 1.77. \) Taking this value one can design the given orthotropic slab without using computer program.

Fig 2 - Orthotropic slab and equivalent isotropic slab as per Theorems VI and VII of Johansen
Table 1

<table>
<thead>
<tr>
<th>Failure Patterns</th>
<th>OLC (20)</th>
<th>OLD (18)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening Sizes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>( r_5 = 0.3, r_6 = 0.2, a = 0.2, \beta = 0.1, r = 1.3 )</td>
<td>( r_5 = 0.3, r_6 = 0.1, a = 0.1, \beta = 0.3, r = 1 )</td>
</tr>
<tr>
<td>2</td>
<td>( r_5 = 0.4, r_6 = 0.1, a = 0.1, \beta = 0.2, r = 1.4 )</td>
<td>( r_5 = 0.4, r_6 = 0.4, a = 0.1, \beta = 0.1, r = 1 )</td>
</tr>
<tr>
<td>3</td>
<td>( r_5 = 0.2, r_6 = 0.1, a = 0.3, \beta = 0.2, r = 1.3 )</td>
<td>( r_5 = 0.2, r_6 = 0.1, a = 0.3, \beta = 0.2, r = 1.5 )</td>
</tr>
<tr>
<td>4</td>
<td>( r_5 = 0.3, r_6 = 0.3, a = 0.2, \beta = 0.2, r = 1.1 )</td>
<td>( r_5 = 0.4, r_6 = 0.4, a = 0.1, \beta = 0.1, r = 1.5 )</td>
</tr>
<tr>
<td>5</td>
<td>( r_5 = 0.2, r_6 = 0.2, a = 0.3, \beta = 0.1, r = 1 )</td>
<td>( r_5 = 0.1, r_6 = 0.1, a = 0.1, \beta = 0.4, r = 1.5 )</td>
</tr>
<tr>
<td>6</td>
<td>( r_5 = 0.2, r_6 = 0.2, a = 0.1, \beta = 0.3, r = 1.8 )</td>
<td>( r_5 = 0.3, r_6 = 0.3, a = 0.1, \beta = 0.2, r = 2 )</td>
</tr>
<tr>
<td>7</td>
<td>( r_5 = 0.2, r_6 = 0.1, a = 0.1, \beta = 0.1, r = 1 )</td>
<td>( r_5 = 0.1, r_6 = 0.1, a = 0.1, \beta = 0.1, r = 2 )</td>
</tr>
<tr>
<td>8</td>
<td>( r_5 = 0.2, r_6 = 0.1, a = 0.1, \beta = 0.1, r = 1.8 )</td>
<td>( r_5 = 0.4, r_6 = 0.1, a = 0.1, \beta = 0.1, r = 2 )</td>
</tr>
<tr>
<td>9</td>
<td>( r_5 = 0.4, r_6 = 0.3, a = 0.1, \beta = 0.2, r = 1.1 )</td>
<td>( r_5 = 0.2, r_6 = 0.3, a = 0.1, \beta = 0.3, r = 1 )</td>
</tr>
<tr>
<td>10</td>
<td>( r_5 = 0.2, r_6 = 0.1, a = 0.1, \beta = 0.2, r = 1 )</td>
<td>( r_5 = 0.3, r_6 = 0.3, a = 0.1, \beta = 0.1 )</td>
</tr>
<tr>
<td>11</td>
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<td>( r_5 = 0.2, r_6 = 0.1, a = 0.1, \beta = 0.3, r = 1 )</td>
</tr>
<tr>
<td>12</td>
<td>( r_5 = 0.4, r_6 = 0.1, a = 0.1, \beta = 0.2, r = 1.2 )</td>
<td>( r_5 = 0.2, r_6 = 0.1, a = 0.4, \beta = 0.4, r = 1.5 )</td>
</tr>
<tr>
<td>13</td>
<td>( r_5 = 0.2, r_6 = 0.1, a = 0.2, \beta = 0.4, r = 1 )</td>
<td>( r_5 = 0.2, r_6 = 0.1, a = 0.3, \beta = 0.4, r = 1.5 )</td>
</tr>
<tr>
<td>14</td>
<td>( r_5 = 0.4, r_6 = 0.2, a = 0.1, \beta = 0.3, r = 1.2 )</td>
<td>( r_5 = 0.4, r_6 = 0.3, a = 0.1, \beta = 0.2, r = 1.8 )</td>
</tr>
<tr>
<td>15</td>
<td>( r_5 = 0.1, r_6 = 0.1, a = 0.4, \beta = 0.3, r = 1 )</td>
<td>( r_5 = 0.2, r_6 = 0.2, a = 0.3, \beta = 0.3, r = 1.5 )</td>
</tr>
<tr>
<td>16</td>
<td>( r_5 = 0.3, r_6 = 0.2, a = 0.2, \beta = 0.2, r = 1 )</td>
<td>( r_5 = 0.2, r_6 = 0.3, a = 0.2, \beta = 0.2, r = 1 )</td>
</tr>
<tr>
<td>17</td>
<td>( r_5 = 0.7, r_6 = 0.1, a = 0.1, \beta = 0.1, r = 1 )</td>
<td>( r_5 = 0.2, r_6 = 0.3, a = 0.2, \beta = 0.1, r = 1 )</td>
</tr>
<tr>
<td>18</td>
<td>( r_5 = 0.2, r_6 = 0.3, a = 0.3, \beta = 0.1, r = 1 )</td>
<td>( r_5 = 0.2, r_6 = 0.3, a = 0.3, \beta = 0.2, r = 1 )</td>
</tr>
<tr>
<td>19</td>
<td>( r_5 = 0.2, r_6 = 0.3, a = 0.3, \beta = 0.2, r = 1 )</td>
<td>( r_5 = 0.2, r_6 = 0.3, a = 0.3, \beta = 0.1, r = 1.3 )</td>
</tr>
<tr>
<td>20</td>
<td>( r_5 = 0.2, r_6 = 0.1, a = 0.3, \beta = 0.1, r = 1 )</td>
<td>( r_5 = 0.2, r_6 = 0.2, a = 0.2, \beta = 0.3, r = 1 )</td>
</tr>
</tbody>
</table>

