Performance Prediction of H-Type Darrieus Turbine by Single Stream Tube Model for Hydro Dynamic Application

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Abstract

A Hydrodynamic model that can accurately predict the performance of a device can be used to optimize the design parameters more rapidly and at a much lower cost than carrying out the design studies using scale model tests. Tidal turbine models are principally dependent on the Hydrodynamic models developed for wind turbines. These models are crucial for deducing optimum design parameters and also for predicting the performance before fabricating the Darrieus hydro turbine. The method of analysis used is Single stream tube model in that induced velocity is assumed as a constant and it is obtained by actuator disc theory and blade momentum theory. Single stream tube model can predict the coefficient of performance easily before experiment of the turbine. In this paper Comparison of coefficient of power and co-efficient of torque by single stream tube model and experimental result, and its strengths and weaknesses are discussed.

Keywords: Low head power generation, Darrieus turbine, Aerodynamic model, VAWT.

1. Introduction

The Darrieus vertical axis wind turbine concept attracted considerable research interest in the late 1970s and 1980s, but has never competed successfully with horizontal axis wind turbines. In recent years there has been growing interest in Darrieus straight blade hydro turbine for low head application which is based on Darrieus wind turbine.

Figure 1: Darrieus Hydro Turbine
In 1974 Templin proposed the single stream tube model which is the first and most simple prediction method for the calculation of Aerodynamic performance characteristics of curve blade Darrieus Vertical axis wind turbine [1]. Stream tube models are momentum models based on Glauert’s Blade element theory [2]. In stream tube models the change in fluid momentum in the flow direction is equated to the stream wise forces on the aerofoil blades.

In this model the entire turbine is assumed to be enclosed within a single stream tube. The objective of the model is to determine the performance coefficient of rotor.

2. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Projected frontal area of turbine</td>
</tr>
<tr>
<td>C</td>
<td>Blade chord</td>
</tr>
<tr>
<td>C_d</td>
<td>Blade drag coefficient</td>
</tr>
<tr>
<td>C_d0</td>
<td>Reference zero-lift-drag coefficient</td>
</tr>
<tr>
<td>C_p</td>
<td>Turbine overall drag coefficient = ( \frac{F_d}{\rho A V^2} )</td>
</tr>
<tr>
<td>C_s0</td>
<td>Rotor drag coefficient = ( \frac{F_D}{\rho A V^2} )</td>
</tr>
<tr>
<td>C_l</td>
<td>Blade lift coefficient</td>
</tr>
<tr>
<td>C_n</td>
<td>Normal force coefficient</td>
</tr>
<tr>
<td>C_p</td>
<td>Turbine overall power co-efficient( = \frac{P_o}{\rho A V^2} )</td>
</tr>
<tr>
<td>C_t</td>
<td>Turbine overall torque coefficient( = \frac{T_B}{\rho A V^2 R} )</td>
</tr>
<tr>
<td>D</td>
<td>Blade drag force</td>
</tr>
<tr>
<td>F_n</td>
<td>Normal force</td>
</tr>
<tr>
<td>F_t</td>
<td>Tangential force</td>
</tr>
</tbody>
</table>

\( H \) Height of turbine blade

\( L \) blade lift force

\( N \) Number of blade

\( R \) Turbine radius

\( Re \) Local Reynolds number = \( \frac{\rho V C}{\mu} \)

\( V_a \) Induced velocity

\( V_c \) or \( V_t \) Chordal velocity component

\( V_n \) Normal velocity component

\( V_w \) Wake velocity

\( W \) Relative velocity

\( \rho \) Density of fluid

\( \theta \) Azimuth position of blade

\( \alpha \) Angle of attack

3. Axial Induction Factor

The axial induction factor, ‘ \( a \) ’ is the fractional decrease in water velocity between the free stream and the rotor plane, so it is defined as [11],

\[
a = \frac{N_c R \omega}{2 \pi R V_{\infty} \sin \theta}
\]

We can define induction factor by reference [5],

\[
a = \frac{V - V_n}{V_n}
\]

Figure: 2: Actuator disc model for Darrieus rotor.
The value for $V_a$ can be obtained by Gluert Actuator Disk theory [2], the expression of the uniform velocity through the rotor is,

$$ V_a = \frac{V_o + V_w}{2} $$
(3)

From equation (1) and (2) we can find induced and wake velocity,

$$ V_i = V_e (1 - \alpha) $$
(4)

$$ V_w = V_e (1 - 2\alpha) $$
(5)