**COEFFICIENTS**

\[
K'x = 1.33, K'y = 0.33, I = 0, I_2 = 2.33, I_3 = 0, I_4 = 0, \mu = 0.5, \Sigma K = 4
\]

\[
K'x = 0.9, K'y = 0.5, I = 1.5, I_2 = 1.1, I_3 = 1.5, I_4 = 0, \mu = 1.5, \Sigma K = 4
\]

---

**COEFFICIENTS**

\[
K'x = 2, K'y = 0.5, I = 0, I_2 = 1.5, I_3 = 0, I_4 = 0, \mu = 1, \Sigma K = 4
\]

\[
K'x = 0.67, K'y = 1.33, I = 0.67, I_2 = 1.33, I_3 = 0.67, I_4 = 0, \mu = 0.50, \Sigma K = 4
\]
Table 2

Numerical examples based on Theorem VI & VII of Affine Theorem of Johansen. K.W.\(^1\)

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Opening Size</th>
<th>Orthogonal Moment Co-efficients</th>
<th>Aspect Ratio (r)</th>
<th>Strength (W_{ult}/L_{ult})</th>
<th>Aspect Ratio ((r^*))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a=0.2,\beta=0.3)</td>
<td>(K_x=0.25, K_y=1.0, I_x=I_y=0.25, I_z=1.0, \mu=0.25, \Sigma K=2.5)</td>
<td>1.0</td>
<td>23.04303</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>(a=0.2,\beta=0.2)</td>
<td>(K_x=0.5, K_y=1.0, I_x=I_y=0.5, I_z=1.0, \mu=0.5, \Sigma K=3.0)</td>
<td>1.4</td>
<td>22.6074</td>
<td>1.98</td>
</tr>
<tr>
<td>3</td>
<td>(a=0.2,\beta=0.1)</td>
<td>(K_x=0.667, K_y=1.0, I_x=I_y=0.667, I_z=1.0, \mu=0.667, \Sigma K=3.33)</td>
<td>1.6</td>
<td>22.87023</td>
<td>1.96</td>
</tr>
<tr>
<td>4</td>
<td>(a=0.1,\beta=0.3)</td>
<td>(K_x=1, K_y=1.0, I_x=I_y=1.0, I_z=1.0, \mu=1, \Sigma K=4.0)</td>
<td>1.3</td>
<td>32.98775</td>
<td>1.3</td>
</tr>
<tr>
<td>5</td>
<td>(a=0.1,\beta=0.2)</td>
<td>(K_x=1.5, K_y=1.0, I_x=I_y=1.5, I_z=1.0, \mu=1.5, \Sigma K=5.0)</td>
<td>1.9</td>
<td>27.71505</td>
<td>1.55</td>
</tr>
<tr>
<td>6</td>
<td>(a=0.3,\beta=0.2)</td>
<td>(K_x=2.0, K_y=1.0, I_x=I_y=2.0, I_z=1.0, \mu=2, \Sigma K=6.0)</td>
<td>1.7</td>
<td>35.05538</td>
<td>1.20</td>
</tr>
<tr>
<td>7</td>
<td>(a=0.3,\beta=0.3)</td>
<td>(K_x=4.0, K_y=1.0, I_x=I_y=4.0, I_z=1.0, \mu=4, \Sigma K=10.0)</td>
<td>2.0</td>
<td>44.61118</td>
<td>1.0</td>
</tr>
</tbody>
</table>

NOTE: (1) \(r^*\)=equivalent isotropic slab aspect ratio, (2) \(I_y/I_x=K_y/K_x=1.0\), (3) \(r_y=6\), (4) \(I_x=0\).