3. Blade Angle of Attack and Relative Velocity

The flow velocities in the upstream and downstream sides of the Darrieus rotor are constant as seen in figure 3. From this figure one can observe that the flow is considered to occur in the axial direction. The tangential velocity (or chordal velocity) component $V_t$ (or $V_c$) in tangential direction of blade profile and the normal velocity component $V_n$ is normal to blade profile. Tangential velocity ($V_t$) and Normal velocity ($V_n$) of blade which is given by [3],

$$ V_t = R\omega + V_a \cos \theta $$
(6)

$$ V_n = V_a \sin \theta $$
(7)

Angle of attack $\alpha$ is angle between relative velocity $W$ and tangential velocity $V_t$ is obtained from the following expressions:

$$ \alpha = \tan^{-1}\left(\frac{V_t}{V_r}\right) $$
(8)

Substituting the values of (6) and (7) in equation (8),

$$ \alpha = \tan^{-1}\left[\frac{\sin \theta}{(R\omega / V_r) + \cos \theta}\right] $$
(9)

The relative flow velocity ($W$) can be obtained as,

$$ W = \sqrt{V_t^2 + V_n^2} $$
(10)

Inserting the values of $V_t$ and $V_n$, and non-dimensionalizing, one can find velocity ratio as,

$$ \frac{W}{V_a} = \sqrt\left(\frac{R\omega}{V_a}\right)^2 + 2\left(\frac{R\omega}{V_a}\right)\cos \theta + 1 $$
(11)

Local relative water dynamic pressure ($q$) is given by:

$$ q = \frac{1}{2} \rho W^2 $$
(12)

4. Blade Element Force and Drag Co-Efficient

Since the disk acts as a drag device, the source of drag must be a pressure difference across the disk and this drag manifests itself as thrust loading along the axis normal to the disk. Rewriting $N_2$ in terms of momentum:

$$ \sum F_i = m(\Delta V) $$
(13)

According to glauert’s theory [2] the velocity through a wind mill disk $V_a$ is the arithmetic mean of the undisturbed velocity $V_o$ and the velocity in the wake. The turbine drag $D$ is given by:
\[ D = m(V_\infty - V_a) \]  

(14)

\[ D = 2\rho AV_a(V_\infty - V_a) \]  

(15)

The disk drag coefficient \( C_{\text{disk}} \) based on dynamic pressure and the disk area is defined as:

\[ C_{\text{disk}} = \frac{D}{\frac{1}{2} \rho AV_a^2} \]  

(16)

And from equation (13) and (14)

\[ C_{\text{disk}} = 4 \left( \frac{V_\infty}{V_a} - 1 \right) \]  

(17)

\[ \frac{V_\infty}{V_a} = 1 + \frac{1}{4} C_{\text{disk}} \]  

(18)

For structural design purpose, a more convenient drag co-efficient \( C_n \) is based on the ambient dynamic pressure [4]:

\[ C_n = \frac{D}{\frac{1}{2} \rho V_a^2 A} \]  

(19)

\[ C_D = C_{\text{disk}} \left( \frac{V_D}{V_\infty} \right)^2 \]  

(20)

\[ C_D = \frac{C_{\text{disk}}}{\left( 1 + \frac{1}{4} C_{\text{disk}} \right)^2} \]  

(21)

For a given turbine geometry and rotational speed, turbine power and rotor drag are calculate using the blade element theory [4]. To calculate the blade element forces, the local Hydrodynamic angle of attack and the local relative dynamic pressure are required. The resultant velocity vector, relative vector, relative to the moving blade element, is resolved into two perpendicular components; one parallel to the span wise and other in blade profile plane as shown in figure 4. The velocity component parallel to the blade span wise has no effect on the blade element Hydrodynamic forces.

Assuming that the instantaneous local blade element lift and drag coefficients \( C_l \) and \( C_d \) are function of angle of attack in steady flow, the normal force coefficient \( C_n \) and tangential force coefficient \( C_t \) are given by following expressions:

\[ C_l = C_t \sin \alpha - C_d \cos \alpha \]  

(22)

\[ C_n = C_t \cos \alpha + C_d \sin \alpha \]  

(23)

The net tangential and normal forces defined as:

\[ F_t = C_t \frac{1}{2} \rho CHW^2 \]  

(24)

\[ F_n = C_n \frac{1}{2} \rho CHW^2 \]  

(25)

Blade element of chord \( C \) is subjected to an elemental normal force \( dN \) and a forward thrust force \( dT \) given by the relation:

\[ dF_n = C_n qCH \]  