Table 3

One Long side continuous slab(OLD), based on the principle \(\mu=r^2\)

<table>
<thead>
<tr>
<th>Coefficient of orthotropy ((\mu))</th>
<th>Aspect Ratio of Slab (r=\sqrt{\mu})</th>
<th>Orthogonal moment coefficients (affine)</th>
<th>(W_{ult}/L_{ult}^2)</th>
<th>Failure pattern for all aspect ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.5</td>
<td>(K_x=0.25, K_y=1.0, I_x=I_y=0.25, I_z=1.0)</td>
<td>44.61118</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.707</td>
<td>(K_x=0.5, K_y=1.0, I_x=I_y=0.5, I_z=1.0)</td>
<td>44.61118</td>
<td></td>
</tr>
<tr>
<td>0.667</td>
<td>0.817</td>
<td>(K_x=0.667, K_y=1.0, I_x=I_y=0.667, I_z=1.0)</td>
<td>44.61118</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>(K_x=1, K_y=1.0, I_x=I_y=1.0, I_z=1.0)</td>
<td>44.61118</td>
<td></td>
</tr>
<tr>
<td>1.44</td>
<td>1.2</td>
<td>(K_x=1.44, K_y=1.0, I_x=I_y=1.44, I_z=1.0)</td>
<td>44.61118</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>1.225</td>
<td>(K_x=1.5, K_y=1.0, I_x=I_y=1.5, I_z=1.0)</td>
<td>44.61118</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>1.414</td>
<td>(K_x=2, K_y=1.0, I_x=I_y=2, I_z=1.0)</td>
<td>44.61118</td>
<td></td>
</tr>
<tr>
<td>2.25</td>
<td>1.5</td>
<td>(K_x=2.25, K_y=1.0, I_x=I_y=2.25, I_z=1.0)</td>
<td>44.61118</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>2.0</td>
<td>(K_x=4, K_y=1.0, I_x=I_y=4, I_z=1.0)</td>
<td>44.61118</td>
<td></td>
</tr>
</tbody>
</table>

Notation:
- Continuous edge
- Simply supported edge
- Free edge
- Negative yield line
- CS: A slab supported on all sides continuously (restrained)
- SS: A slab simply supported on all sides
- OLC: A slab restrained on one long sides and simply supported on other three sides
- OLD: A slab restrained on three sides and simply supported on one long sides
- \(K_y m_{ult}\): Positive ultimate yield moment per unit length provided by bottom tension

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Bars placed parallel to X-axis

$K_{ymult}$ Positive ultimate yield moment per unit length provided by bottom tension bars placed parallel to Y-axis

$I_{ymult}$ and $I_{ymult}$ Negative ultimate yield moment per unit length provided by top tension reinforcement bars placed parallel to x-axis.

$I_{ymult}$ and $I_{ymult}$ Negative ultimate yield moment per unit length provided by top tension reinforcement bars placed parallel to y-axis.

$K_1$, $K_2$, $I_1$, $I_2$, $I_3$, $I_4$, $a$, $b$, $m_{ult}$, $r$, $r_1$, $r_2$, $r_3$, $r_4$, $r_5$, $r_6$, $u_{dl}$, $W_{ult}$, $r$, $\mu$

Slab dimensions in X and Y directions respectively

Coefficients of opening in the slab

Ultimate Yield moment per unit length of the slab

Aspect ratio of slab defined by $L_x/L_y$. 

Non-dimensional parameters of yield line propagation

Non-dimensional parameters of opening distances

Uniformly Distributed Load

Ultimate uniformly distributed load per unit area of slab.

Coefficient of orthotropy $\mu = \frac{K_y' + I_1}{K_y' + I_2}$

Appendix-A
Appendix-B

Chart-1: OLD, K2=1

Strength (Wyle^2/mL^2)

α = β

r=1
K1=1
K1=2

r=1.5

r=2

r=3

r=1
K1=3
K1=5

r=1.5

r=2

r=3

r=1
K1=3
K1=5

r=1.5

r=2

r=3

r=1
K1=3
K1=5

r=1.5

r=2

r=3

r=1
K1=3
K1=5

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K1=3
K1=5

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K1=3
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K1=5

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r=1.5

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K1=3
K1=5

r=1.5

r=2

r=3

r=1
K1=3
K1=5

r=1.5

r=2

r=3
Isotropic slab with a interior opening basing on Theorem's VI & VII of affine theorem

Chart-3: OLD, K2=2

Chart-4: OLD, K2=2

Chart-5: Strength Vs Aspect Ratio
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