(26)

\[ dF_t = C_t qCH \]  

(27)

The elemental drag at any blade position ‘\( \theta \)’ as shown in figure 5 is:

\[ dD = (dF_n \sin \theta - dF_t \cos \theta) d\theta \]  

(28)

The total drag value is obtained by integration on a full revolution (\( 0 \leq \theta \leq 2\pi \)). Thus the total drag of a rotor with \( N \) blades of constant chord \( C \) is given by the relation:
\[
D = \frac{NCH}{2\pi} \int_{\theta=0}^{\theta=\pi} q\left(C_s \sin \theta - C_t \cos \theta\right) d\theta
\]  

(29)

\[
T_s = \frac{NCHR}{2\pi} \int_{\theta=0}^{\theta=\pi} qC_t d\theta
\]

(33)

Torque co-efficient is defined as:

\[
C_t = \frac{T_s}{\frac{1}{2} \rho V_s^2 A}
\]

(34)

From equation (34) and (33),

\[
C_t = \frac{NC}{2\pi \rho RV_s^2} \int_{\theta=0}^{\theta=\pi} qC_t d\theta
\]

(35)

The shaft power is then,

\[
P = \omega T_s
\]

(36)

For straight blade vertical axis rotor of diameter \(D\) and height \(H\) only one part of the total wind kinetic energy flow is converted into useful shaft power. Maximum possible power of the swept rotor area is:

\[
P_{\text{max}} = \frac{1}{2} \rho V_s^3 A
\]

(37)

The power coefficient \(C_p\) can be defined as the ratio of actual power \(P\) given by equation to the maximum value \(P_{\text{max}}\) given by [5],

\[
C_p = \frac{P}{P_{\text{max}}}
\]

(38)

Put value of \(P_{\text{max}}\) and \(P\) in above equation,

\[
C_p = \left(\frac{NC\omega}{2\pi \rho V_s^2}\right) \int_{\theta=0}^{\theta=\pi} qC_t d\theta
\]

(39)

Coefficient of power for rotor is obtained by integration on a full revolution \(0\leq \theta \leq 2\pi\). The relation with coefficient of torque and coefficient of power are given by:

\[
C_p = 2C_t
\]

(40)

We can find co-efficient of power from above equation for a given turbine geometry and for each specified value of tip speed ratio \(R\omega/V_s\).
5. Experimental Setup

The set-up consists of a Darrieus rotor and structure. The structure is fabricated using studs and mild steel plates. Geometric details of the rotors considered in the present study are given in Table 1. Darrieus rotor geometry detail is given. Calculation of performance coefficient is based on data for both blade profiles.

Table 1: Darrieus rotor geometry detail

<table>
<thead>
<tr>
<th>Sr No.</th>
<th>Blade Profile</th>
<th>D (mm)</th>
<th>H (mm)</th>
<th>C (mm)</th>
<th>V (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NACA 0018</td>
<td>250</td>
<td>150</td>
<td>100</td>
<td>0.46</td>
</tr>
<tr>
<td>2</td>
<td>NACA 4415</td>
<td>370</td>
<td>150</td>
<td>100</td>
<td>0.395</td>
</tr>
</tbody>
</table>

The hydrofoils used in turbine rotors are made from rapid prototype machine. Fibber Reinforced Plastic (FRP) is used to make hydrofoils. The mild steel plates are held in place by means of washers and nuts for easy replacement of different diameter rotors using same structure. Rotors are mounted on two bearings (UC 204, NTN make). A self-aligned bearing is used as upper side support, to avoid unwanted frictional torque arising due to minor misalignment between upper and lower rotor support shaft. A rope brake dynamometer type arrangement is used for measuring torque and subsequently power developed by Darrieus rotor. The accuracy of spring balance used is 10gms. The ultrasonic flow meter used to measure volumetric flow rate of water through the channel. Velocity of water is calculated using the continuity equation by using cross sectional flow area. Cross sectional flow area is measured by scale with least count of 1 mm. Details of the open channels used in the present study are given in Table 2.

Table 2: Details of the open channels used

<table>
<thead>
<tr>
<th>Channel Width (mm)</th>
<th>Water level (mm)</th>
<th>Average velocity (m/s²)</th>
<th>Blade Profile used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>750</td>
<td>300</td>
<td>0.46</td>
</tr>
<tr>
<td>2</td>
<td>750</td>
<td>300</td>
<td>0.395</td>
</tr>
</tbody>
</table>

To take care about same frictional condition, bearings are always washed in diesel before mounting in experimental set-up. This exercise is carried out at the start of the experiment to maintain same mechanical efficiency for every experimental result. Bearings are periodically lubricated by lubricating oil. Single unit rotor experiments are performed with constant velocity given in table.2 for above NACA profiles are used for study effect tip speed ratio on co-efficient of power and co-efficient of torque. Water level is maintained such that the rotor is always in submerged condition (i.e., just above the top plate of the rotor). To measure the torque, the rotor is loaded gradually to record spring balance reading and angular speed of the rotor. Mostly, each result is repeated thrice.

5 Results and Discussion

5.1 Angle of Attack

Here Angle of attack \( \alpha \) for Darrieus rotor blade is calculated for different tip speed ratio between 1 and 2 for different azimuth angle of blade between 0 to \( 2\pi \). Blade profile of rotor is taken as NACA0018 for calculation of angle of attack and Geometry detail and velocity is given in table 1. Figure 6 shows the variation in angle of attack \( \alpha \) as the azimuthally position \( \theta \) of blade for different tip speed ratio. The
angle of attack become smaller as the tip speed ratio increases; this can be appreciated in the following graph of the angle of attack variation of a blade in a full revolution.

Lift coefficient for azimuth position of blade of 0 to 30 degree there is more variation in the lift of the blade which is shown in figure 8. When the Reynolds number increases there is increment in the lift coefficient of the blade.

5.2 Lift Co-efficient

Figure 7 shows lift coefficient versus different angle of attack for different Reynolds number. Here lift coefficient drawn for Blade azimuth position from 0 to 180 degree for blade profile NACA0018. For blade azimuth position from 30 to 180 degree there is no change in the lift of the blade is shown in figure 7.

5.3 Drag co-efficient

Figure 9 shows Drag coefficient versus different angle of attack for different Reynolds number. Here, drag coefficient drawn for Blade azimuth position from 0 to $2\pi$ degree for blade profile NACA0018.
It is noted that Blade azimuth position from 15 to 180 degree there is no change in the drag of the blade but at azimuth position of blade for 0 to 15 degree there is more variation in the drag of the blade. When the Reynolds number increases there is increment in the drag coefficient of the blade that shown in figure 10.

5.4 Comparison between Tangential and Normal force coefficient

Here, the variation of tangential and normal force coefficient with reference to the angle of attack is drawn for bale azimuth position between 0 to π degree and Reynolds number of 40000 for blade profile is NACA0018.

5.5 Torque coefficient

Experiment is done for torque coefficient for different tip speed ratio for two different NACA profile. The graph of torque coefficient for two NACA profile, NACA0018 and NACA4415, with reference to the tip speed ratio and with their experimental result is shown. Geometrical data and velocity for both blade profiles are given in table 1. Torque coefficient is accurately predicted by single stream tube model for lower tip speed ration but when the tip speed ratio increases there is more variation in the experimental and analytical result which is shown by figure 13 and figure 14.
In figure 13 and 14 Variation in torque coefficient with reference to tip speed ratio and their comparison with the experimental result is shown.

Here, maximum power coefficient developed for blade profile NACA0018 and NACA4415 is 0.3322 and 0.091241042 accordingly, but their experimental result is 0.3215 and 0.0845 accordingly which is more than single stream tube result.

5.6 Power coefficient

Figure 15 and 1.6 shows the comparison of power coefficient between experimental result and analytical result with reference to tip speed ratio for two NACA profile, NACA0018 and NACA 4415. Power coefficient is accurately predicted by single stream tube model for lower tip speed ration but when the tip speed ratio increases there is more variation in the experimental and analytical result.

6. Conclusion and future work

Single stream tube model is simple prediction of coefficient of performance of rotor. Time require for calculation is less. This model can predict the overall performance of low tip speed ration of rotor and a lightly loaded turbine but according to the inquest, it always predicts higher power than the experimental
results. It does not predict the water velocity variations across the rotor. These variations gradually increase with the increase of the blade solidity and tip speed ratio. Single stream tube model does not take into account the difference in the induced velocities between the upstream and downstream halves of the rotor or any difference in velocities across the rotor such as those due to water shear. The main drawback of these models is that they become invalid for large tip speed ratios and also for high rotor solidities because the momentum equations in these particular cases are inadequate.

In future work we can develop another model based on single stream tube model which can be accurately predict the performance coefficient for darrieus rotor for higher tip speed ratio and higher solidity rotor.

7. References: